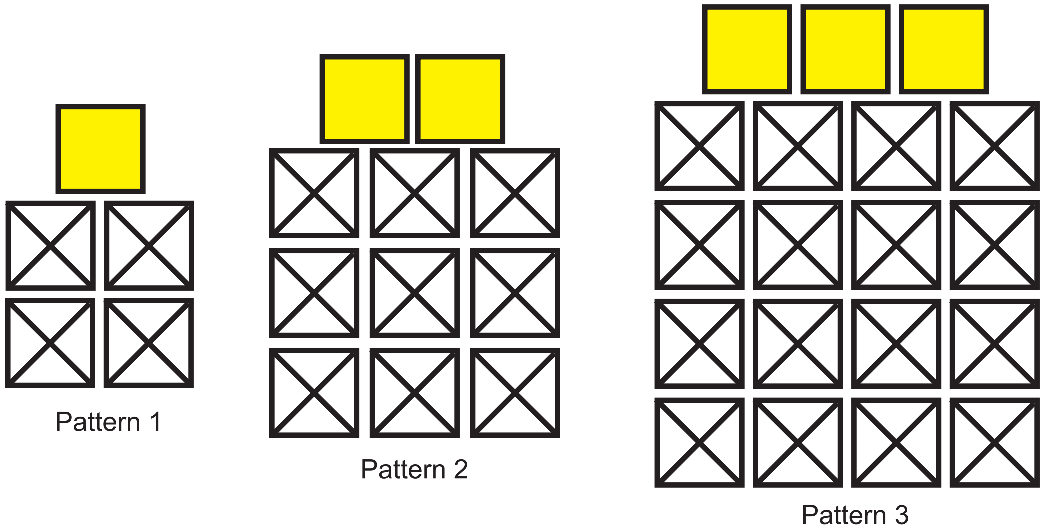
NCEA LEVEL 1 MATHEMATICS

MCAT 1.2 - AS91027

Apply Algebraic Procedures in Solving Problems



🟨	Tiles	1	2	3
☒	Tiles	$4 = (2^2)$	$9 = (3^2)$	$16 = (4^2)$

The relationship between tiles (T) and the pattern number n is $T = n + (n + 1)^2$

Questions and Answers

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NCEA Level 1 Mathematics, Questions & Answers
AS91027 Apply Algebraic Procedures in Solving Problems
Kim Freeman

This edition is Part 1 of an eBook series designed to help you study towards NCEA.

Note: Calculators are not to be used in the actual AS91027 exam.
ALL other Achievement Standards allow the use of appropriate technology.

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Contents

AS91027 Apply Algebraic Procedures in Solving Problems

Expanding and Factorising

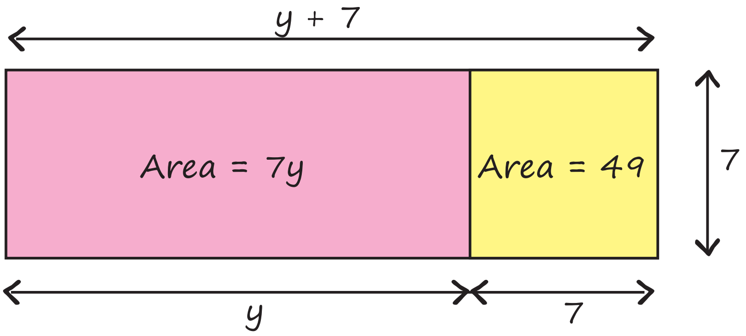
An equation or formula may contain brackets, e.g. $A = \frac{1}{2}(a + b)$

Removing the brackets from such an expression is known as expanding. Each term inside the brackets must be multiplied by the number or variable outside.

$$7(y + 7) = 7 \times y + 7 \times 7$$
$$= 7y + 49$$

The diagram below gives an illustration of the first example.

- Note that the variables x and x^2 are different.
- The + and - signs go with the term which follows.



Examples: Expand the following:

a. $a(a - 2) = a \times a - 2 \times a$
$= a^2 - 2a$

b. $-2(5 - 3b) = -2 \times 5 + (-2) \times (-3b)$
$= -10 + 6b$

c. $x(x - 2) + 5(2x + 1) = (x \times x) - (x \times 2) + (5 \times 2x) + (5 \times 1)$
$= x^2 - 2x + 10x + 5$
$= x^2 + 8x + 5$

The reverse process of putting the brackets in is known as factorising. To factorise an expression it is necessary to identify all the numbers and variables that are factors of the expression.

$10x + 2 \quad = 2(5x + 1)$

Both terms can be divided by 2.

$12x - 20 \quad = 4(3x - 5)$

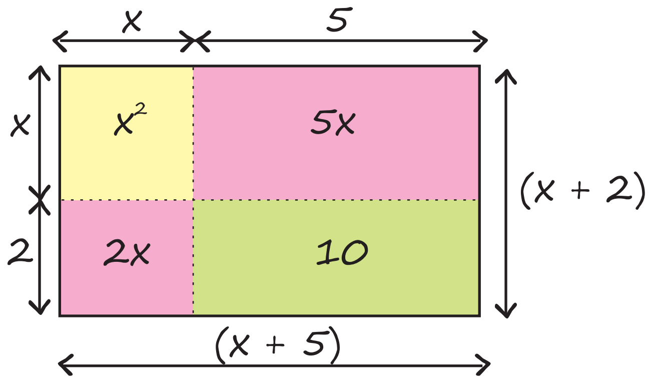
Both terms can be divided by 4.

$5x^2 - 35x \quad = 5x(x - 7)$

Both terms can be divided by x and 5.

Therefore 5x was placed outside the brackets.

Sometimes a situation will require terms in a bracket to be multiplied by another set of terms in a bracket. In this case each of the terms has to be multiplied by each other.



Examples: Expand the following:

d. $(x + 2)(x + 5) \quad = x(x + 5) + 2(x + 5)$

$= x^2 + 5x + 2x + 10$

$= x^2 + 7x + 10$

e. $(x + 5)(x - 3) \quad = x(x - 3) + 5(x - 3)$

$= x^2 - 3x + 5x - 15$

$= x^2 + 2x - 15$

f. $(x + 4)^2$

$$\begin{aligned} &= (x + 4)(x + 4) \\ &= x(x + 4) + 4(x + 4) \\ &= x^2 + 4x + 4x + 16 \\ &= x^2 + 8x + 16 \end{aligned}$$

g. $(4x - 3)(2x - 7)$

$$\begin{aligned} &= 4x(2x - 7) - 3(2x - 7) \\ &= 8x^2 - 28x - 6x + 21 \\ &= 8x^2 - 34x + 21 \end{aligned}$$

h. $(2x - 2)(x + 3)$

$$\begin{aligned} &= 2x(x + 3) - 2(x + 3) \\ &= 2x^2 + 6x - 2x - 6 \\ &= 2x^2 + 4x - 6 \end{aligned}$$

Most factorising is achieved by trial and error.
e.g. Factorise $x^2 + 7x + 6$.

- x^2 means the completed answer will be of the form $(x \quad)(x \quad)$.
- The + 6 comes from multiplying two numbers.
- The + 7 comes adding the same two numbers.
- Factors of +6 are: 6,1; 3,2; -6,-1; -3,-2.
- Of these $6 + 1 = 7$.

∴ Factorise: $x^2 + 7x + 6 = (x + 6)(x + 1)$

Look at how these have been factorised.

$x^2 + 4x - 21 = (x + 7)(x - 3)$

$y^2 - 7y + 10 = (y - 5)(y - 2)$

$a^2 - 4a - 5 = (a - 5)(a + 1)$

Note how the larger of the two numbers in the factorised expression has the same sign as the middle term in the expanded expression. i.e. $y^2 \underset{\wedge}{-} 12y + 32$

$= (y \underset{\wedge}{-} 8)(y - 4)$

Exercises

Expand the following:

1. $u(u + 1)$

2. $v(v - 6)$

3. $-w(3w - 2)$

4. $x(4x + 5)$

5. $3y(2y - 3)$

6. $-z(-5z + 3)$

7. $3 + 2(x - 8)$

8. $5(x + 7) - 12$

9. $3(x - 6) + 2(4x - 5)$

10. $4(a + 6) - 2(a - 2)$

11. $2x(x + 1) - x(7 - x)$

12. $x^2(x + 1)$

13. $\frac{1}{2}(4x + 12)$

14. $\frac{2}{3}(12x - 6)$

. . . .

15. $3x(2x^2 - 4)$

. . . .

16. $x(x^2 + 4) + x(3x + 2)$

. . . .

. . . .

Factorise the following:

17. $6x + 24$

. . . .

18. $5x - 25$

. . . .

19. $11x^2 - 66x$

. . . .

20. $10x + 25xy$

. . . .

21. $100x + 20y$

. . . .

22. $27 - 33x$

. . . .

23. $5x^2 + x$

. . . .

24. $6a^2 + 3a$

. . . .

25. $15b^2 - 30b$

. . . .

26. $14y^2 + 21y$

. . . .

27. $5 + 5n^2$

. . . .

28. $6x^2 + 18xy$

. . . .

29. $2xy - 4ab$

. . . .

30. $3p^2 - 9pq$

. . . .

Expand and simplify:

31. $(x + 1)(x + 6)$

. . . .

. . . .

32. $(x + 2)(x + 8)$

. . . .

. . . .

33. $(x - 5)(x + 7)$

. . . .

. . . .

34. $(x - 2)(x + 9)$

. . . .

. . . .

35. $(x + 4)(x - 5)$

. . . .

. . . .

36. $(x + 7)(x - 3)$

. . . .

. . . .

37. $(x - 10)(x - 15)$

. . . .

. . . .

38. $(x - 8)(x - 11)$

. . . .

. . . .

39. $(x + 6)^2$

. . . .

. . . .

40. $(x - 9)^2$

. . . .

. . . .

41. $(x + 1)^2 + 10$

. . . .

. . . .

42. $(x - 5)^2 - 20$

. . . .

. . . .

Factorise each expression:

43. $x^2 + 10x + 21$

44. $x^2 + x - 12$

45. $x^2 - 2x - 15$

46. $x^2 - 14x + 40$

47. $x^2 + 11x + 30$

48. $x^2 + x - 2$

49. $x^2 - 3x - 10$

50. $x^2 - 4x - 96$

51. $x^2 - 5x - 14$

52. $x^2 - 16$

53. $x^2 - 81$

54. $(x - 3)^2 - 16$

55. $x^2 + 2x = 15$

56. $x^2 = 6x - 8$

57. $2x^2 - 2x = 220$

58. $4x^2 - 100$

Algebraic Expressions Involving Exponents

Exponents are a useful way of writing expressions in a shorter format.

e.g. $(2x)^5 \ggg 2x \times 2x \times 2x \times 2x \times 2x = 32x^5$

$$\frac{a^3 \times a^7}{a^2 \times a^4} = \frac{a\times a\times a \times a\times a\times a\times a\times a\times a\times a}{a\times a \times a\times a\times a\times a}$$

$$= \frac{\cancel{a}\times\cancel{a}\times\cancel{a} \times \cancel{a}\times\cancel{a}\times\cancel{a}\times a\times a\times a\times a}{\cancel{a}\times\cancel{a} \times \cancel{a}\times\cancel{a}\times\cancel{a}\times\cancel{a}}$$

$$= a^4$$

or $$\frac{a^3 \times a^7}{a^2 \times a^4} = \frac{a^{3+7}}{a^{2+4}}$$

$$= \frac{a^{10}}{a^6}$$

$$= a^{10-6}$$

$$= a^4$$

The following rules apply whenever exponents (indices) are used:

$$a^m \times a^n = a^{m+n}$$

$$\frac{a^n}{a^m} = a^{n-m}$$

$$a^0 = 1$$

$$(a^m)^n = a^{m\times n}$$

$$a^{-1} = \frac{1}{a}$$

$$a^{-n} = \frac{1}{a^n}$$

Using these rules is much quicker – especially if the indices are large.

Examples:

a. Simplify $\frac{12xy}{8x}$

$$= \frac{4x(3y)}{4x\,(2)}$$

$$= \frac{3y}{2}$$

b. Simplify $\frac{20x}{5xy}$

$$= \frac{5x(4)}{5x(y)}$$

$$= \frac{4}{y}$$

c. Simplify $\frac{24xy^2}{3x^2y}$

$= \frac{3xy(8y)}{3xy(x)}$

$= \frac{8y}{x}$

With these problems you need to simplify by:
(1) Factorising the top and bottom
(2) Using the exponent rules
(3) Using both 1 and 2.

d. Simplify $\frac{2g^2 - 12gh}{6g^2}$

$= \frac{2g(g - 6h)}{2g\,(3g)}$

$= \frac{g - 6h}{3g}$

e. Simplify $\frac{3x^2 + 15xy}{6x^2}$

$= \frac{3x(x + 5y)}{3x(2x)}$

$= \frac{x + 5y}{2x}$

e. Simplify $\frac{3x^2}{30xy}$

$= \frac{3x(x)}{3x(10y)}$

$= \frac{x}{10y}$

Simplify means find and eliminate the common factors.

g. Simplify $\frac{14a^5}{7a^2}$

$= \frac{2a^3(7a^2)}{7a^2}$

$= 2a^3$

h. Simplify $(6x^3y^2)^2 = 36x^6y^4$

Exercises

Simplify each expression:

1. $(4x^2)^2$

. . . .

2. $(8x^2y)^2$

. . . .

3. $\left(\frac{x^2}{y}\right)^2$

. . . .

4. $\frac{4x^5}{8x^{10}}$

. . . .

5. $\frac{9x^5}{12x^3}$

. . . .

6. $\frac{8x^2 - 10xy}{2x^2}$

. . . .

. . . .

7. $\frac{3a - 15ab}{6ab}$

. . . .

. . . .

8. $5y^2 \times 4y^n = 20y^8$

What is the value of n?

. . . .

. . . .

. . . .

9. $(5a^n)^2 = 25a^8$

What is the value of n?

. . . .

. . . .

. . . .

10. $a^6 \div a^n = 1$

What is the value of n?

. . . .

. . . .

. . . .

Substituting Values into Formulae

The process of replacing the letters in a formula with numbers is known as substitution. Some examples follow. Write out the formulae with all the values then use a calculator.

a. The length of a metal rafter is L (metres). The length of the rafter can change with temperature variations.
The length can be found by the formula: $L = 20 + 0.02t$
t = the temperature (ºC)
Find the length of the rod when $t = 29°$ and $t = -10°$.

Using $t = 29°$ $\quad L = 20 + 0.02 \times 29$

$= 20.58\text{m}$

Using $t = -10°$ $\quad L = 20 + 0.02 \times (-10)$

$= 19.8\text{m}$

b. If $P = 2\sqrt{\frac{x^2}{y}}$ and $x = 10$, $y = 4$; find P

$$P = 2\sqrt{\frac{(10)^2}{4}}$$

$$= 2\sqrt{\frac{100}{4}}$$

$$= 10$$

c. At a garage the cost C ($) for car repairs is determined by the formula:
$C = 100 + p + 35t$ where p = cost ($) of the parts
t = time (hours) spent on the repairs
Find the cost of brake repairs if parts cost $175 and time spent on the repairs is 1hr 30min.

Remember 1hr 30min = 1.5 hours

$C = 100 + 175 + 35 \times 1.5$

$C = \$327.50$

Exercises

1. $s = \frac{1}{2}(u + v)t$

Find s when:

(i) $u = -4$, $v = 10$, $t = 2$

. . . .

(ii) $u = 1.6$, $v = 2.8$, $t = 3.2$

. . . .

2. $L = 20 - 0.8F$

Find L when $F = 15$

. . . .

3. $V = p^2 + q^2$

Find V when $p = 8$, $q = 4.5$

. . . .

4. $Z = 2(x + y)$

Find Z when $x = 10.2$, $y = 6.8$

. . . .

. . . .

5. $P = \frac{x + y}{2}$

Find P when $x = 4$, $y = -10$

. . . .

. . . .

6. $Q = \frac{a}{b}$

Find Q when $a = -100$, $b = -4$

. . . .

7. $W = \frac{a + 2b + c}{5}$

Find W when $a = 2.5$, $b = -5$ and $c = -8.5$

. . . .

8. $C = \frac{xy}{x + y}$

Find C when $x = 10$, $y = -5$

. . . .

9. $A = \frac{xy^2}{z}$

Find A when $x = 2$, $y = 3$, $z = 100$

. . . .

10. $D = \frac{5(x + y)}{2y}$ $\quad x = 9.8$, $y = 5.3$

(i) Find the approximate value of D without using a calculator.

. . . .

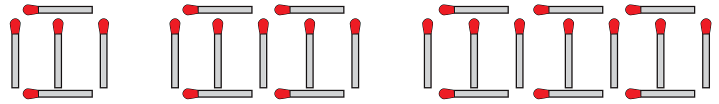
(ii) Use a calculator to find the correct value to 2 decimal places.

. . . .

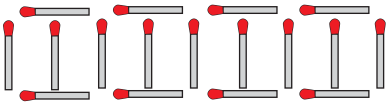
Describing Linear Patterns

This section looks at how terms of a sequence are related.

a. A matchstick pattern is shown below:



i. Draw a diagram of Pattern number 4.



Pattern 4

ii. Draw a table that gives the Patterns 1 to 5 and the number of matchsticks needed for each.

Pattern	1	2	3	4	5
Matches	5	9	13	17	21

iii. Which pattern needs exactly 41 matchsticks?

Look for a relationship between the pattern number and the number of matches. In this case the difference between each number is 4. When there is a common difference the formula for the nth term is: nth term $= dn + (a - d)$ where d = common difference, a = the first term of the sequence. In the case above nth term

$$= dn + (a - d)$$
$$= 4n + (5 - 4)$$
$$= 4n + 1$$

Therefore the relationship is: $M = 4P + 1$

Pattern number 10 will give 41 matches ($41 = 4\times10 + 1$)

iv. How many matchsticks are needed for Pattern 50?

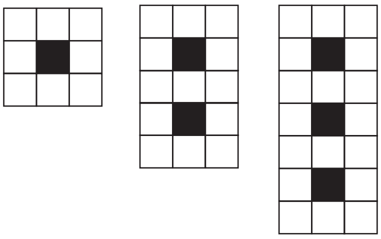
Using $M = 4P + 1$, $M = 4\times50 + 1$

i.e. number of matches = 201

Exercises

1. A shower wall is tiled using the pattern below:

a. Complete the table that gives the number of black tiles compared to the number of white tiles.



black tiles	white tiles
1	
2	
3	
4	

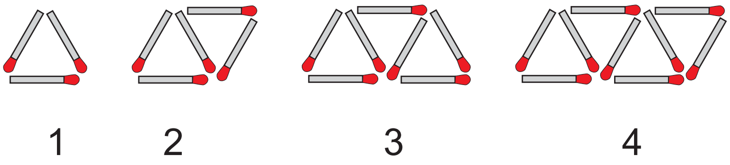
b. The rule for the number of white tiles (w) in terms of the number of black tiles (b) is:

w = ...

. . . .

. . . .

2. Patterns can be made of matchsticks.



a. Complete the table:

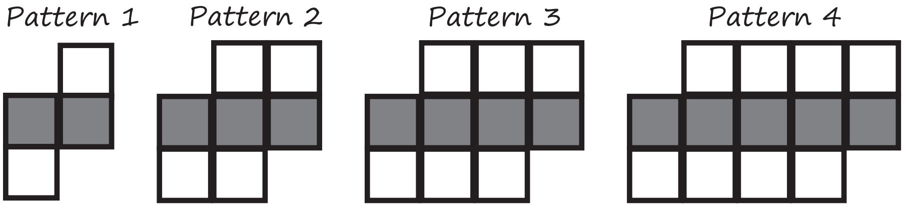
Pattern (P)	1	2	3	4	5	6
Matches (M)						

b. The rule for calculating total matches in each pattern is to multiply the pattern number by 2 and add 1.

How many sticks will there be in pattern 10?

. . . .

3. Look at the pattern below.



Write the rule for Pattern n (white squares, shaded squares, total squares).

. . . .

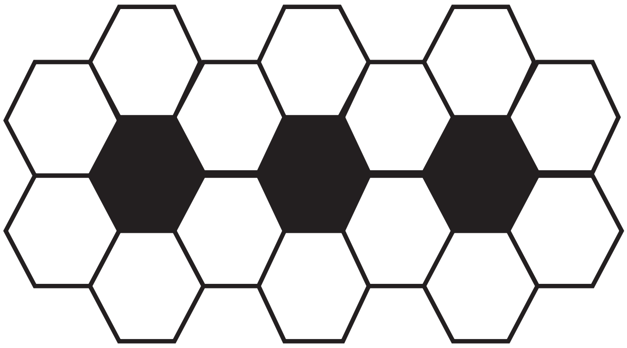
. . . .

4. A number pattern begins: 4, 8, 12, 16, 20, 24

Describe this number pattern.

Term n = ...

5. A landscape gardener is designing a garden path. It is to have hexagonal black and white paving and is to be laid according to the pattern below.



a. Draw up a table that shows the number of black pavers and white pavers needed.

b. How many white pavers are needed if 100 black pavers are ordered?

. . . .

. . . .

Solving Linear Equations

Most equations require a number of steps before they can be solved. Each step must be logical. Whatever you do to one side of the equation you must do to the other. Follow the steps below to see how the following equations are solved.

a. $4x + 7 = 19$
b. $5x - 8 = 15$
c. $\frac{x}{5} - 2 = 3$
d. $4x + 6 = 3x + 16$
e. $6 - 2x = 12$
f. $3(x + 2) = 5(x - 2)$

The Answers

a. $4x + 7 = 19$
$4x = 12$ Subtract 7 from both sides
$x = 3$ Divide both sides by 4

b. $5x - 8 = 15$
$5x = 23$ Add 8 to both sides
$x = 4.6$ Divide both sides by 5

c. $\frac{x}{5} - 2 = 3$
$\frac{x}{5} = 5$ Add 2 to both sides
$x = 25$ Multiply both sides by 5

d. $4x - 6 = 3x + 16$
$4x = 3x + 22$ Add 6 to both sides
$x = 22$ Subtract 3x from both sides

e. $6 - 2x = 12$
$-2x = 6$ Subtract 6 from both sides
$x = -3$ Divide both sides by -2

f. $3(x + 2) = 5(x - 2)$
$3x + 6 = 5x - 10$ Expand the brackets
$3x = 5x - 16$ Subtract 6 from both sides
$-2x = -16$ Subtract 5x from both sides
$x = 8$ Divide both sides by -2

Exercises

Solve each equation:

1. $5x - 6 = 39$

. . . .

2. $6x + 12 = 20$

. . . .

3. $-4x - 18 = 6$

. . . .

4. $3x + 6 = 1$

. . . .

5. $4(2x + 3) = -8$

. . . .

. . . .

6. $6x - 8 = -26$

. . . .

7. $\frac{2x}{5} + 1 = 3$

. . . .

. . . .

8. $4x - 8 = 5x - 2$

. . . .

9. $6x + 7 = 2x + 20$

. . . .

10. $x + 6 = 2x - 8$

. . . .

11. $3x + 7 = 2x + 11$

. . . .

12. $10x + 2 = 8x + 22$

. . . .

13. $3(x + 2) = 5(x - 2)$

. . . .

. . . .

14. $4 = 8 - \frac{x}{3}$

. . . .

. . . .

Solving Factorised Equations

If two factors are multiplied together to give 0 then either one of them must be 0, i.e. $xy = 0$, either $x = 0$ or $y = 0$.
Look at the examples below and see how each are solved.

a. $(x - 5)(x + 1) = 0$

either $x - 5 = 0$ or $x + 1 = 0$

$\therefore$ $x = 5$ or $x = -1$

b. $(3x - 6)(x - 4) = 0$

either $3x - 6 = 0$ or $x - 4 = 0$

$\therefore$ $x = 2$ or $x = 4$

c. $(2x - 1)(x + 5) = 0$

either $2x - 1 = 0$ or $x + 5 = 0$

$\therefore$ $x = 0.5$ or $x = -5$

d. $(x - 4)^2 = 0$

$x - 4 = 0$

$\therefore$ $x = 4$

e. $(4x + 6)(x + 2) = 0$

either $4x + 6 = 0$ or $x + 2 = 0$

$\therefore$ $x = -1.5$ or $x = -2$

f. $(2x + 5)(x - 10) = 0$

either $2x + 5 = 0$ or $x - 10 = 0$

$\therefore$ $x = -2.5$ or $x = 10$

g. $5x(2x - 9) = 0$

either $5x = 0$ or $2x - 9 = 0$

$\therefore$ $x = 0$ or $x = 4.5$

Exercises

Solve each of the factorised equations:

1. $(x - 5)(x - 10) = 0$

. . . .

2. $(x + 3)(x - 8) = 0$

. . . .

3. $(x - 9)(x + 4) = 0$

. . . .

4. $(x + 15)^2 = 0$

. . . .

5. $(2x - 5)(x + 7) = 0$

. . . .

6. $(3 + x)(3x + 12) = 0$

. . . .

. . . .

7. $(7 - 2x)(3 + 4x) = 0$

. . . .

. . . .

. . . .

8. $x(2x + 9) = 0$

. . . .

9. $(2x + 8)(4x - 10) = 0$

. . . .

10. $(3x - 8)(3x + 8) = 0$

. . . .

These final two questions are in advance of achievement level. This is because they require a little more work.

11. $(x + 3)^2 - 25 = 0$

. . . .

. . . .

. . . .

. . . .

12. $(x - 2)^2 - 9 = 0$

. . . .

. . . .

. . . .

. . . .

Algebraic Methods - Achievement Examples

a. Expand and simplify: $6(x + 4) - 4(x + 5)$

Multiply and expand the brackets then collect all the like terms

$$= 6x + 24 - 4x - 20$$

$$= 2x + 4$$

b. Factorise: $x^2 - 2x - 48$

Find two numbers that multiply to give -48 and add to give -2

$$= (x - 8)(x + 6)$$

c. Anderson knows that $8x^2 \times 4x^n = 32x^{10}$. What is the value of n?

The rule is $x^a \times y^b = xy^{a+b}$.

This means that $2 + n = 10$ and $n = 8$.

d. Solve: $15a - 10 = 12a + 5$

Try and get all the a's on the LHS

and all the numbers on the RHS of the = sign.

$$15a - 12a = 5 + 10$$

$$3a = 15$$

$$a = 5$$

e. Solve: $(3x - 3)(x + 8) = 0$

Either $3x - 3 = 0$ or $x + 8 = 0$.

Solve both of these to find your two answers.

$x = 1$ or $x = -8$

f. Solve: $\frac{5x}{2} - 8 = 0$

With equations get all the variables (letters) on the LHS and all the numbers on the RHS of the = sign.

$$\frac{5x}{2} = 8$$

$\frac{5x}{2} \times \frac{8}{1}$ cross multiply

$5x = 16$

$x = 3.2$ or $3\frac{1}{5}$

Exercises

1. Solve $3(x - 9) = 9$

. . . .

. . . .

2. Solve $5x + 3 = x - 6$

. . . .

. . . .

3. Solve: $5x(x + 9) = 0$

. . . .

. . . .

4. Expand and simplify $(2x + 7)(x - 5)$

. . . .

. . . .

5. Simplify $\frac{15x^5}{3x^2}$

. . . .

. . . .

6. $Y = \frac{x(x + 5)}{2}$ Find Y when $x = 5$

. . . .

. . . .

7. Solve $(x + 3)(x - 8) = 0$

8. Solve $17x - 9 = 12x + 4$

9. Solve: $\frac{2x + 6}{5} = 4$

10. Expand and simplify: $(2x - 2)(x + 1)$

11. Factorise completely: $x^2 - 5x - 14$

12. $F = \frac{N}{2}(3N - 5)$. Find F, when N = 11.

13. Solve $(3x - 1)(x + 7) = 0$

. . . .

. . . .

14. Solve $6x - 3 = 2x + 8$

. . . .

. . . .

15. Solve: $\frac{5x}{2} + 8 = 33$

. . . .

. . . .

16. Expand and simplify: $(2x - 1)(3x + 5)$

. . . .

. . . .

17. Factorise completely: $x^2 + 5x - 24$

. . . .

18. Simplify: $\frac{15x^{12}}{5x^3}$

. . . .

19. Simplify $5^{13} \div 5^{10}$

. . . .

20. $R = 0.45DT$. Calculate R when $D = 27.8$ and $T = 3.6$

. . . .

Simplify or Solve Rational Expressions

When fractions are added or subtracted they must have the same denominator.

Simplify: $\frac{x}{6} + \frac{x}{5} = \frac{5x}{30} + \frac{6x}{30}$

$= \frac{11x}{30}$

Express: $\frac{3}{x} + \frac{5}{x+1}$ as a single fraction

$= \frac{3(x+1)}{x(x+1)} + \frac{5x}{x(x+1)}$ each has a common denominator

$= \frac{3x + 3 + 5x}{x(x+1)}$ add the numerators

$= \frac{8x+3}{x(x+1)}$ the final answer

Simplify: $\frac{x^2 + 8x + 15}{x+3}$

$= \frac{(x+5)(x+3)}{x+3}$

$= x + 5$

Solve: $\frac{4x+1}{11} = 3$ $\Rightarrow \frac{4x+1}{11} = \frac{3}{1}$ cross multiply

$\Rightarrow 4x + 1 = 33$

$\Rightarrow 4x = 32$

$\Rightarrow x = 8$ check $(4 \times 8 + 1) \div 11 = 3$

Solve: $\frac{10x}{2} + 3.5 = 16$ $\Rightarrow \frac{10x}{2} + \frac{7}{2} = \frac{16}{1}$

$\Rightarrow 10x + 7 = 32$ multiply both sides by 2

$\Rightarrow 10x = 25$ now solve

$\Rightarrow x = 2.5$

Exercises

Simplify:

1. $\frac{4}{x} + \frac{2}{y}$

. . . .

. . . .

2. $\frac{5}{3a} - \frac{1}{2b}$

. . . .

. . . .

3. $\frac{3x}{9x + 6}$

. . . .

. . . .

4. $\frac{x^2 - 5x + 6}{x^2 - 4}$

. . . .

. . . .

5. $\frac{-4xy \times -2xy}{6x^2y}$

. . . .

. . . .

Solve:

6. $\frac{3}{4}k = 9$

. . . .

. . . .

7. $\frac{m}{8} + 2 = \frac{1}{2}$

. . . .

. . . .

8. $\frac{2t}{5} + 8 = 4$

. . . .

. . . .

9. $\frac{7e}{5 - e} = 10.5$

. . . .

. . . .

10. $\frac{x}{5} + \frac{x}{2} = -14$

. . . .

. . . .

Describing Quadratic Patterns

Term (n) 1 2 3 4 5 6

Look at this sequence of numbers: 2, 6, 12, 20, 30, 42 ...

The difference between each number is: 4 6 8 10 12

The difference between these numbers is: 2 2 2 2

If the first difference between each number changes, then it could be a quadratic sequence. When the second difference is constant, you have a quadratic sequence - i.e., there is an n^2 term.

If the second difference is 2, start with n^2.
If the second difference is 4, you start with $2n^2$.
If the second difference is 6, you start with $3n^2$.

The formula for the sequence 2, 6, 12, 20, 30, 40 ... starts with n^2 as the second difference is 2. Use n^2 as a starting point to calculate the formula.

Term (n)	1	2	3	4	5	6
Sequence:	2,	6,	12,	20,	30,	42
n^2	1	4	9	16	25	36

The difference between the sequence and n^2 is n, i.e 2-1 =1, 6-4=2.

Therefore the formula for the pattern = $n^2 + n$

a. Write down the next two terms of the sequence: 5, 12, 23, 38, _ , _

The first differences are: 7, 11, 15, The second difference is 4.

Continuing the sequence, the differences between each term will be: 15 + 4 = 19 and 19 + 4 = 23

Therefore the next 2 terms in the sequence will be: 38 + 19 = 57 and 57 + 23 = 80. The sequence will be: 5, 12, 23, 38, 57, 80

b. Find a formula for the nth term of the sequence: 5, 12, 23, 38, _ , _

The second difference is 4. Therefore the formula will start $2n^2$.

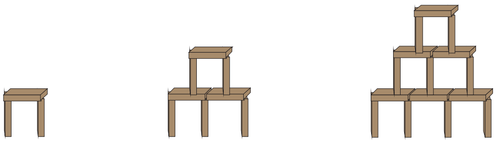
nth term:	1	2	3	4	5	6
Sequence:	5	12	23	38	57	80
$2n^2$	2	8	18	32	50	72

The difference between $2n^2$ and original number is n + 2

Therefore the formula for the nth term is $2n^2 + n + 2$

Exercises

1. The following structures were made with slabs of wood.



a. Complete the table to give the number of slabs needed for each structure.

Storeys (x)	1	2	3	4	5	6	7	8
Slabs needed (y)	3	8	15					

b. Give the rule for the relationship between the number of storeys and slabs of wood needed.

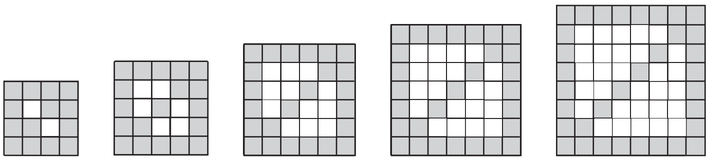
. . . .

. . . .

c. If you wanted to build a structure with 25 storeys, how many slabs of wood would be needed?

. . . .

2. Look at the tile pattern below then complete the table to give the formula for the number of white tiles. NOTE: Total tiles = grey tiles + white tiles.



Number of Tiles on the bottom line	Total Number of Tiles	Number of Grey Tiles	Number of White Tiles
n	n^2	$5n - 6$	

3. Sequence q = 3, 8, 15, 24, 35, ….

a. Calculate the sixth term of the sequence.

. . . .

. . . .

b. The nth term of sequence q is $n^2 + kn$, where k represents a number. Find the value of k.

. . . .

. . . .

4. The first three terms of a sequence are: (3×4)+1, (4×5)+2, (5×6)+3. Find the next two terms and the rule for the nth term.

. . . .

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5. The first four terms of a sequence are: 4, 9, 16, 25, ...
Find the next two terms and the rule for the nth term.

. . . .

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. . . .

Rearranging Formulae

Sometimes a formula needs to be rearranged to be more useful. A common formula is the one that converts °F to °C, i.e °F = 1.8°C + 32.
To convert °C to °F rearrange the formula to make °C the subject.

$F = 1.8C + 32$

$F - 32 = 1.8C$ subtract 32 from both sides

$\frac{F - 32}{1.8} = C$ divide each side by 1.8

$C = \frac{F - 32}{1.8}$ rearrange the formula

a. The mean of x, y, z can be found using the formula: $M = \frac{x + y + z}{3}$
Rearrange the formula to make z the subject.

$M = \frac{x + y + z}{3}$

$3M = x + y + z$ multiply both sides by 3

$3m - x - y = z$ subtract (x + y) from both sides

$z = 3m - x - y$ rearrange the formula

b. Make x the subject of ax - c = 4x + b.

$ax - c = 4x + b$

$ax - 4x - c = b$ subtract 4x from both sides

$x(a - 4) = b + c$ factorise to isolate the x

$x = \frac{b + c}{a - 4}$ divide both sides by (a - 4)

c. Make b the subject of the formula: $P = \frac{2b}{a - b}$

$P(a - b) = 2b$ multiply both sides by (a - b)

$Pa - Pb = 2b$ expand

$Pa = 2b + Pb$ add Pb to both sides

$Pa = b(2 + P)$ factorise to isolate the b

$\frac{Pa}{2 + P} = b$ divide both sides by 2 + P

$b = \frac{Pa}{2 + P}$

Exercises

Rearrange to make x the subject

1. $y = 10x + 5$

. . . .

. . . .

2. $-2x - 8y = 7$

. . . .

. . . .

3. $P = \frac{x}{V}$

. . . .

. . . .

4. $y = \frac{x + 5}{2}$

. . . .

. . . .

5. $y = \frac{(3 + x)a}{4}$

. . . .

. . . .

. . . .

6. Make v the subject: $S = \frac{(u + v)t}{2}$

. . . .

. . . .

7. Make c the subject: $a^2 = b^2 + c^2$

. . . .

. . . .

8. Make a the subject: $v^2 = u^2 + 2as$

. . . .

. . . .

. . . .

9. Make r the subject: $V = \pi r^2 h$

. . . .

. . . .

. . . .

10. Make a the subject: $A = \frac{1}{2}(a + b)h$

. . . .

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. . . .

Solving Equations

An equation is the equivalent of a mathematical sentence. Within this sentence, two expressions have the same value. If you add, subtract, multiply, or divide one side of the equation, then you have to do exactly the same operation to the other side of the equation.

e.g. Solve each of the following equations:

a. $3x + 4 = 25$

$3x = 21$ subtract 4 from both sides

$x = 7$ divide each side by 3

b. $6x + 7 = 4x + 19$

$2x + 7 = 19$ subtract 4x from both sides

$2x = 12$ subtract 7 from both sides

$x = 6$ divide each side by 2

c. $\frac{3}{4}a = 36$

$a = 48$ multiply each side by $\frac{4}{3}$

d. $a = 5(a - 2) + 3$

$a = 5a - 10 + 3$ expand the brackets & simplify

$a = 5a - 7$

$-4a = -7$ divide by -4

$a = 1.75$ or $\frac{7}{4}$

e. $\frac{5x}{2} - 5 = 3$

$2(\frac{5x}{2} - 5) = 2(3)$ multiply both sides by 2

$5x - 10 = 6$ add 10 to both sides

$5x = 16$ divide both sides by 5

$x = 3.2$ or $3\frac{1}{5}$

Exercises

Solve these equations:

1. $3a - 4 = 23$

. . . .

2. $8x - 6 = -26$

. . . .

3. $13x + 7x = 10$

. . . .

4. $4x + 6 = 3x + 10$

. . . .

5. $\frac{a}{3.7} = 10$

. . . .

6. $\frac{2}{3} = \frac{6}{x}$

. . . .

7. $x + 9 = \frac{x + 6}{4}$

. . . .

. . . .

. . . .

8. $7 + 3(x - 1) = 19$

. . . .

. . . .

9. $\frac{5x}{2} + 3x = 33$

. . . .

. . . .

10. $3x - 2 = x + 7$

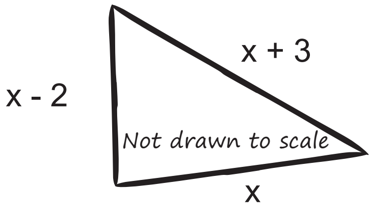
. . . .

. . . .

11. $2x = 2^6$

. . . .

12. Triangle perimeter = 22 cm.
Calculate each side length.



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Inequations

An inequation has a greater than (>) or less than (<) sign. This means that both sides of the equation do not equal each other. Calculating values in an inequation is much the same as with normal equations (i.e. those with = signs). But when multiplying or dividing both sides of an inequation by a negative number then you must change the direction of the sign. The simple example below illustrates how multiplying or dividing by a negative number changes the “sense” of an inequation:

$$5 > 2$$

Multiply both sides by -1 => $-5 < -2$

Exercises

1. $2y + 3 > 4$

. . . .

2. $-3x + 4 < 16$

. . . .

3. $\frac{-y}{2} \geq 4$

. . . .

4. $3 - 4x > 11$

. . . .

5. $\frac{2x-9}{9} > 7$

. . . .

6. $3x + 7 < 2x - 6$

. . . .

7. $-2x > \frac{2}{3}$

. . . .

8. $3(x - 2) \leq 5$

. . . .

9. $-x < 3x + 8$

. . . .

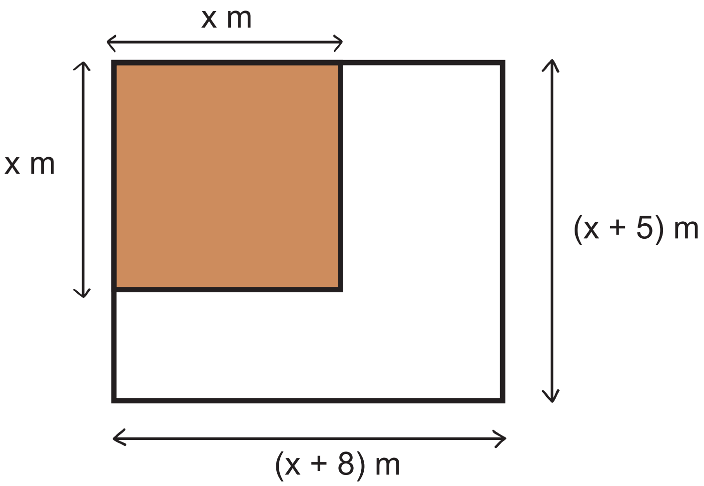
10. $5(x + 3) - 6x \geq 12$

. . . .

Solving Quadratic Problems

Factorising a quadratic equation can make it easier to solve.

a. The sides of an existing square warehouse are to be extended by 5 metres and 8 metres. The area of the new extended warehouse will be 340m^2. The existing warehouse (shaded) and planned extension are shown in the diagram below.



Solve the equation (x + 8)(x + 5) = 340 to find the new dimensions.

$(x + 8)(x + 5) = 340$

$x^2 + 5x + 8x + 40 = 340$ expand the equation

$x^2 + 13x - 300 = 0$ subtract 340 from both sides

$(x + 25)(x - 12) = 0$ factorise the new equation

$x = -25$ or $x = 12$

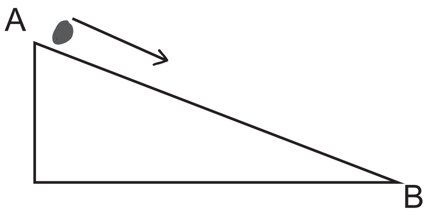
The original warehouse was 12m × 12m (x)

(as it can't be -25m)

The new dimensions will 20m (x + 8) and 17m (x + 5)

Testing the answer with the values: 20 × 17 = 340

b. A ball bearing rolls down a slope labeled AB. The time, t seconds, for the ball bearing to reach B is the solution to the equation t^2 + 5t = 36. How long does it take for the ball bearing to reach B?



$t^2 + 5t - 36 = 0$

$(t + 9)(t - 4) = 0$

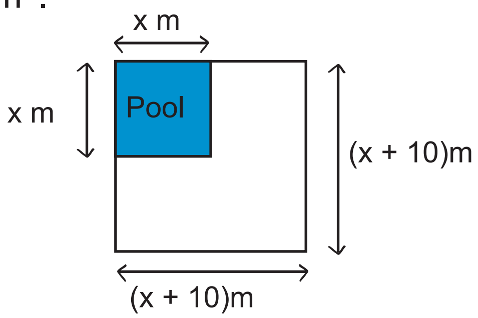
$T = -9, t = 4$

It takes 4 seconds for the ball to reach B.

(It could not be a negative time).

Exercises

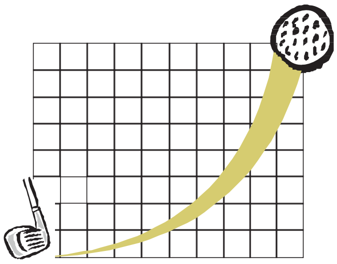
1. The diagram shows a square courtyard and square pool in one corner. The courtyard extends 10m on two sides of the pool. The courtyard and pool take up 225m^2.



Solve the equation $225 = (x + 10)^2$ to find the side length of the pool.

2. A field is 40 m longer than it is wide. The area of the field is 3200 m^2
What is the length and width of the field?

3. A golf ball is hit into the air. Its flight can be calculated by the equation: $h = 40t - 8t^2$ where h = height from the ground and t = time in the air. Find the time taken for the ball to reach a height of 48 metres. Explain why there are two possible values.



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. . . .

4. To find two positive consecutive odd integers whose product is 99 we can use the following logic:

x is the first integer

x + 2 is the second integer

therefore $x(x + 2) = 99$

Continue with the logic to find the answer.

. . . .

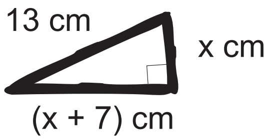
. . . .

. . . .

5. The hypotenuse of a certain right angled triangle is 13 cm. The other two lengths are x and (x + 7) cm. Complete the logic below to find the lengths of the other two sides.

Using Pythagoras: $13^2 = (x + 7)^2 + x^2$

$169 = x^2 + 14x + 49 + x^2$



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Solving Pairs of Simultaneous Equations

Some questions give two equations with two unknowns. These questions will ask you the values of the unknowns. To solve, you can find the intersection points of their graphs or you could use one of the following algebraic methods:

a. Comparison: This method can be used if both equations have the same subject. e.g. Solve for x and y when $y = 90 - x$ and $y = 63 + \frac{1}{2}x$.

$90 - x = 63 + \frac{1}{2}x$

$-x = -27 + \frac{1}{2}x$ subtract 90 from both sides

$-1\frac{1}{2}x = -27$ subtract ½x from both sides

$\underline{x = 18}$ divide both sides by −1½

$y = 90 - 18$ put the x value into one of the equations

$\underline{y = 72}$ solve for y

b. Substitution: This method can be used if one of the equations has a single variable as the subject. e.g. Solve the simultaneous equations: $y = 3x - 9$
$4x - y = 13$

The first equation can be substituted into the second

$4x - (3x - 9) = 13$

$4x - 3x + 9 = 13$

$x + 9 = 13$

$\underline{x = 4}$

$y = 3(4) - 9$ put $x = 4$ into the other equation

$\underline{y = 3}$

c. Elimination: Use this method if the co-efficients of either x or y are the same in both equations. e.g. $4y - 3x = -4$

$\underline{8y + 3x = 28}$ add the equations to eliminate x

$12y = 24$

$\underline{y = 2}$

Put the solution for y (i.e. $y = 2$) into one of the equations:

$4(2) - 3x = -4$ => $8 - 3x = -4$

$-3x = -12$

$\underline{x = 4}$

Exercises

Solve the following Simultaneous Equations using the Comparison Method.

1. $y = 2 + 4x$

$y = 3 + 2x$

2. $y = 2x + 3$

$y = {}^{-}x + 6$

3. $y = x + 5$

$y = {}^{-}x - 3$

4. $y = 2x - 1$

$y = 3 - 6x$

Solve the following Simultaneous Equations using the Substitution Method.

5. $2y + x = 12$

$y = x - 6$ . . . .

6. $y = 4x - 2$

$y - 2x = 1$ . . . .

7. $y = 2x + 3$

$x = 6 - y$ . . . .

8. $y = 370 - x$

$8x + 5y = 2330$ . . . .

Solve the following Simultaneous Equations using the Elimination Method.

9. $x + y = 6$

$4x + y = 12$

10. $3y - 2x = 9$

$y + 2x = 7$

11. $2x + 4y = 2$

$2x - 2y = 17$

12. $x + y = 20$

$8x + 5y = 120$

Algebraic Methods - Merit Examples

a. Simplify fully: $\frac{2x^2 - 12xy}{6x^2}$

$= \frac{2x(x - 6y)}{2x(3x)}$ factorise then simplify

$= \frac{x - 6y}{3x}$

b. Rewrite the formula $A = \pi\sqrt{\frac{W}{G}}$ to make W the subject.

$A^2 = \pi^2\frac{W}{G}$ square both sides

$GA^2 = \pi^2W$ multiply both sides by G

$\frac{GA^2}{\pi^2} = W$ divide each side by π^2

$W = \frac{GA^2}{\pi^2}$

c. Solve the equations for x and y: $2y + 3y = 15$
$-4x - 3y = 3$

$5y = 15$, therefore $y = 3$

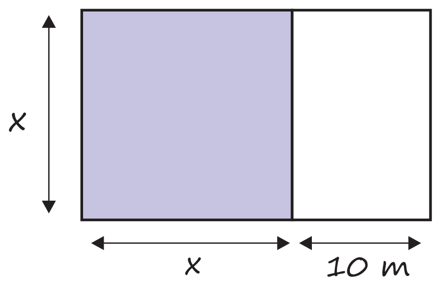
$-4x - (3 \times 3) = 3$ put $y = 3$ into the 2nd equation

$-4x - 9 = 3$

$-4x = 12$

$x = -3$, Therefore $x = -3$, $y = 3$

d. A square warehouse is extended by 10 metres at one end.
The area of the extended warehouse is 375m²
Find the original area of the warehouse.



$x(x + 10) = 375$

$x^2 + 10x = 375$

$x^2 + 10x - 375 = 0$

$(x + 25)(x - 15)$

$x = -25$ or $x = 15$

Therefore the original warehouse size must be 15 x 15 m²

Therefore the original area = 225m²

Exercises

1. Simplify: $\frac{x^2 - 6x - 16}{(x + 2)}$

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2. Elton has more than twice as many CDs as Robbie. Altogether they have 56 CDs. Write a relevant equation and use it find the least number of CDs that Elton could have.

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. . . .

3. Elton purchases some DVDs from the mall. He buys four times as many music DVDs as movie DVDs. The music DVDs are $2.50 each. The movie DVDs are $1.50 each. Altogether he spends $92.
Solve the equations to find out how many music DVDs that he purchased.

$$S = 4V$$
$$2.5S + 1.5V = 92$$

. . . .

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. . . .

4. Simplify: $\frac{x}{2} + \frac{x}{8}$

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5. One of the solutions of $4x^2 + 8x + 3 = 0$ is $x = -1.5$

Use this solution to find the second solution of the equation $4x^2 + 8x + 3 = 0$

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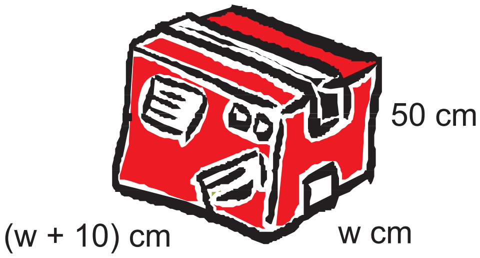
. . . .

6. The volume of the box shown is 60 litres.

Find the dimensions of the box.



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7. The triangle drawn below is equilateral. The perimeter is 30 cm. Write down two equations and solve them simultaneously to find the values of x and y.

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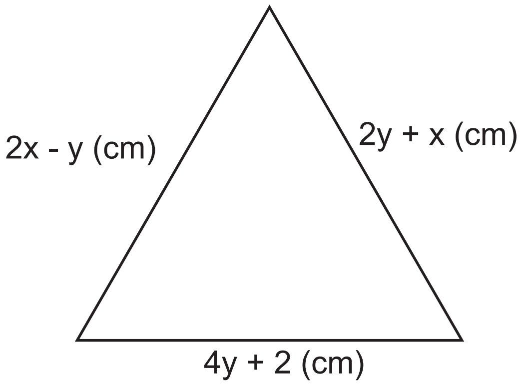

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8. Simplify: $\frac{x^2 - 4y^2}{x^2 - 2xy}$

. . . .

. . . .

9. Express as a single fraction: $\frac{x}{2} + \frac{3x}{5}$

. . . .

. . . .

10. Solve the equation $x^2 + 2x = 255$

Hint: two factors of 255 are 15 and 17.

. . . .

. . . .

11. Simplify: $\frac{2m}{3} + \frac{m}{4}$

. . . .

. . . .

. . . .

12. There are V litres in Claudia's water tank. There are d "drippers" on the irrigation hose from the tank to the garden. Each dripper uses x litres of water per day.

(a) Write an expression to show the total amount of water, T, left in the tank after one day.

. . . .

(b) At the end of the day on the 1st of April there were 150 litres of water in the tank. The next day, 4 drippers were used to irrigate the garden and at the end of the day there were 60 litres of water left.

Use the expression you gave above to show how much water each dripper used on that day.

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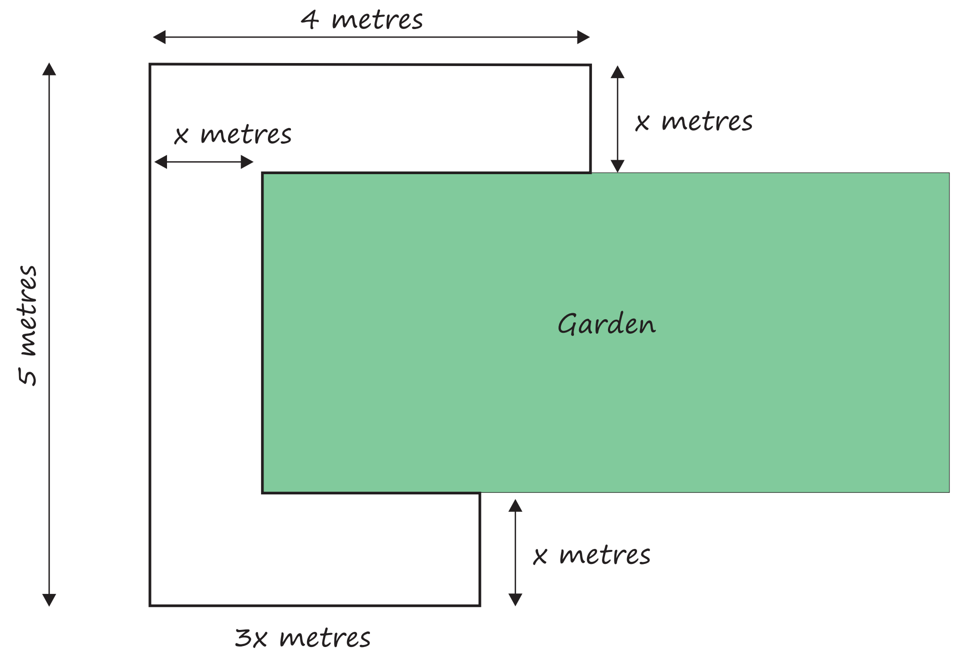
Amount of water, T, used by each dripper = ... Litres.

13. Graeme is designing a path around the front of his garden.
His design is shown below.



The width of the path is x metres.

Graeme has sufficient paving to make a path with a total area of 22 m^2.

The area of the path can be written as $4x + 3x^2 + (5 - 2x)x = 22$.

Rewrite the equation and then solve to find the width of the path around the front of the garden.

Algebraic Methods - Excellence Examples

a. Zahara is five years old and Maddox is four years older.
Form a relevant equation. Use it to find how many years it will take until Zahara's and Maddox's ages in years, multiplied together make 725 years.

Let Z = Zahara's age: $Z(Z + 4) = 725$

$$Z^2 + 4Z - 725 = 0$$

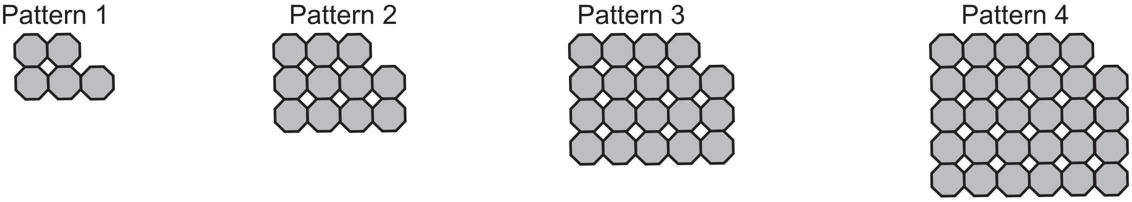
$$(Z + 29)(Z - 25) = 0$$

$$Z = -29 \text{ or } Z = 25$$

Zahara will be 25 and Maddox will be 29. $(25 \times 29 = 725)$

As Zahara is now 5 it will take another 20 years.

b. Holmsey is using octagonal tiles to make patterns.



Holmsey has 271 octagonal tiles. He wants to use all the tiles in a pattern as above. Write an equation to show the relationship between the pattern number (n) and the number of tiles used (t).
Solve the equation to find the pattern number that would have 271 tiles.

Pattern number	(n)	1	2	3	4
Tiles	(t)	5	11	19	29

The first difference between the terms is: 6, 8, 10.

The second difference is 2. This means the equation will start n^2.

Look at the relationship between n, n^2 and t:

n	1	2	3	4
n^2	1	4	9	16
t	5	11	19	29

Possible equations are $t = n^2 + 3n + 1$ **or** $t = (n + 1)^2 + n$

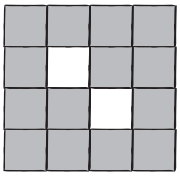
Using $t = n^2 + 3n + 1$ $\quad n^2 + 3n + 1 = 271$

$$n^2 + 3n - 270 = 0$$

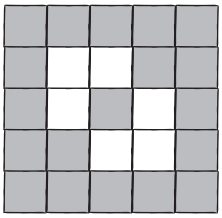
$$(n + 18)(n - 15) = 0$$

Pattern number (n) = 15

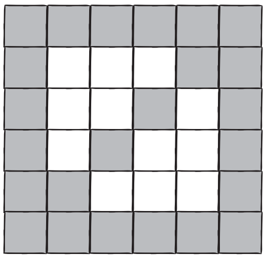
Exercises



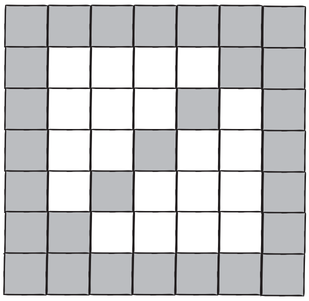
Pattern 1



Pattern 2



Pattern 3



Pattern 4

1. The design above can be modeled by the following formulae where n = the number of square tiles on the bottom line.

Total number of square tiles = n^2.

Total number of grey tiles = $5n - 6$.

a. Write the formula for the number of white tiles.

b. A square courtyard is to be tiled using the design above. Each side of the courtyard requires 25 tiles.
Give the total number of grey and white tiles required.

2. At the local garden centre, Mr Rose makes two rectangular garden plots.
Plot 1 is 5 metres longer than it is wide and has an area of 18.75m^2.
Plot 2 is 3 metres longer than it is wide and has an area of 22.75m^2.
The combined width of both gardens is 6 metres.

Find the length and width of each garden.

Show any equations you need to use.
Show all working.
Set out your work logically.
Use correct mathematical statements.

3. Students from Mahobe High School are about to be transported to a sports game in two mini buses - A and B. They are all seated in the mini buses ready to depart.

- If 3 students in bus A are moved to bus B then each bus will have the same number of students.
- If 2 students in bus B are moved to bus A then bus A will have twice the number of students that are in bus B.

Use the information given to find the total number of students in the mini buses. You must show all your working and give at least one equation that you used to get your final answer.

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Level 1 CAT Practice

AIMING FOR ACHIEVEMENT

QUESTION ONE

Expand and simplify.

1. $(5x - 4)(x + 3)$

2. $(x - 3)^2$

3. $5(x + 2) + 2(x - 3)$

4. $(3x - 1)(2x + 4)$

5. $2(x - 10) - 4(x + 2)$

6. $3(x - 4)^2$

QUESTION TWO

Factorise completely.

1. $x^2 - 7x - 30$

2. $7x^2 - 21x$

3. $2x^2 - 8x - 24$

4. $x^2 - 9x + 8$

5. $4x^2 - 9$

6. $x^2 + 5x - 50$

QUESTION THREE

Simplify.

1. $\dfrac{4x^2}{16x^5}$

2. $6x^4 . 5x^3$

3. $\dfrac{5x^2 - 20}{x + 2}$

4. $\dfrac{2x^2 . x^5}{2x^4}$

5. $(2x^2)^3$

6. $\dfrac{2x^2 - 10x - 28}{x + 2}$

QUESTION FOUR

Find the value of n.

1. $3y^5 \times 5y^n = 15y^{10}$

2. $4(y^2)^n \times 3y^4 = 12y^{16}$

3. $\dfrac{8x^9}{4x^n} = 2x^4$

QUESTION FIVE

The cost of sending a parcel via Mahobe Post depends on the weight of the parcel and the distance it has to travel. The rates are $1.50 per kilogram and 25 cents per kilometre. The cost of sending a parcel can therefore be calculated by the formula: $C = 1.5w + 0.25d$ where w = the weight in kg
d = the distance in km.

The distance from Auckland to Whangarei is 160 km.
Find the cost of sending an 8kg parcel from Auckland to Whangarei.

QUESTION SIX

1. What number should replace x in the pattern:

$4^{a-1} = 1,\ 4^{a} = 4,\ 4^{a+1} = 16,\ 4^{a+2} = 64,\ 4^{a+3} = x$

2. In a tile pattern the number of coloured tiles used is 3^{x}, where x is the row number. Calculate the number of tiles that would be used in rows 4, 5 and 6 the tile pattern.

3. The formula $P = I^2R$ gives the amount of power (in Watts) that is lost through the electrical cable. I is the current in amps and R is the resistance in ohms. Resistance in wiring leading to an electrical oven can be 10 ohms. If the oven has a current of 15 amps how much power is lost through the cable?

4. Ashton opens a savings account for University Study. He makes an initial deposit when opening the account and his parents deposit $120 each month. After 3 months there is $1000 in the account. How much did Ashton initially deposit when he opened the account?

QUESTION SEVEN

Solve the following equations.

1. $(x + 4)(x - 7) = 0$

2. $(3x - 1)(x + 4) = 0$

3. $(2x + 1)(x - 5) = 0$

4. $6x(x - 8) = 0$

5. $(1 - 2x)(x - 5) = 0$

6. $2(x + 4) = 18$

7. $3x(x + 8) = 60$

8. $6x - 3 = 2x + 9$

9. $3x + 5 = x - 4$

10. $7.1x + 5.4x = 100$

11. $\frac{2x}{3} = \frac{9}{2}$

12. $\frac{4x + 5}{5} = 3$

QUESTION EIGHT

Simplify.

1. $\frac{4a^2 - 12ab}{8a^2}$

2. $\frac{12xy + 2x^2}{6x^2}$

3. $\frac{x}{2} + \frac{x}{7}$

4. $\frac{2a}{3} + \frac{4a}{5}$

5. $\frac{x^2 + 5x - 24}{x^2 - 9}$

6. $\frac{2x^2 + 14x + 20}{x + 2}$

Questions **ONE** to **SEVEN** are all basic Achievement type questions. Question **EIGHT** is Merit standard.

Level 1 CAT Practice

AIMING FOR MERIT

QUESTION ONE

1. The formula $A = 4¶r^2$ gives the surface area of a ball.

A = the surface area and r = the radius of the ball.

Rearrange the formula to make r the subject.

2. The perimeter of a rectangle can be calculated by the formula $P = 2L + 2W$ where L is the length of the rectangle and W is the width.

Rearrange the formula to make L the subject.

3. A grandfather clock keeps accurate time due to the pendulum length and gravity. The formula used is $T = 2¶\sqrt{\frac{L}{g}}$

Rearrange the formula to make L the subject.

4. Vassily is using the equation $y = 3x^2 - 5$

Rearrange the equation to make x the subject.

QUESTION TWO

1. Taz is designing a path to run from the front to the back of the house.

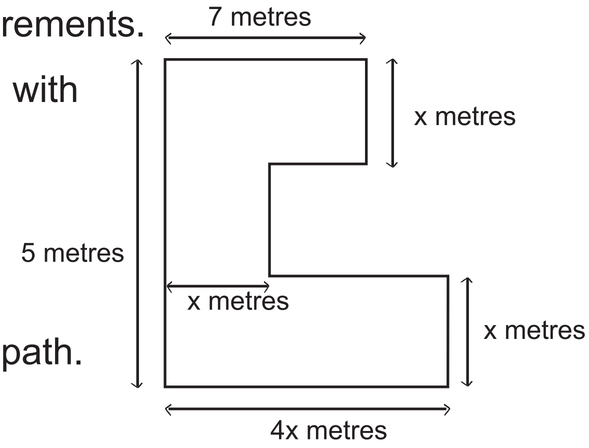
The diagram shows the shape and measurements.

Taz has sufficient concrete to make a path with a total area of 32 m^2.

The area of the path can be written as:

$7x + 4x^2 + (5 - 2x)x = 32$

Solve the equation to find the width of the path.



2. The sides of a square warehouse are extended by 5 metres along one side and 3 metres on the other side. The new floor area is 63 m^2.

What was the area of the original warehouse?

QUESTION THREE

1. A maths problem that states: “Five minus three times a mystery number is less than twenty”.

Write an inequality and use it to find all the possible values for the mystery number.

2. Thorpe saves \$12 000 to go to the Olympics. He wants to purchase as many tickets as he can for the athletics. Each ticket to the athletics costs \$240. Travel, food and accommodation costs \$10 200. Use this information to write an equation or inequation. What is the greatest number of tickets to the athletics that Thorpe can buy?

3. An isosceles triangle has a perimeter of 218 mm. The third side of the triangle is shorter than the two equal sides by 25 mm. How long is the third side?

4. Cindy works at Pac and Slave and earns \$12.50 per hour. She also does baby sitting for 2½ hours on a Friday night for which she earns \$40. Cindy wants to earn at least \$100 per week. How many hours does she have to work at Pak and Slave to achieve this?

5. Perlman opens a book and notes that the two page numbers add up to 265. What are the numbers of the pages he is looking at?

6. The formula for the sum (S) of the first n counting numbers is: $S = \frac{n(n - 1)}{2}$

Calculate the sum of the first 100 counting numbers.

7. Tweeter and Toots buy a pizza for \$9.40.

They split the cost in the ratio of 2:3 with Tweeter paying the larger portion.

How much does each person pay?

8. Buster, Todd and Cal win \$2400 between them.

Buster gets a share of \$x

Todd gets twice as much as Buster.

Cal’s share is \$232 less than Busters.

Write an equation for each the amounts in terms of x then calculate the amounts that each person will receive.

Level 1 CAT Practice

AIMING FOR MERIT

QUESTION ONE

Solve the simultaneous equations:

1. $x + 2y = 9$

$4x + 3y = 16$

2. $3y - 8x = 30$

$3y + 2x = 15$

3. $\frac{x}{2} + 3y = 2$

$10y + x + 4 = 0$

4. $4x + 5y = 25$

$x + y = 5$

5. $3y - 5x - 12 = 26$

$\frac{1}{4}y + 4x - 15 = -3$

6. $x + y = \frac{1}{2}(y - x)$

$y - x = 4$

QUESTION TWO

Relative speed is the speed of one body with respect to another.

For example if a boat is sailing at 10 km/hour down a river that is also running at 10 km/hour then the boat will be sailing at 20 km/hour. If the boat tries to sail upstream then the current is acting against it and to move forward it would have to sail at a speed greater than 10 km/hour.

A passenger plane takes 3 hours to fly the 2100 km from Sydney to Auckland in the same direction as the jetstream. The same plane takes 3.5 hours to fly back (against the jetstream) from Auckland to Sydney.

Using the variables:

P = plane speed
W = wind speed

and the equations:

$(P + W)(3) = 2100$
$(P - W)(3.5) = 2100$

Calculate the plane speed and the wind speed of the plane.

QUESTION THREE

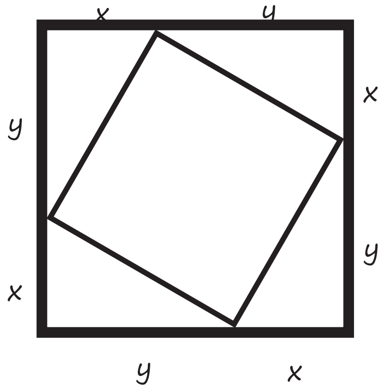
1. The diagram below shows a large square with side lengths (x + y).

Inside this square is a smaller square with side lengths of z. The area of the large square can be written: z^2 + area of the 4 triangles

$= z^2 + \frac{1}{2}xy + \frac{1}{2}xy + \frac{1}{2}xy + \frac{1}{2}xy$

$= z^2 + 2xy$

Use this information to prove the Pythagoras Theorem for right angled triangles.



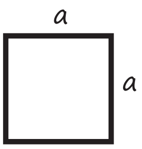
2. In this question you are to find the dimensions of a rectangular warehouse space. The length of the warehouse is 12 metres longer than its width. The warehouse is to be built on a section measuring 25 × 40 metres. It will also have an office attached measuring 6 metres × 10 metres.

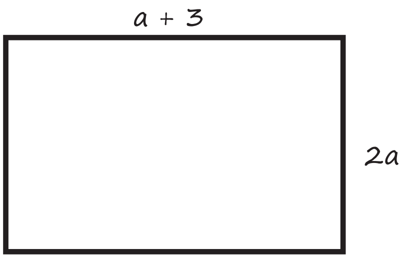
Council regulations state that only 70% of the land area can be used.

Find the maximum allowable length and width of the warehouse.

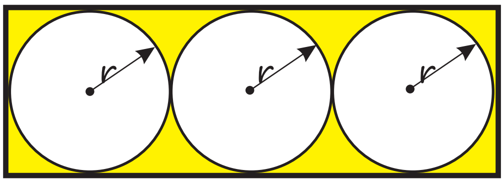
3. The sponsor of the school year book has asked that the length and width of their advertisement be increased by the same amount so that the area of the advertisement is double that of last years. If last year's advertisement was 12 cm wide × 8 cm long what will be the width and length of the enlarged advertisement?

4. The area of the square below is $4x^2 - 56x + 196$. Use this to develop an expression for the area of the rectangle.





5. Write an expression in factored form for the shaded area of the shape below.



Level 1 CAT Practice

AIMING FOR MERIT

QUESTION ONE

1. Expand and simplify:

a. $(2x + 1)(x - 3)$

b. $3(y + 5) - 2(y - 8)$

c. $12 - 2(x + 2)$

2. Factorise:

a. $x^2 + 9x - 36$

b. $x^2 - 14x + 49$

3. Simplify:

a. $(2x^4)^3$

b. $(4y^3)^2$

4. Solve for x:

a. $x^3 = -64$

b. $2^x = 64$

5. Simplify:

a. $\frac{4x}{3} + \frac{5x}{8}$

b. $\frac{x^2 - 81}{2x + 18}$

c. $\frac{24x^9}{8x^3}$

An operation * is defined by:

$$a * b = \frac{10ab}{(a + b)^2}$$

6. Find the value of $3 * -4$

QUESTION TWO

1. Solve these equations:

a. $7x + 25 = 5 - x$

b. $4y(y + 2) = 0$

c. $3z^5 = 96$

2. Factorise fully: $a^2 + 3a - 40$

3. Write in simplest form: $\frac{a^2 + 3a - 40}{a^2 + 8a}$

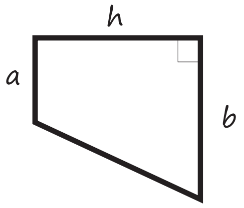
4. What is the value of k if:

$(2a)^4 \times a^k = 16a^8$,

5. Expand and simplify $(2a + 4)(a - 1)$.

6. The area of a trapezium is given by:

$$\text{Area} = \frac{(a + b)h}{2}$$



If a = 15 cm, b = 9 cm

and area = 36 cm^2 find the value of h.

7. Clearly state the range and possible values of a if $(2a + 8)(a - 2) < 4a + 2$.

8. A rectangular swimming pool is 30 metres × 10 metres. Around the pool is a concrete path that is w metres wide. The total area of the pool and surrounding path is 800 m^2. Using the information form an equation and solve it to find the width (w) of the path around the pool.

QUESTION THREE

1. Last year the Mahobe Football club sold pizzas.

   There were 250 pizzas sold for a total raised of $1730.

   Large pizzas sold for $8 each and small ones sold for $5 each.

   There were x large pizzas and y small pizzas.

   Solve the simultaneous equations below to find the number of each sized pizza that was sold.

$$x + y = 250$$

$$5x + 8y = 1730$$

2. **a.** There are many interesting properties of consecutive numbers.

Consecutive numbers are numbers such as 21, 22, 23, 24.

For example, choose any 5 consecutive numbers. Take the middle number and multiply it by 5. The answer will be the same as if you summed all 5 of the numbers.

Write an expression that represents five consecutive numbers and use this expression to show that if you multiply the middle number by five you get a result the same as if you summed all five numbers.

b. In another example take three consecutive whole numbers.

- Square each number and sum the three squares.
- Subtract two from the sum and divide the result by three.

Write down an expression of represent any three consecutive numbers. Use this expression to show that if you follow the steps outlined above with any set of three consecutive numbers you will always get as a result the square of the second of the numbers that you first started with.

3. To find Sung's birth month multiply it by 4. Add to this product the difference between 12 and his birth month. Subtract from this result twice the sum of 5 and his birth month. If you successfully follow this equation you should end up with 10. What must Sung's birth month be?

Level 1 CAT Practice

AIMING FOR EXCELLENCE

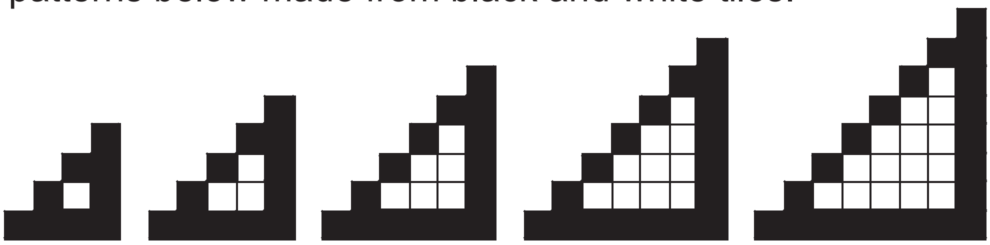
QUESTION ONE

You are to explore the sequence of numbers given by the rule: $2n^2 + 3n - 1$

Find the rule for the difference between any two consecutive terms for the sequence $2n^2 + 3n - 1$. You should show all your working.

QUESTION TWO

Look at the patterns below made from black and white tiles.



n = the number of square tiles on the bottom row.

$0.5n(n + 1)$ = the total number of squares used.

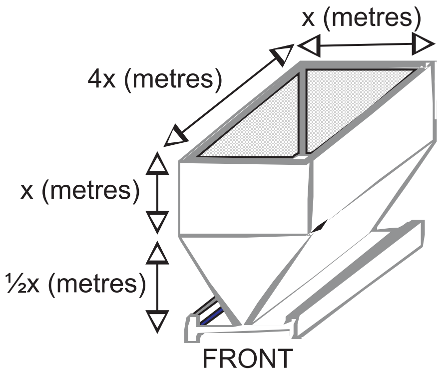
$0.5n^2 - 2.5n + 3$ = the total number of white squares used.

1. One particular design has a total of 171 squares.
   How many squares does this pattern have on the bottom row?
2. One particular pattern contains 42 black squares.
   How many squares does this pattern have on the bottom row?

QUESTION THREE

The diagram shows a rice hopper.

The volume of the hopper can be calculated by finding the cross sectional area of the front and multiplying it by the length (4x metres).



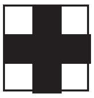
If the hopper can hold 40 m³ of rice, calculate the size of x.

QUESTION FOUR

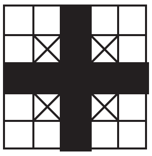
An artist uses tiles to create different designs.

For the design below, she uses square tiles some of which are black, others have crosses and the rest are white.

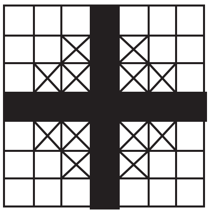
The first four designs are shown below.



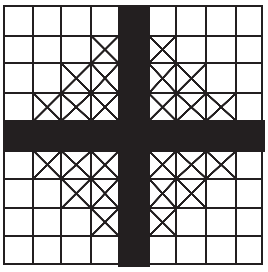
Design 1



Design 2



Design 3



Design 4

There are 3 equations that can be formed to calculate the number of black, crossed and white tiles for each design (n).

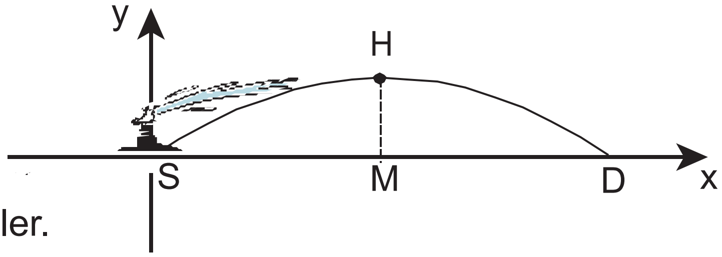
The equations are: Black Tiles = $4n + 1$

Crossed tiles = $2(n^2 - n)$

White tiles = $2(n^2 + n)$

Prove that the total number of tiles in any of the designs, n, is given by the equation $(2n + 1)^2$.

QUESTION FIVE



The diagram shows the path of a jet of water from a park's water sprinkler.

The furthest distance that the water travels is 50 metres and can be described by the equation: $y = 0.5x - 0.01x^2$, where x is the horizontal distance traveled and y is the vertical maximum height that the water reaches.

1. The point mid-way between S and D is the highest point of the water (H). Find the greatest height (MH) that the water reaches.

2. At one end of the park is a 2.25 m high fence. The water is just managing to go over this fence. If the park caretaker moves the sprinkler so that the water just reaches the base of the fence how far will the sprinkler have to be moved?

The Answers

Page 9 – Expanding

1. $u(u + 1) = u^2 + u$
2. $v(v - 6) = v^2 - 6v$
3. $-w(3w - 2) = -3w^2 + 2w$
4. $x(4x + 5) = 4x^2 + 5x$
5. $3y(2y - 3) = 6y^2 - 9y$
6. $-z(-5z + 3) = 5z^2 - 3z$
7. $3 + 2(x - 8) = 3 + 2x - 16$
   $= 2x - 13$
8. $5(x + 7) - 12 = 5x + 35 - 12$
   $= 5x + 23$
9. $3(x - 6) + 2(4x - 5)$
   $= 3x - 18 + 8x - 10$
   $= 11x - 28$
10. $4(a + 6) - 2(a - 2)$
    $= 4a + 24 - 2a + 4$
    $= 2a + 28$
11. $2x(x + 1) - x(7 - x)$
    $= 2x^2 + 2x - 7x + x^2$
    $= 3x^2 - 5x$
12. $x^2(x + 1) = x^3 + x^2$
13. $\frac{1}{2}(4x + 12) = 2x + 6$

Page 10

14. $\frac{2}{3}(12x - 6) = 8x - 4$
15. $3x(2x^2 - 4) = 6x^3 - 12x$
16. $x(x^2 + 4) + x(3x + 2)$
    $= x^3 + 4x + 3x^2 + 2x$
    $= x^3 + 3x^2 + 6x$
17. $6x + 24 = 6(x + 4)$
18. $5x - 25 = 5(x - 5)$
19. $11x^2 - 66x = 11x(x - 6)$
20. $10x + 25xy = 5x(2 + 5y)$
21. $100x + 20y = 20(5x + y)$
22. $27 - 33x = 3(9 - 11x)$
23. $5x^2 + x = x(5x + 1)$
24. $6a^2 + 3a = 3a(2a + 1)$
25. $15b^2 - 30b = 15b(b - 2)$
26. $14y^2 + 21y = 7y(2y + 3)$
27. $5 + 5n^2 = 5(1 + n^2)$
28. $6x^2 + 18xy = 6x(x + 3y)$
29. $2xy - 4ab = 2(xy - 2ab)$
30. $3p^2 - 9pq = 3p(p - 3q)$

Page 11

31. $(x + 1)(x + 6) = x^2 + 7x + 6$
32. $(x + 2)(x + 8) = x^2 + 10x + 16$
33. $(x - 5)(x + 7) = x^2 + 2x - 35$
34. $(x - 2)(x + 9) = x^2 + 7x - 18$
35. $(x + 4)(x - 5) = x^2 - x - 20$
36. $(x + 7)(x - 3) = x^2 + 4x - 21$
37. $(x - 10)(x - 15) = x^2 - 25x + 150$
38. $(x - 8)(x - 11) = x^2 - 19x + 88$
39. $(x + 6)^2 = x^2 + 12x + 36$
40. $(x - 9)^2 = x^2 - 18x + 81$
41. $(x + 1)^2 + 10 = x^2 + 2x + 11$
42. $(x - 5)^2 - 20 = x^2 - 10x + 5$

Page 12

43. $x^2 + 10x + 21 = (x + 7)(x + 3)$
44. $x^2 + x - 12 = (x + 4)(x - 3)$
45. $x^2 - 2x - 15 = (x - 5)(x + 3)$
46. $x^2 - 14x + 40 = (x - 10)(x - 4)$
47. $x^2 + 11x + 30 = (x + 6)(x + 5)$
48. $x^2 + x - 2 = (x + 2)(x - 1)$
49. $x^2 - 3x - 10 = (x - 5)(x + 2)$
50. $x^2 - 4x - 96 = (x - 12)(x + 8)$
51. $x^2 - 5x - 14 = (x - 7)(x + 2)$
52. $x^2 - 16 = (x - 4)(x + 4)$
53. $x^2 - 81 = (x - 9)(x + 9)$
54. $(x - 3)^2 - 16 = x - 6x + 9 - 16$
    $= x - 6x - 7$
    $= (x - 7)(x + 1)$

Page 12 (continued)

55. $x^2 + 2x = 15$

$= x^2 + 2x - 15$

$= (x + 5)(x - 3)$

56. $x^2 = 6x - 8$

$= x^2 - 6x + 8$

$= (x - 4)(x - 2)$

57. $2x^2 - 2x = 220$

$= 2x^2 - 2x - 220$

$= 2(x^2 - x - 110)$

$= 2(x - 11)(x + 10)$

58. $4x^2 - 100 \quad = 4(x^2 - 25)$

$= 4(x - 5)(x + 5)$

Page 15

1. $16x^4$

2. $64x^4y^2$

3. $\frac{x^4}{y^2}$

4. $\frac{1}{2x^5}$ (divide top & bottom by $4x^5$)

5. $\frac{3x^2}{4}$ (divide top & bottom by $3x^3$)

6. $\frac{4x - 5y}{x}$ (divide top & bottom by $2x$)

7. $\frac{1 - 5b}{2b}$ (divide top & bottom by $3a$)

8. $2 + n = 8$ therefore $n = 6$

9. $2n = 8$ therefore $n = 4$

10. $a^0 = 1$, $6 - n = 6$, therefore $n = 6$

Page 17

1. i. $\tfrac{1}{2}(-4 + 10) \times 2 = 6$

ii. $\tfrac{1}{2}(1.6 + 2.8) \times 3.2 = 7.04$

2. $20 - 0.8 \times 15 = 8$

3. $8^2 + 4.5^2 = 84.25$

4. 34

5. $(4 + -10) \div 2 = -3$

6. $-100 \div -4 = 25$

7. $(2.5 + 2\times(-5) + (-8.5)) \div 5 = -3.2$

8. $(10 \times -5) \div (10 + -5) = -10$

9. $(2 \times 9) \div 100 = 0.18$

10. i. $(5(10 + 5)) \div 10 = 7.5$

ii. $(5(9.8 + 5.3)) \div (2 \times 5.3) = 7.12$

Page 19

1. a. White titles = 8, 13, 18, 23

b. Rule $= dn + (a - d)$

$= 5n + (8 - 5)$

$= 5n + 3$

2. a. $M = 3, 5, 7, 9, 11, 13$

b. Matches $= 2p + 1$

$= 2 \times 10 + 1$

$= 21$

Page 20

3. For pattern n, shaded squares $= n + 1$

Form pattern n, white squares $= 2n$

Total squares $= n + 1 + 2n$

$= 3n + 1$

4. Term $n = 4n$

5. a. Black, 1, 2, 3, 4, 5, 6

White, 6, 10, 14, 18, 22, 26

b. Formula W = white, B = Black

$W = 4B + 2$, If $B = 100$ black pavers, order 402 white pavers.

Page 23

1. $5x = 45$ $\quad x = 9$

2. $6x = 8$ $\quad x = 1\frac{1}{3}$

3. $-4x = 24$ $\quad x = -6$

4. $3x = -5$ $\quad x = \frac{-5}{3}$

Page 23 (continued)

5. $8x + 12 = -8$

$8x = -20$

$x = \frac{-20}{8}$ or -2.5

6. $6x = -18$

$x = -3$

7. $2x = 10$

$x = 5$

8. $4x - 5x = -2 + 8$

$-1x = 6$

$x = -6$

9. $6x - 2x = 20 - 7$

$4x = 13$

$x = \frac{13}{4}$ or 3.25

10. $x - 2x = -8 - 6$

$-x = -14$

$x = 14$

11. $3x - 2x = 11 - 7$

$x = 4$

12. $10x - 8x = 22 - 2$

$2x = 20$

$x = 10$

13. $3x + 6 = 5x - 10$

$3x - 5x = -10 - 6$

$-2x = -16$

$x = 8$

14. $3(4 - 8) = -x$

$-12 = -x$

$x = 12$

Page 25

1. $x = 5$ or $x = 10$

2. $x = -3$ or $x = 8$

3. $x = 9$ or $x = -4$

4. $x = -15$

5. $x = 2.5$ or $x = -7$

6. $x = -3$ or $x = -4$

7. $x = 3.5$ or $x = -0.75$

8. $x = -4.5$ or $x = 0$

9. $x = -4$ or $x = 2.5$

10. $x = 8/3$ or $x = -8/3$

11. $x^2 + 6x + 9 - 25 = 0$

$x^2 + 6x - 16 = 0$

$(x + 8)(x - 2)$

$x = -8$ or $x = 2$

12. $x^2 - 4x + 4 - 9 = 0$

$x^2 - 4x - 5 = 0$

$(x - 5)(x + 1)$

$x = 5$ or $x = -1$

Pages 27 - 29 Achievement Exercises

1. $3x - 27 = 9$

$3x = 36$

$x = 12$

2. $5x - x = -6 - 3$

$4x = -9$

$x = -2.25$

3. $x = 0$ or $x = -9$

4. $2x(x - 5) + 7(x - 5)$

$= 2x^2 - 10x + 7x - 35$

$= 2x^2 - 3x - 35$

5. $5x^3$

6. $y = 5(5 + 5) \div 2 = 25$

7. $x = -3$ or $x = 8$

8. $17x - 12x = 4 + 9$

$5x = 13$

$x = \frac{13}{5}$ or 2.6

Page 28 (cont)

9. $2x + 6 = 20$

$2x = 14$

$x = 7$

10. $2x(x + 1) - 2(x + 1)$

$2x^2 + 2x - 2x - 2$

$2x^2 - 2$

11. $(x - 7)(x + 2)$

12. $F = (11 \div 2) \times (3 \times 11 - 5)$; $F = 154$

Page 29

13. $x = \frac{1}{3}$ or $x = -7$

14. $6x - 2x = 8 + 3$

$4x = 11$

$x = \frac{11}{4}$ or 2.75

15. $\frac{5x}{2} = 25$

$5x = 50$

$x = 10$

16. $2x(3x + 5) - 1(3x + 5)$

$= 6x^2 + 10x - 3x - 5$

$= 6x^2 + 7x - 5$

17. $(x + 8)(x - 3)$

18. $3x^9$

19. $5^3 = 125$

20. $R = 0.45 \times 27.8 \times 3.6$; $R = 45.036$

Page 31

1. $\frac{4y + 2x}{xy}$

2. $\frac{10b - 3a}{6ab}$

3. $\frac{x}{3x + 2}$

4. $\frac{(x - 3)(x - 2)}{(x + 2)(x - 2)} = \frac{(x - 3)}{(x + 2)}$

5. $\frac{8x^2y^2}{6x^2y} = \frac{2x^2y(4y)}{2x^2y(3)}$

$= \frac{4y}{3}$

6. $3k = 36$

$k = 12$

7. $\frac{m + 16}{8} = \frac{4}{8}$

$m = -12$

8. $\frac{2t}{5} = -4$

$2t = -20$

$t = -10$

9. $7e = 50.5 - 10.5e$

$17.5e = 52.5$

$e = 3$

10. $\frac{7x}{10} = \frac{-14}{1}$

$7x = -140$

$x = -20$

Pages 33-34

1. a. Slabs = 24, 35, 48, 63, 80

b. Look at the pattern between the storeys and the slabs.

1x3=3, 2x4=8, 3x5=15,
4x6=24, 5x7=35

$y = x(x + 2)$

$y = x^2 + 2x$

Using this formula, 25 storeys would need 675 slabs of wood.

2. Total = Grey + white

$n^2 = 5n - 6 +$ white

white $= n^2 - 5n + 6$

3. a. Sixth term = 48

b. Using term 1, $n^2 + kn = 3$

$1^2 + k = 3$

$k = 2$

4. Next two terms are (6×7)+4, (7×8)+5

$(n + 2)(n + 3) + n$

$= n^2 + 2n + 3n + 6 + n$

$= n^2 + 6n + 6$

5. Next two terms are 36, 49

Terms:	1	2	3	4	5	6
Sequence:	4	9	16	25	36	49

Rule $= (n + 1)^2$

Page 37

1. $y - 5 = 10x$

$x = \frac{y - 5}{10}$

2. $-2x = 7 + 8y$

$x = \frac{7 + 8y}{-2}$

Page 37 (cont)

3. $x = PV$

4. $2y = x + 5$

$x = 2y - 5$

5. $4y = 3a + xa$

$4y - 3a = xa$

$x = \frac{4y - 3a}{a}$

6. $2s = tu + tv$

$v = \frac{2s - tu}{t}$

7. $a^2 - b^2 = c^2$

$c = \sqrt{a^2 - b^2}$

8. $v^2 - u^2 = 2as$

$a = \frac{v^2 - u^2}{2s}$

9. $r^2 = \frac{V}{\pi h}$

$r = \sqrt{\frac{V}{\pi h}}$

10. $A = \frac{ah + bh}{2}$

$2A = ah + bh$

$a = \frac{2A - bh}{h}$

Page 39

1. $3a = 27, a = 9$

2. $8x = -20, x = -2.5$

3. $20x = 10, x = 0.5$

4. $x = 4$

5. $a = 37$

6. $2x = 18, x = 9$

7. $4x + 36 = x + 6$

$3x = -30$

$x = -10$

8. $7 + 3x - 3 = 19$

$3x = 15$

$x = 5$

9. $11x = 66, x = 6$

10. $2x = 9, x = 4.5$

11. $2x = 64, x = 32$

12. $(x - 2) + (x + 3) + x = 22$

$3x = 21$

$x = 7$

Triangle sides are 5, 7, 10

Page 40

1. $2y > 1$

$y > 0.5$

2. $-3x < 12$

$x > -4$

3. $-y \geq 8$

$y \leq -8$

4. $-4x > 8$

$x < -2$

5. $2x - 9 > 63$

$2x > 72$

$x > 36$

6. $x < -13$

7. $-6x > 2$

$x < -\frac{1}{3}$

8. $3x - 6 \leq 5$

$3x \leq 11$

$x \leq 3\frac{2}{3}$

9. $-x - 3x < 8$

$-4x < 8$

$x > -2$

10. $5x + 15 - 6x \geq 12$

$-x \geq -3$

$x \leq 3$

Page 42

1. $x^2 + 20x + 100 = 225$

$x^2 + 20x - 125 = 0$

$(x + 25)(x - 5) = 0$

$x = -25$ or $x = 5$

Side length of pool is positive

$(x + 10) = 15$m

2. $x(x + 40) = 3200$

$x^2 + 40x - 3200 = 0$

$(x + 80)(x - 40) = 0$

$x = -80$ or $x = 40$

Field length can only be positive

Length is $(x + 40) = 80$ m

Width $= 40$ m

Page 43

3. At $h = 48$, $48 = 40t - 8t^2$

$8t^2 - 40t + 48 = 0$

$8(t^2 - 5t + 6) = 0$

$8(t - 3)(t - 2)$

Height at 48m is at $t = 3$ and $t = 2$ sec

The reason for 2 values is that the ball passes through 48 m on the way up and again on the way down.

4. $x^2 + 2x = 99$

$x^2 + 2x - 99 = 0$

$(x + 11)(x - 9) = 0$

$x = 9$ (positive integer)

Two consecutive positive integers are 9 and 11.

5. $x^2 + 14x + 49 + x^2 - 169 = 0$

$2x^2 + 14x - 120 = 0$

$2(x^2 + 7x - 60) = 0$

$2(x + 12)(x - 5) = 0$

$x = -12$ or $x = 5$

As the lengths have to be positive $x = 5$

Side lengths as 5, 12, 13

Page 45

1. $2 + 4x = 3 + 2x$

$2x = 1$

$x = 0.5$

$y = 2 + 4(0.5)$

$y = 4$

check other equation $4 = 3 + 2(0.5)$

2. $2x + 3 = -x + 6$

$3x = 3$

$x = 1$

$y = 2(1) + 3$

$y = 5$

check other equation $5 = (-1) + 6$

3. $x + 5 = -x - 3$

$2x = -8$

$x = -4$

$y = (-4) + 5$

$y = 1$

check other equation $1 = -(-4) - 3$

4. $2x - 1 = 3 - 6x$

$8x = 4$

$x = 0.5$

$y = 2(0.5) - 1$

$y = 0$

check other equation $0 = 3 - 6(0.5)$

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5. $2(x - 6) + x = 12$

$2x - 12 + x = 12$

$3x = 24$

$x = 8$

$y = 8 - 6$

$y = 2$

check other equation $2(2) + 8 = 12$

6. $(4x - 2) - 2x = 1$

$2x - 2 = 1$

$x = 1.5$

$y = 4(1.5) - 2$

$y = 4$

check other equation $4 - 2(1.5) = 1$

7. $y = 2(6 - y) + 3$

$y = 12 - 2y + 3$

$3y = 15$

$y = 5$

$x = 6 - 5$

$x = 1$

check other equation, $5 = 2 + 3$

8. $8x + 5(370 - x) = 2330$

$8x + 1850 - 5x = 2330$

$3x = 480$

$x = 160$

$y = 370 - 160$

$y = 210$

check other equation

$8(160) + 5(210) = 2330$

$1280 + 1050 = 2330$

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9. $x + y = 6$

$4x + y = 12$

Subtract

$-3x = -6$

$x = 2$

$x + y = 6$

$2 + y = 6$

$y = 4$

check with other equation $4x + y = 12$

$4(2) + (4) = 12$

10. $3y - 2x = 9$

$y + 2x = 7$

Add

$4y = 16$

$y = 4$

$3(4) - 2x = 9$

$-2x = -3$

$x = 1.5$

check with other equation $y + 2x = 7$

$4 + 2(1.5) = 7$

11. $2x + 4y = 2$

$2x - 2y = 17$

Subtract

$6y = -15$

$y = -2.5$

$2x + 4(-2.5) = 2$

$2x = 12$

$x = 6$

check with one of the equations

$2x - 2y = 17$

$2(6) - 2(-2.5) = 17$

$12 - -5 = 17$

12. $5x + 5y = 100$

$8x + 5y = 120$

Subtract

$-3x = -20$

$x = 6.67$ (2 dp)

$5 \times (6.67) + 5y = 100$

$y = 13.33$ (2 dp)

Don't forget to check your answer with another equation.

Page 49, Merit Exercises

1. $\frac{(x-8)(x+2)}{(x+2)} = x - 8$

2. Equations that can be formed are:

$E > 2R$

$E + R = 56$

or $E = 56 - R$

Using substitution $56 - R > 2R$

$56 > 3R$

$R < 18\frac{2}{3}$

Elton has at least 38 CDs

3. Substitute $S = 4V$ into the other equation

$2.5(4V) + 1.5V = 92$

$10V + 1.5V = 92$

$11.5V = 92$

$V = 8$

Substituting $V = 8$ into an equation

$S = 4(8)$

$\Rightarrow S = 32$ (he purchased 32 music DVDs)

Page 50

4. $\frac{8x + 2x}{16} = \frac{5x}{8}$

5. Factorising the equation will be either:

$(4x + 2)(x + 1.5)$ or $(4x + 1.5)(x + 2)$

Of the two the correct factorisation is

$(4x + 2)(x + 1.5)$

Therefore the other solution must be -0.5

6. $V = 50 \times (w + 10) \times w$

$V = (50w + 500) \times w$

$V = 50w^2 + 500w$

This is the formula for the volume

60 litres = 60,000 cm^3

$50w^2 + 500w = 60000$

$50w^2 + 500w - 60000 = 0$

$50(w^2 + 10w - 1200) = 0$

$50(w + 40)(w - 30) = 0$

$w = -40$ or $w = 30$

i.e. $w = 30$, $w + 10 = 40$, height $= 50$

Dimensions are 30cm × 40cm × 50cm

= 60,000 cm^3

= 60 litres

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7. If the perimeter is 30 cm the length of each side = 10 cm (equilateral triangle)

$2x - y = 10$

$2y + x = 10$ or $x = 10 - 2y$

Substituting

$2(10 - 2y) - y = 10$

$20 - 4y - y = 10$

$20 - 5y = 10$

$-5y = -10$

$y = 2$

If $y = 2$ then $2x - 2 = 10$

$2x = 12$

$x = 6$

Substituting the values into the equations

$2(6) - 2 = 10$

$2(2) + 6 = 10$

$4(2) + 2 = 10$

8. $\dfrac{(x - 2y)(x + 2y)}{x(x - 2y)} = \dfrac{x + 2y}{x}$

9. $\dfrac{5x + 6x}{10} = \dfrac{11x}{10}$

10. $x^2 + 2x - 255 = 0$

$(x + 17)(x - 15)$

See page 8 – the sign of the largest factor is the same as middle value (+ 2x)

$x = -17$ or $x = 15$

Page 52

11. $\dfrac{2m}{3} + \dfrac{m}{4} = \dfrac{8m + 3m}{12} = \dfrac{11m}{12}$

12. **a.** $T = V - dx$

T = Total volume remaining

V = initial volume

d = number of drippers

x = amount used by each dripper

b. $T = V - dx$

$60 = 150 - 4x$

$4x = 90$

$x = 22.5$

Amount of water used by each dripper is 22.5 litres.

Page 53

13. $4x + 3x^2 + 5x - 2x^2 = 22$

$9x + x^2 = 22$

$x^2 + 9x - 22 = 0$

$(x + 11)(x - 2) = 0$

$x = -11$ or $x = 2$

Width of path = 2m

Page 55

1. Total = grey + white

$n^2 = 5n - 6 +$ white

White = $n^2 - 5n + 6$

Total tiles (n^2) = 625

Grey tiles ($5n - 6$) = 119

White tiles ($n^2 - 5n + 6$) = 506

Page 56

2. Plot 1, $x(x + 5) = 18.75$

x = width of Plot 1

$x^2 + 5x = 18.75$

Plot 2, $y(y + 3) = 22.75$

y = width of Plot 2

$x + y = 6$

$y = 6 - x$

$(6 - x)(6 - x + 3) = 22.75$

$(6 - x)(9 - x) = 22.75$

$54 - 15x + x^2 = 22.75$

$x^2 - 15x + 54 = 22.75$

and $x^2 + 5x = 18.75$

$-20x + 54 = 4$ (subtract)

$-20x = -50$

$x = 2.5$

$x + y = 6, \quad x = 2.5, y = 3.5$

Plot 1 is $2.5 \times 7.5 \text{ m}^2$

Plot 2 is $3.5 \times 6.5 \text{ m}^2$

Page 57

3. First scenario $A - 3 = B + 3$

$A = B + 6$

Second scenario $A + 2 = 2(B - 2)$

$A = 2B - 4 - 2$

$A = 2B - 6$

Using $A = B + 6$ and $A = 2B - 6$

$B + 6 = 2B - 6$

$12 = B$

Using $B = 12$, $A = 18$ (as $A = B + 6$)

Testing the numbers

Initial Bus		Move 1	
A	B	A	B
18	12	15	15

Initial Bus		Move 2	
A	B	A	B
18	12	20	10

Don't forget – the question asks for the total number of students in the two buses.

Total number of students = 30

Page 59

QUESTION ONE

1. $(5x - 4)(x + 3)$

$= 5x^2 + 15x - 4x - 12$

$= 5x^2 + 11x - 12$

2. $(x - 3)^2$

$= (x - 3)(x - 3)$

$= x^2 - 3x - 3x + 9$

$= x^2 - 6x + 9$

3. $5(x + 2) + 2(x - 3)$

$= 5x + 10 + 2x - 6$

$= 7x + 4$

4. $(3x - 1)(2x + 4)$

$= 6x^2 + 12x - 2x - 4$

$= 6x^2 + 10x - 4$

5. $2(x - 10) - 4(x + 2)$

$= 2x - 20 - 4x - 8$

$= -2x - 28$

Page 59 (cont)

6. $3(x - 4)^2$

$= 3(x - 4)(x - 4)$

$= 3(x^2 - 4x - 4x + 16)$

$= 3(x^2 - 8x + 16)$

$= 3x^2 - 24x + 48$

QUESTION TWO

1. $x^2 - 7x - 30$

$= (x - 10)(x + 3)$

2. $7x^2 - 21x$

$= 7x(x - 3)$

3. $2x^2 - 8x - 24$

$= 2(x^2 - 4x - 12)$

$= 2(x - 6)(x + 2)$

4. $x^2 - 9x + 8$

$= (x - 8)(x - 1)$

5. $4x^2 - 9$

$= (2x + 3)(2x - 3)$

6. $x^2 + 5x - 50$

$= (x + 10)(x - 5)$

QUESTION THREE

1. $\frac{4x^2}{16x^5} = \frac{1}{4x^3}$

2. $6x^4 . 5x^3 = 30x^7$

3. $\frac{5x^2 - 20}{x + 2} = \frac{5(x^2 - 4)}{x + 2}$

$= \frac{5(x - 2)(x + 2)}{x + 2}$

$= 5(x - 2)$

4. $\frac{2x^2 . x^5}{2x^4} = \frac{2x^7}{2x^4}$

$= x^3$

5. $(2x^2)^3 = (2x^2)(2x^2)(2x^2)$

$= 8x^6$

6. $\frac{2x^2 - 10x - 28}{x + 2} = \frac{2(x^2 - 5x - 14)}{x + 2}$

$= \frac{2(x - 7)(x + 2)}{x + 2}$

$= 2x - 14$

Page 59 (cont)

QUESTION FOUR

1. $3y^5 \times 5y^n = 15y^{10}$

   Exponents: $5 + n = 10$, $n = 5$

2. $4(y^2)^n \times 3y^4 = 12y^{16}$

   $(y^2)^n \times y^4 = y^{16}$

   $(y^2)^n = y^{12}$ therefore $n = 6$

3. $\frac{8x^9}{4x^n} = 2x^4$ $9 - n = 4$,

   $n = 5$

QUESTION FIVE

$C = \$1.5 \times 8 + \$0.25 \times 160$
$C = \$12 + \$40$
$C = \$52$

Page 60

QUESTION SIX

1. $a = 1$, $x^4 = 256$

2. $3^4 = 81$, $3^5 = 243$, $3^6 = 729$

3. $P = 15^2 \times 10 = 2250$ watts

4. $1000 = x + 120 \times 3$
   $1000 - 360 = x$
   $x = 640$

QUESTION SEVEN

1. $x = -4$ or $x = 7$

2. $x = \frac{1}{3}$ or $x = -4$

3. $x = -\frac{1}{2}$ or $x = 5$

4. $x = 0$ or $x = 8$

5. $x = \frac{1}{2}$ or $x = 5$

6. $x = 5$

7. $3x^2 + 24x - 60 = 0$
   $3(x^2 + 8x - 20) = 0$
   $3(x + 10)(x - 2) = 0$
   $x = -10$, $x = 2$

8. $6x - 3 = 2x + 9$
   $4x = 12$, $x = 3$

9. $3x + 5 = x - 4$
   $2x = -9$
   $x = -4.5$

10. $12.5x = 100$
    $x = 8$

11. $4x = 27$ (found by cross multiplying)
    $x = 6\frac{3}{4}$ (6.75)

12. $15 = 4x + 5$ (found by cross multiplying)
    $10 = 4x$
    $x = 2\frac{1}{2}$ (2.5)

QUESTION EIGHT

1. $\frac{4a(a - 3b)}{4a \cdot 2a} = \frac{a - 3b}{2a}$

2. $\frac{2x(6y + x)}{2x \cdot 3x} = \frac{6y + x}{3x}$

3. $\frac{7x + 2x}{14} = \frac{9x}{14}$

4. $\frac{10a + 12a}{15} = \frac{22a}{15}$

5. $\frac{(x + 8)(x - 3)}{(x + 3)(x - 3)} = \frac{(x + 8)}{(x + 3)}$

6. $\frac{2(x^2 + 7x + 10)}{x + 2} = \frac{2(x + 5)(x + 2)}{x + 2}$

   $= 2(x + 5)$ or $2x + 10$

Page 61, CAT Practice 2

QUESTION ONE

1. $r^2 = \frac{A}{4\pi}$ and $r = \sqrt{\frac{A}{4\pi}}$

2. $2L = P - 2W$ and $L = \frac{P - 2W}{2}$

3. $T^2 = 4\pi^2 \times \frac{L}{g}$

   $T^2 = \frac{4\pi^2 L}{g}$

   $gT^2 = 4\pi^2 L$

   $L = \frac{gT^2}{4\pi^2}$

Page 63 QUESTION ONE (cont)

4. $y + 5 = 3x^2$

$x^2 = \frac{y + 5}{3}$

$x = \sqrt{\frac{y + 5}{3}}$

QUESTION TWO

1. $7x + 4x^2 + (5 - 2x)x = 32$

$7x + 4x^2 + 5x - 2x^2 = 32$

$12x + 2x^2 = 32$

$2x^2 + 12x - 32 = 0$

$2(x^2 + 6x - 16) = 0$

$2(x + 8)(x - 2) = 0$

$x = -8$ or $x = 2$

Therefore path width = 2 m

2. If the warehouse is square then each side can have a length of x.

$(x + 5)(x + 3) = 63$

$x^2 + 8x + 15 = 63$

$x^2 + 8x - 48 = 0$

$(x + 12)(x - 4) = 0$

$x = -12$ or $x = 4$

As the warehouse length cannot be negative the old sides were 4 m

Area of the original warehouse was 16 m²

Page 62

QUESTION THREE

1. $5 - 3x < 20$

$5 - 20 < 3x$

$-15 < 3x$

$3x > -15, \quad x > -5$

2. $10\,200 + 240x < 12\,000$

$240x < 1800$

$x < 7.5$

The greatest number of tickets he can purchase is 7.

3. $x + x + (x - 25) = 218$

$3x = 243$

$x = 81$

$81 - 25 = 56$ mm

isosceles triangle sides are 81, 81, 56 mm

Page 62 (cont)

4. $12.5x + 40 = 100$

$12.5x = 60$

$x = 4.8$

Therefore Cindy will have to work a minimum of 5 hours at Pac and Slave.

5. $n + (n + 1) = 265$

$2n = 264$

$n = 132$

Perlman is at pages 132 and 133

6. $S = 100(100 - 1) \div 2$

$S = 4950$

7. $\$9.40 \div 5 = 1.88$

Tweeter pays $3 \times 1.88 = \$5.64$

Toots pays $2 \times 1.88 = \$3.76$

8. $x + 2x + (x - 232) = 2400$

$4x = 2632$

$x = 658$

Buster get $\$658$

Todd gets $\$1316$

Cal gets $\$426$

Page 63

QUESTION ONE

1. $x = 9 - 2y$ substitute into other equation

$4(9 - 2y) + 3y = 16$

$36 - 8y + 3y = 16$

$-5y = -20$

$y = 4$

Using $y = 4$, $x = 9 - 2(4)$

$x = 1$

Double checking with other equation.

$4(1) + 3(4) = 16$

2. $3y - 8x = 30$

$-\underline{3y + 2x = 15}$

$-10x = 15$

$x = -1.5$

$3y - 8(-1.5) = 30$

$3y = 30 - 12$

$y = 6$

Double check with other equation

$3(6) + 2(-1.5) = 15$

Page 63 (cont)

3. Multiply the first equation by 2, rearrange the second then subtract.

$x + 6y = 4$

$\underline{x + 10y = -4}$

$-4y = 8$ therefore $y = -2$

$x + 6(-2) = 4$, therefore $x = 16$

Checking x and y with other equation:

$16 + 10(-2) = -4$

Therefore $x = 16$, $y = -2$

4. $4x + 5y = 25$

$x = 5 - y$

$4(5 - y) + 5y = 25$

$20 - 4y + 5y = 25$

$y = 5$

$x = 5 - 5,$

Therefore $x = 0$, $y = 5$

5. Multiply equation 2 by 4 and rearrange the equations

$3y - 5x = 38$

$y + 16x - 60 = -12$ or $y = 48 - 16x$

Substitute y into equation 1.

$3(48 - 16x) - 5x = 38$

$144 - 48x - 5x = 38$

$-53x = -106$

$x = 2$

Calculate x using $y = 48 - 16(2)$

$y = 16$

Checking with other equation

$3(16) - 5(2) = 38$

Therefore $x = 2$, $y = 16$

6. Multiply equation 1 by 2 and simplify

$2x + 2y = y - x$

$3x + y = 0$

Rearrange equation 1 then substitute into 2.

$y - 4 = x$, therefore $3(y - 4) + y = 0$

$3y - 12 + y = 0$

$4y - 12 = 0$

Therefore $y = 3$

If $y = 3$, $3x + 3 = 0$, making $x = -1$

Check with other equation $-2 + 6 = 3 - -1$

Therefore $x = -1$, $y = 3$

Page 63 (cont)

QUESTION TWO

$3P + 3W = 2100$

$P + W = 700$

$P = 700 - W$

$3.5P - 3.5W = 2100$

$3.5(700 - W) - 3.5W = 2100$

$2450 - 3.5W - 3.5W = 2100$

$2450 - 7W = 2100$

$-7W = -350$

$W = 50$

If $W = 50$, $3P + 3(50) = 2100$

$3P = 1950$

$P = 650$

Plane Speed = 650 km / h

Wind Speed = 50 km / hr

Page 64

QUESTION THREE

1. $(x + y)^2 = z^2 + 2xy$

$x^2 + 2xy + y^2 = z^2 + 2xy$

$x^2 + y^2 = z^2$ (Pythagorus Theorem)

2. If width = x, length = x + 12

Area = $x(x + 12)$

Add to this the area of the office

Area = $x(x + 12) + 60$

70% of the section area

$= 0.7 \times 25 \times 40$

= 700 (this is the total allowable area)

$x(x + 12) + 60 = 700$

$x^2 + 12x - 640 = 0$

$(x + 32)(x - 20) = 0$

Therefore maximum width = 20 m

and maximum length = 32 m

3. Old area = 96 cm²

New area = 192 cm²

$(x + 12)(x + 8) = 192$

$x^2 + 20x + 96 = 192$

$x^2 + 20x - 96 = 0$

$(x + 24)(x - 4) = 0$

$x = -24$ or $x = 4$

Increase L and W each by 4 cm

New size = 16 × 12 cm

Page 64 (cont)

4. $a^2 = 4x^2 - 56x + 196$

$a^2 = 4(x^2 - 14x + 49)$

$a^2 = 4(x - 7)^2$

$a = 2(x - 7)$

$a = 2x - 14$

Therefore values of a in the rectangle.

$a + 3 = 2x - 14 + 3$

$= 2x - 11$

$2x = 2(2x - 14)$

$= 4x - 28$

Area of rectangle

$= (2x - 11)(4x - 28)$

$= 8x^2 - 56x - 44x - 308$

$= 8x^2 - 100x - 308$

5. Length of rectangle = $6r$

Width = $2r$

Area = $12r^2$

Area of a circle = πr^2

Area of 3 circles = $3\pi r^2$

Shaded area $= 12r^2 - 3\pi r^2$

$= 3r^2(4 - \pi)$

Page 65, CAT Practice 4

QUESTION ONE

1\. a. $2x^2 - 6x + x - 3$

$= 2x^2 - 5x - 3$

b. $3y + 15 - 2y + 16$

$= y + 31$

c. $12 - 2x - 4$

$= 8 - 2x$

2\. a. $x^2 + 9x - 36 = (x + 12)(x - 3)$

b. $x^2 - 14x + 49 = (x - 7)^2$

3\. a. $8x^{12}$

b. $16y^6$

4\. a. $x = -4$

b. $x = 6$

5\. a. $\dfrac{32x + 15x}{24} = \dfrac{47x}{24}$

b. $\dfrac{(x - 9)(x + 9)}{2(x + 9)} = \dfrac{(x - 9)}{2}$

c. $3x^6$

Page 65 (Question One cont)

6\. $(10 \times 3 \times -4) \div (3 + -4)^2$

$= -120 \div 1$

$= -120$

QUESTION TWO

1\. a. $7x + 25 = 5 - x$

$8x = -20$

$x = -2.5$

b. $4y(y + 2) = 0$

$y = 0$ or $y - -2$

c. $3z^5 = 96$

$z^5 = 32$

$z = 2$

2\. $a^2 + 3a - 40 = (a + 8)(a - 5)$

3\. $\dfrac{a^2 + 3a - 40}{a^2 + 8a} = \dfrac{(a + 8)(a - 5)}{a(a + 8)}$

$= \dfrac{a - 5}{a}$

4\. $16a^4 \times a^k = 16a^8$, $k = 4$

5\. $(2a + 4)(a - 1) = 2a^2 + 2a - 4$

6\. $36 = h(15 + 9) \div 2$

$36 = 24h \div 2$

$36 = 12h$, therefore $h = 3$

7\. $(2a + 8)(a - 2) < 4a + 2$

$2a^2 + 4a - 16 < 4a + 2$

$2a^2 - 18 < 0$

$2(a^2 - 9) < 0$

$2(a - 3)(a + 3) < 0$

If $a = 3$ or $a = -3$ then equation = 0

therefore $x < 3$ and $x > -3$

8\. Path + pool length = $2x + 30$

Path + pool width = $2x + 10$

$(2x + 30)(2x + 10) = 800$

$4x^2 + 20x + 60x + 300 = 800$

$4x^2 + 80x - 500 = 0$

$4(x^2 + 20x - 125) = 0$

$4(x + 25)(x - 5) = 0$

$x = -25$ or $x = 5$

Path is 5 m wide.

Page 66

QUESTION THREE

1. $y = 250 - x$

$5x + 8(250 - x) = 1730$

$5x + 2000 - 8x = 1730$

$-3x = -270$

$x = 90$

$x + y = 250$,

therefore numbers sold:

x (large) = 90

y (small) = 160

2. a. let the consecutive numbers

$= a-2, a-1, a, a+1, a+2$

summing the numbers together

$= a-2 + a-1 + a + a+1 + a+2$

$= 5a$

b. consecutive numbers $= a, a+1, a+2$

Square and sum each.

$a^2 + (a+1)^2 + (a+2)^2$

$= a^2 + a^2 + 2a + 1 + a^2 + 4a + 4$

$= 3a^2 + 6a + 5$

Subtract 2, divide the result by 3

$= 3a^2 + 6a + 3$

$= a^2 + 2a + 1$

$= (a + 1)^2$

3. Sung's birth month equation is:

$4x + (12 - x) - 2(5 + x) = 10$

Simplifying gives $x + 2 = 10$ or $x = 8$

Kim's birth month is 8 (August)

Page 67, CAT Practice 5

QUESTION ONE

n	$2n^2 + 3n - 1$	Differences	
0	-1		
1	4	5	
2	13	9	4
3	26	13	4
4	43	17	4
5	64	19	4

The rule for the difference between any term is $D = 4n + 5$ where D is the difference between n and n+ 1

Page 67, (cont)

Proving this algebraically using $(n+1) - (n)$

$[2(n+1)^2 + 3(n+1) - 1] - 2n^2 + 3n - 1$

$= [2(n^2+2n+1) + 3n + 3 - 1] - [2n^2+3n - 1]$

$= [2n^2 + 4n + 2 + 3n + 2] - [2n^2 + 3n - 1]$

$= [2n^2 + 7n + 4] - [2n^2 + 3n - 1]$

$= 4n + 5$

QUESTION TWO

1. $0.5n(n + 1) = 171$

$0.5n^2 + 0.5n - 171 = 0$

$n^2 + n - 342 = 0$

$(n + 19)(n - 18) = 0$

Design (n) = 18

2. total squares − white squares = black

$[0.5n(n + 1)] - [0.5n^2 - 2.5n + 3] = 42$

$[0.5n^2 + 0.5n] - [0.5n^2 - 2.5n + 3] = 42$

$3n - 3 = 42$

Design (n) = 15

QUESTION THREE

= Area Front Square + Area Triangle

= (length)² + (½ × base. × height)

$= x^2 + \frac{1}{2} . x . \frac{1}{2}x$

$= x^2 + \frac{1}{4}x^2$

Cross Sectional Area $= \frac{5}{4} x^2$

Volume = Cross Sectional Area × Length

$= \frac{5}{4}x^2 \times 4x$

$40 = 5x^3$

$8 = x^3$, therefore size of $x = 2$ m

Page 68

QUESTION FOUR

Total the formulas for each tile type.

$(4n + 1) + (2n^2 - 2n) + (2n^2 + 2n)$

$= 4n + 1 + 4n^2$

$= 4n^2 + 4n + 1$

Compare this to $(2n + 1)^2$

$= (2n + 1)(2n + 1)$

$= 4n^2 + 4n + 1$

Page 68

QUESTION FIVE

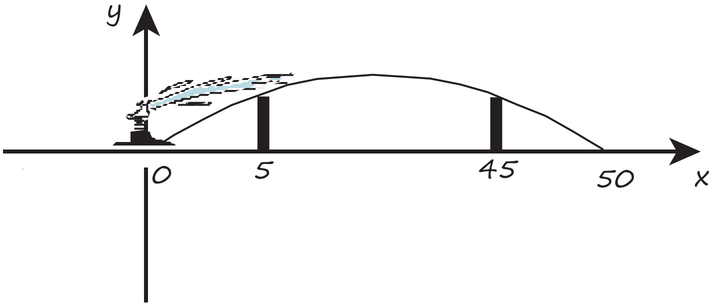
1. Because the graph is quadratic the highest point will be at the mid way point between where the water starts and finishes i.e. when $x = 25$ metres

Therefore $0.5(25) - 0.01(25)^2$

$= 12.5 - 0.01\ (625)$

$= 12.5 - 6.25$

$= 6.25$ metres high



2. $2.25 = 0.5x - 0.01x^2$

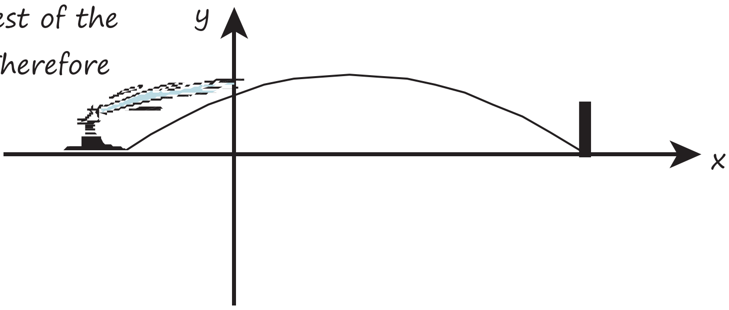
$0.01x^2 - 0.5x + 2.25 = 0$

$x^2 - 50x + 225 = 0$

$(x - 5)(x - 45) = 0$

$x = 5$ or $x = 45$

the fence is 45 metres (the furthest of the two factors) from the sprinkler. Therefore move it back 5 metres.



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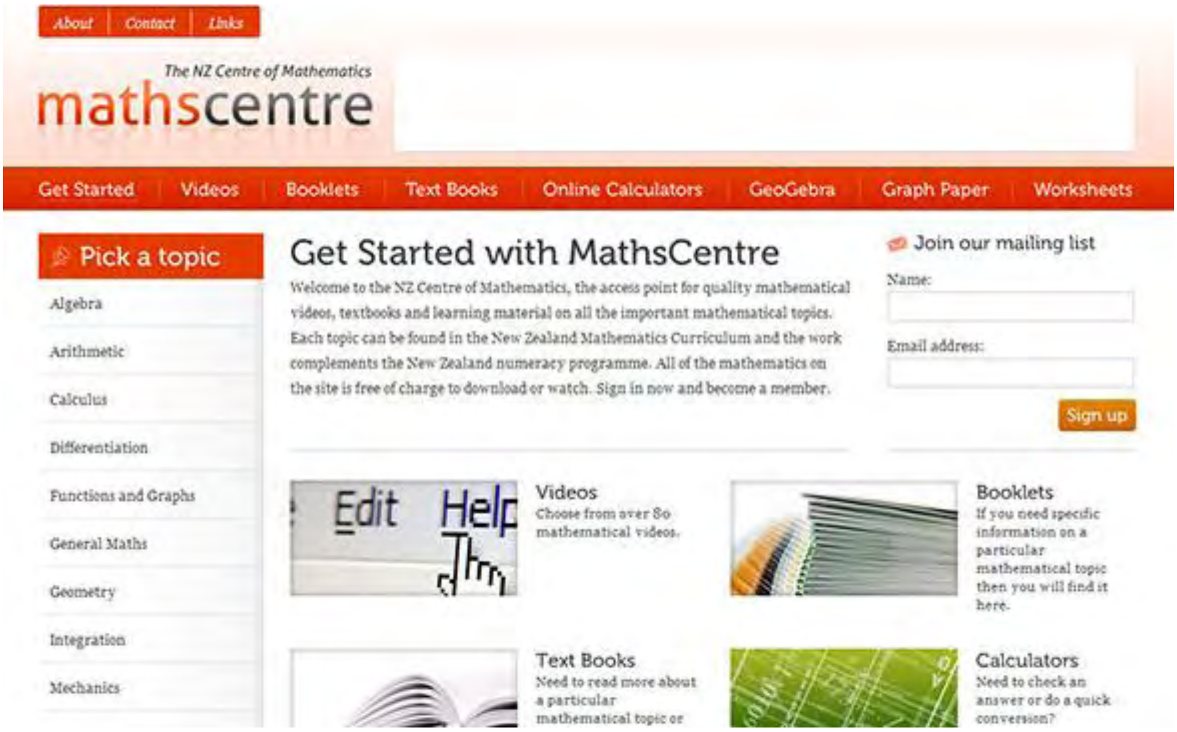

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Level 1 Mathematics and Statistics CAT, 2016

91027 Apply algebraic procedures in solving problems

Tuesday 13 September 2016
Credits: Four

You should attempt ALL the questions in this booklet.

Calculators may NOT be used.

Show ALL working.

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ASSESSOR'S USE ONLY	**Achievement Criteria**	
Achievement	**Achievement with Merit**	**Achievement with Excellence**
Apply algebraic procedures in solving problems.	Apply algebraic procedures, using relational thinking, in solving problems.	Apply algebraic procedures, using extended abstract thinking, in solving problems.
		Overall level of performance ☐

ASSESSOR'S USE ONLY

QUESTION ONE

(a) (i) The area of a rectangle is $x^2 - x - 2$.

If one side has length $x + 1$ metres, give the second side in terms of x.

(ii) What do you know about the value of x for this rectangle?

Explain your answer.

(b) Ranee has more money than Hone.

If Ranee gave Hone \$20, they would have the same amount.

If instead Hone gave Ranee \$22, Ranee would then have twice as much as Hone.

How much money does each person actually have?

(c) $A = 3(n^2 - 4n + 2) + n$

and $B = (2n + 1)(n - 6) + n^2 + 3$

Give an expression for A in terms of B.

(d) For what value(s) of x will $4 \times 2^x = 2^{6x + 3}$?

QUESTION TWO

(a) A parabola has the equation $y = 3x^2 - 2x + 5$.

What is the value of y when $x = 4$?

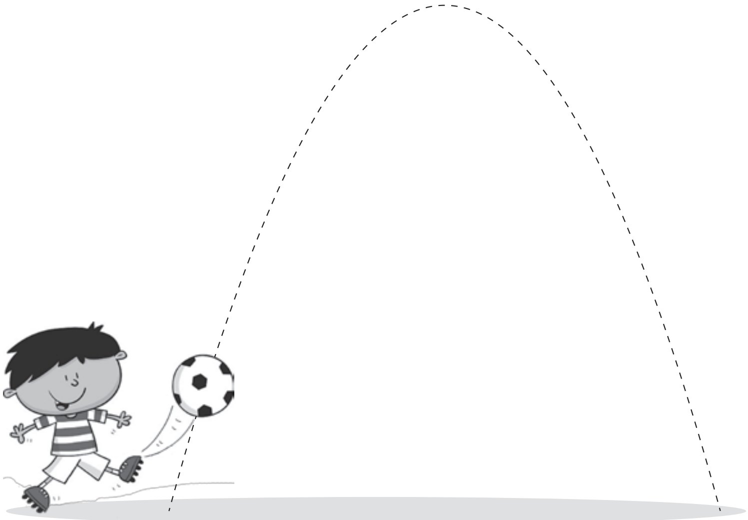
(b) For what values of x is $(x - 2)(x + 2) > (x - 2)(x + 3)$?

(c) If n is a whole number, for what values of n is $6 \times 2^{n+1} > 123$?

(d) Solve $x^2 + 2x - 8 = 0$.

(e) Solve $\dfrac{x^2+2x-8}{(x+2)(x-2)}=\dfrac{x}{2}$.

(f) Raj kicks a ball. The flight path of the ball can be modelled by $y=-(x^2-4x)$ where x and y are measured in metres.



(i) What does x measure?

(ii) For what percentage of the horizontal distance that the ball travels will it be 3 metres or more above the ground?

QUESTION THREE

(a) A rectangle has an area of $x^2 + 4x - 12$.

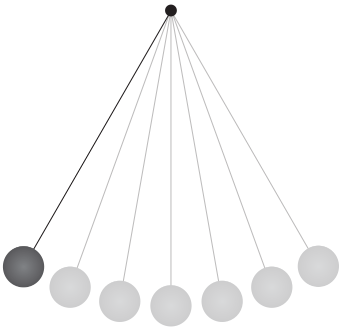
(i) What are the lengths of the sides in terms of x, for all values of x?

(ii) If the area of the rectangle is 128 cm^2, what is the value(s) of x?

(b) Brook knows that the time it takes for a pendulum to swing from one side to the other and back is given by the formula:

$$T = 2\pi\sqrt{\frac{L}{9.8}}$$

where L is the length of the string.



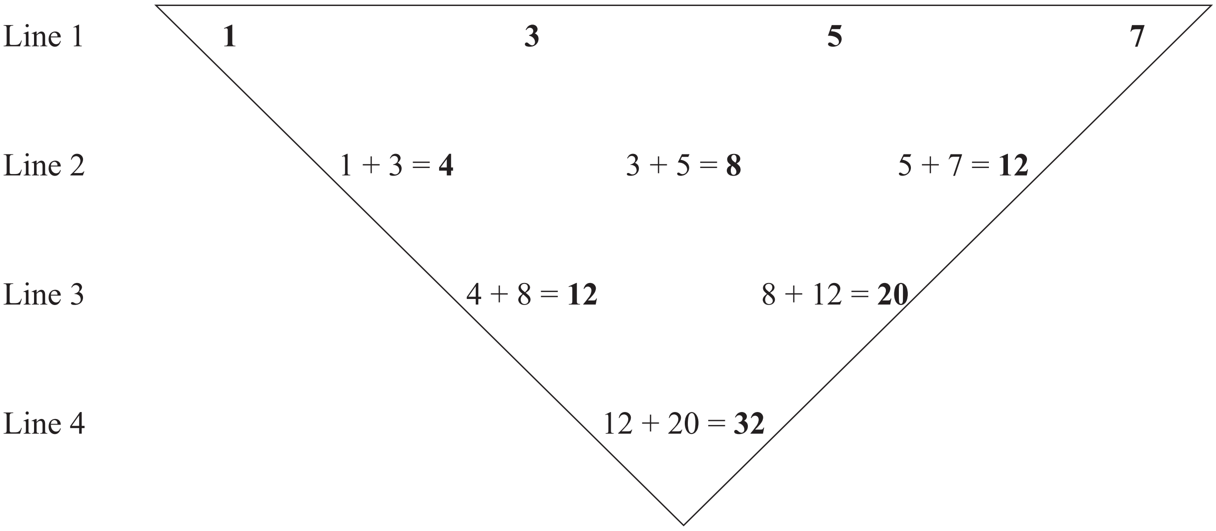
Write a formula that she could use to find the length of the string in terms of the time, T, taken for one swing.

(c) Show that $\frac{2}{x} + \frac{3+x}{5}$ is the same as $\frac{x^2+3x+10}{5x}$.

(d) Jason writes down 4 numbers: 1, 3, 5, and 7.

He adds the pairs of numbers to form a triangle, as shown below.

He stops when he gets to a single number at the bottom of the triangle.



(i) Investigate what happens when Jason changes the order of the numbers in Line 1.

Does he get the same answer in Line 4?

(ii) Find, using algebra, the relationship of the numbers in the first line to the numbers in the fourth line when he changes the order of the numbers in Line 1.

Explain your answer.

(iii) If Jason writes 4 consecutive numbers in order, what do you know about the numbers if the number at the bottom of the triangle is divisible by 3?

Explain your answer.

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THURSDAY

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Level 1 Mathematics and Statistics CAT, 2016

91027 Apply algebraic procedures in solving problems

Thursday 15 September 2016
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Achievement	**Achievement with Merit**	**Achievement with Excellence**
Apply algebraic procedures in solving problems.	Apply algebraic procedures, using relational thinking, in solving problems.	Apply algebraic procedures, using extended abstract thinking, in solving problems.
		Overall level of performance

QUESTION ONE

(a) (i) A rectangle has an area of $x^2 + 5x - 36$.

What are the lengths of the sides of the rectangle in terms of x?

(ii) If the area of the rectangle is 114 cm^2, what is the value(s) of x?

(b) Jake and Mele deliver newspapers.

Jake has more newspapers to deliver than Mele.

If Jake gave Mele 23 newspapers, they would have the same number of newspapers.

If, instead, Mele gave Jake 7 newspapers, Jake would then have twice as many as Mele.

How many newspapers does each person actually have?

(c) Show that $\frac{3}{2x}+\frac{x+4}{4}$ is the same as $\frac{2x^2+8x+12}{8x}$.

(d) For what value of x will $9 \times 3^x = 3^{5x+4}$?

QUESTION TWO

(a) A parabola has the equation $y = 3x^2 - 5x + 7$

What is the value of y when $x = 2$?

(b) For what values of x is $(x - 3)(x + 3) < (x - 4)(x + 2)$?

(c) If p is a whole number, for what values of p is $10 \times 2^{p-1} < 165$?

(d) $M = 5(a^2 - 3a + 4) + a^2$

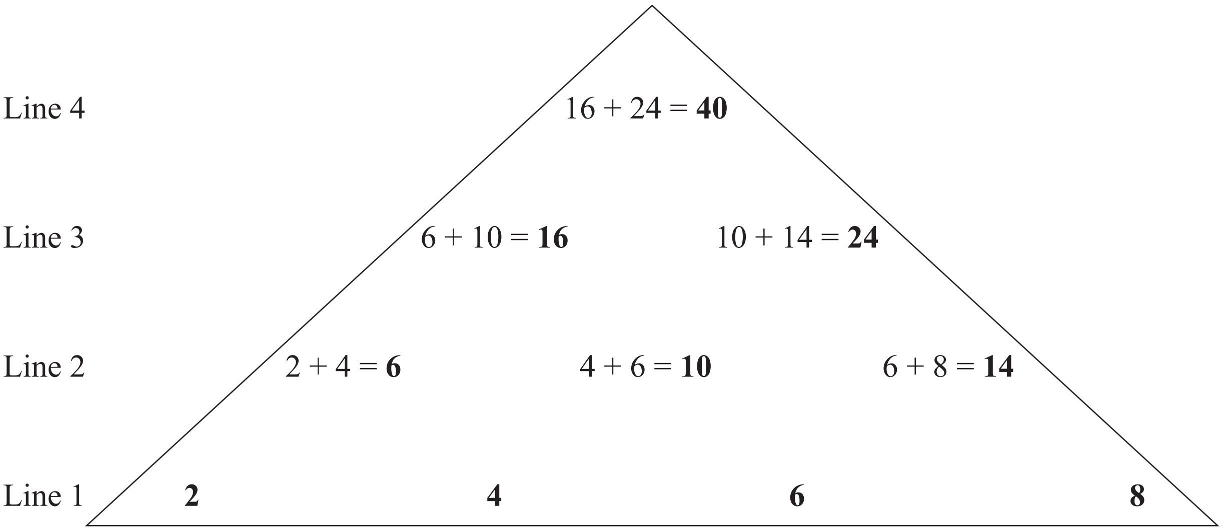
$N = (3a - 5)(2a - 4) + 7a$

Give an expression for *M* in terms of *N*.

(e) Janine writes down 4 numbers: 2, 4, 6, and 8.

She adds the pairs of numbers to form a triangle as shown below.

She stops when she gets to a single number at the top of the triangle.



(i) Investigate what happens when Janine changes the order of the numbers in Line 1.

Does she get the same answer as in Line 4?

(ii) Find, using algebra, the relationship of the numbers in the first line to the numbers in the fourth line when she changes the order of the numbers in Line 1.

Explain your answer.

(iii) If Janine writes 4 consecutive numbers in order, what do you know about the numbers if the number at the top of the triangle is divisible by 3?

Explain your answer.

QUESTION THREE

(a) (i) The area of a rectangle is $n^2 - 4n - 5$, where n is a positive number.

If one side is has length $n + 1$, give the second side in terms of n.

(ii) What do you know about the value of n for this rectangle?

(b) The area of a piece of a circular pizza is given by the formula $A = \frac{3}{4}\pi r^2$.

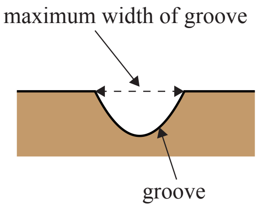
Write the formula that could be used to find the radius of the piece of this circular pizza.

(c) Solve $x^2 - 3x - 10 = 0$.

(d) Solve $\frac{x^2 - 3x - 10}{(x+5)(x-5)} = \frac{x}{2}$.

Question Three continues on the following page.

(e) A game has a groove that a small ball is rolled along.



http://offers.kd2.org/en/gb/lidl/pbaHo/

The groove can be modelled by

$y = x^2 - 4x$, where $0 \leq x \leq 4$, and x and y are measured in centimetres.

(i) What does y measure?

(ii) What percentage of the maximum horizontal width of the groove is the width of the groove when it's at a vertical depth of 3 cm?

Assessment Schedule – 2016

Mathematics and Statistics (CAT): Apply algebraic procedures in solving problems (91027A Day 1)

Assessment Criteria

Achievement	Merit	Excellence
Apply algebraic procedures in solving problems.	Apply algebraic procedures, using relational thinking, in solving problems.	Apply algebraic procedures, using extended abstract thinking, in solving problems.

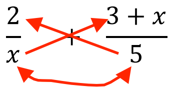
Evidence Statement

ONE	Evidence	Achievement (u)	Merit (r)	Excellence (t)
(a)(i)	$x - 2$ accept $(x + 1)(x - 2)$ even if they continue to solve for = 0	Correct factor.		
(ii)	• $x > 2$ • area cannot be negative (or 0) or the length of side(s) must be positive *ignore the missing 0*	Either bullet.	Both bullets.	
(b)	(If R is the amount Ranee has and H is the amount Hone has, $R > H$) – this statement not necessary $R - 20 = H + 20$ $R = H + 40$ and $2(H - 22) = R + 22$ $2H - 44 = H + 40 + 22$ $H = 106$ $R = 146$ OR Two incorrect equations as a result of a consistent error. eg. If they omit the subtraction of 20 and 22 $R = H + 20$ $R + 22 = 2H$ $H + 20 + 22 = 2H$ $H = 42$ $R = 62$ *(max grade r)* OR Two incorrect equations of similar difficulty to above that relate to the context. *(max grade r)*	At least one equation correct. OR Incorrect equations combined and simplified. OR Incorrect equations combined and simplified.	Amount of Hone or Ranee found with algebraic working. OR Consistent **solutions** with only 1 equation correct. OR Consistent **solutions** from incorrect equations related to the problem. OR Consistent **solutions** from incorrect equations related to the problem.	Correct solutions.
(c)	$A = 3n^2 - 12n + 6 + n$ $= 3n^2 - 11n + 6$ $B = 2n^2 + n - 12n - 6 + n^2 + 3$ $= 2n^2 - 11n - 6 + n^2 + 3$ $= 3n^2 - 11n - 3$ $A = B + 9$	A or B correctly expanded	A and B correctly expanded and simplified A in terms of B	Correct expression for A in terms of B.

			consistent with incorrectly simplified expressions for *A* and *B* as long as both expressions are still quadratics.	
(d)	$2^2 \times 2^x = 2^{6x+3}$ $x + 2 = 6x + 3$ $5x = -1$ $x = -\frac{1}{5}$ OR $2^2 = \frac{2^{6x+3}}{2^x}$ $2^{5x+3} = 2^2$ OR $2^x = \frac{2^{6x+3}}{2^2}$ $2^{6x+1} = 2^x$	Equation established with base 2.	Linear equation formed.	Equation solved from correct algebraic evidence.

TWO	**Evidence**	**Achievement (u)**	**Merit (r)**	**Excellence (t)**
(a)	45	y calculated. No alternative.		
(b)	$x^2 - 4 > x^2 + x - 6$ $-4 > x - 6$ $x < 2$ (or $2 > x$) *Accept with working as an equality and provided inequality inserted at the end.*	One correct expansion.	Both expansions correct and simplified. OR Solved as an equality. OR Consistent solving with 1 incorrect expansion.	Correct solution. Accept $–x > -2$ and ignore further incorrect working.
(c)	$2^{n+1} > \frac{123}{6}$ or $2^{n+1} > 20.5$ $2^4 = 16$ < 20.5 $2^5 = 32$ $n + 1 \geq 5$ or $n > 3$ or $n \geq 4$ Or $n = 4, 5, 6, \ldots\ldots$ OR $2{\times}2^{n+1} > \frac{123}{3}$ $2^{n+2} > 41$ etc	Inequality simplified. OR Correct trialling of at least one number (as the powers of 2 are well known). OR Inequality simplified. OR CAO.	Consistent solution from incorrect working OR Correct simplification leading to $n = 4$ or $n > 4$ OR Correct simplification and ignoring the +1 in finding the solution.	Correct simplification leading to correct inequation
(d)	$(x + 4)(x – 2) = 0$ $x = –4$ or $x = 2$	Factorised correctly *(evidence can come from 2 (e)).* OR Correct answers only. OR Consistently solved from $(x – 4)(x + 2) = 0$	Solved correctly.	

(e)	Either $\frac{(x+4)(x-2)}{(x+2)(x-2)} = \frac{x}{2}$ $\frac{(x+4)}{(x+2)} = \frac{x}{2}$ $2x + 8 = x^2 + 2x$ $x^2 = 8$ $x = \pm\sqrt{8}$ (± not required) or $x = \pm 2\sqrt{2}$ OR $2x^2 + 4x - 16 = x^3 - 4x$ $x^3 -2x^2 - 8x + 16 = 0$ which cannot be solved at NCEA Level 1. *(This method gains highest grade **r**)*	Correctly expanded.	Expression simplified. ($x \neq -2$ not required) to second line of evidence OR Simplified and = 0.	$x^2 = 8$ or $x = \pm\sqrt{8}$ or $x = \pm 2\sqrt{2}$ (± not required)
(f)(i)	The horizontal distance from the point where the ball was kicked.	Defines x in context.		
(f)(ii)	$3 = -x^2 + 4x$ $x^2 - 4x + 3 = 0$ $(x - 3)(x - 1) = 0$ Ball is 3 metres above the ground when $x = 3$ or 1 Therefore 3 m or more above the ground for 2 m. Intercepts are 0 and 4 Total horizontal distance = 4 m Percentage of horizontal distance above 3 m is 50%. May be given as a fraction or decimal.	Equates relationship to 3.	Solves equation.	Percentage calculated showing some working. Accept equivalent solution.

<table>
<tr><th>THREE</th><th>Evidence</th><th>Achievement (u)</th><th>Merit (r)</th><th>Excellence (t)</th></tr>
<tr><td>(a)(i)</td><td>A = $(x + 6)(x - 2)$
(So the sides are $x - 2$ and $x + 6$) – not required.</td><td>Factorised.
Ignore any solving.</td><td></td><td></td></tr>
<tr><td>(ii)</td><td>$x^2 + 4x - 12 = 128$
$x^2 + 4x - 140 = 0$
$(x + 14)(x - 10) = 0$
$x = -14, 10$
$x = 10$
CAO gains u</td><td>Equation rearranged to equal 0.
OR
$x^2 + 4x = 140$</td><td>Factorised and solved giving two correct solutions.</td><td>One positive solution only. This may come directly from factorised form without showing negative solution.</td></tr>
<tr><td>(b)</td><td>$\frac{T}{2\pi} = \sqrt{\frac{L}{9.8}}$
$L = 9.8\left(\frac{T}{2\pi}\right)^2$</td><td>Progress in rearrangement.</td><td>One error in the rearranged formula.
Square root must be rearranged to give squared.</td><td>Correct rearrangement.</td></tr>
<tr><td>(c)</td><td>LHS = $\frac{2\times5+x(3+x)}{5x}$
OR

$\frac{2}{x} + \frac{3+x}{5}$</td><td>Writing over a common denominator, showing some evidence of algebraic working, or lines.</td><td></td><td></td></tr>
<tr><td colspan="5">In part (d) of this question, three grades are to be allocated.
Up to 2t grades may be awarded across part (d) of this question for providing:
1. full explanation of the changing of the terminal numbers when the order of the starting numbers are changed.
2. full explanation of the terminal number being divisible by 3 when 4 consecutive numbers are used to form the triangle.
The rearranged triangles may occur on the original triangle.</td></tr>
<tr><td>(d)(i)</td><td>Numbers rearranged using 1, 3, 5, 7</td><td>Two rearranged triangles set up correctly.
OR
One rearrangement and correct statement of comparison consistent with their triangles.</td><td>At least two different triangles set up resulting in two different terminal numbers and they make a statement as to whether they are the same or different.</td><td></td></tr>
</table>

(d)(ii)	Triangle set up using letters for the initial row. $a \quad b \quad c \quad d$ $a + b \quad b + c \quad c + d$ $a + 2b + c \quad b + 2c + d$ $a + 3b + 3c + d$ 1. It stays the same, unless he swaps a with b or c and/or d with b or c. OR 1. If the outside numbers are swapped, or the middle numbers are swapped, the total does not change, but if one or both of the end numbers are swapped with one of the middle numbers, then the total changes. OR If less general example Involving algebraic terms in the first line e.g. x, $x + 2$, $x + 4$ and x +6	One general triangle correct. One triangle established	Incomplete explanation. Two triangles set up where the terminal expressions are different **and** partial explanation.	Explanation given in general terms.
(d)(iii)	$x \quad x + 1 \quad x + 2 \quad x + 3$ $2x + 1 \quad 2x + 3 \quad 2x + 5$ $4x + 4 \quad 4x + 8$ $8x + 12$ 2. Three is a factor of 12 hence if x is a multiple of 3 so must the total be. OR 2. Using the triangle from part ii) As the middle two numbers in the last line are multiples of 3, then the last line will be a multiple of 3 if the first and last numbers add to a multiple of 3.	Sets up line 1 of triangle algebraically for 4 consecutive numbers.	Partial explanation.	Full explanation.

Assessment Schedule – 2016

Day 2

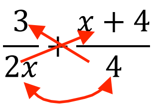
Mathematics and Statistics (CAT): Apply algebraic procedures in solving problems (91027B)

Assessment Criteria

Achievement	Merit	Excellence
Apply algebraic procedures in solving problems.	Apply algebraic procedures, using relational thinking, in solving problems.	Apply algebraic procedures, using extended abstract thinking, in solving problems.

Evidence Statement

ONE	Evidence	Achievement (u)	Merit (r)	Excellence (t)
(a)(i)	$(x + 9)$ and $(x – 4)$ (So the sides are $x + 9$ and $x – 4$) – not required	Factorised. Ignore any solving.		
(ii)	$x^2 + 5x – 36 = 114$ $x^2 + 5x – 150 = 0$ $(x + 15)(x – 10) = 0$ $x = 10, -15$ $x = 10$ CAO gains **u**	Equation rearranged to equal 0. OR $x^2 + 5x = 150$	Factorised and solved giving two correct solutions	One positive solution only. This may come directly from factorised form without showing negative solution.
(b)	(If J is the number of papers for Jake and M is the number for Mele. $J > M$) – this statement is not necessary $J – 23 = M + 23$ $J = M + 46$ and $J + 7 = 2(M – 7)$ $J + 7 = 2M – 14$ $J = 2M – 21$ $M + 46 = 2M – 21$ $M = 67$ $J = 113$ OR Two incorrect equations as a result of a consistent error. e.g If they omit the subtraction of 23 and 7 $J = M + 23$ $J +7 = 2M$ $M + 23 = 2M - 7$ $M = 30$ $J = 53$ *(max grade r)* OR Two incorrect equations of similar difficulty to above that relate to the context. *(max grade r)*	At least one equation correct. OR Incorrect equations combined and simplified. OR Incorrect equations combined and simplified.	Amount of Jake or Mele found with algebraic working. OR Consistent **solutions** with only 1 equation correct. OR Consistent **solutions** from incorrect equations related to the problem. OR Consistent **solutions** from incorrect equations related to the problem	Correct solution.

(c)	LHS = $\frac{3\times4+2x(x+4)}{4\times2x}$ OR  $\frac{3}{2x}+\frac{x+4}{4}$	Writing over a common denominator, showing some evidence of algebraic working, or lines.		
(d)	$9 \times 3^x = 3^{5x+4}$ $3^{x+2} = 3^{5x+4}$ $x + 2 = 5x + 4$ $4x = -2$ $x = -\frac{1}{2}$ OR $3^2 = \frac{3^{5x+4}}{3^x}$ $3^{4x+4} = 3^2$ OR $3^x = \frac{3^{5x+4}}{3^2}$ $3^x = 3^{5x+2}$	Equation established with base 3.	Linear equation formed.	Equation solved from correct algebraic evidence.

TWO	Evidence	Achievement (u)	Merit (r)	Excellence (t)
(a)	9	y calculated. No alternative.		
(b)	$x^2 - 9 < x^2 - 2x - 8$ $-9 < -2x - 8$ $-1 < -2x$ $-\frac{1}{2} < -x$ Or $x < \frac{1}{2}$ *Accept with working as an equality and provided inequality inserted at the end.*	One correct expansion.	Both expansions correct and simplified. OR Solved as an equality. OR Consistent solving with 1 incorrect expansion.	Solved to $-\frac{1}{2} < -x$ or equivalent Ignore further incorrect working.
(c)	$10 \times 2^{p-1} < 165$ $2^{p-1} < 16.5$ $2^4 = 16$ $2^5 = 32$ $p - 1 \leq 4$ $p \leq 5$ or $p < 6$ or $p = 0, 1, 2, 3, 4, 5$ OR $2 \times 2^{p-1} < 33$ $2^p < 33$ $p \leq 5$ or $p < 6$	Inequality simplified. OR Correct trialling of at least one number (as powers of 2 are well known). OR Inequality simplified. OR CAO	Consistent solution from incorrect working OR Correct simplification leading to $p = 5$ or $p < 5$ OR Correct simplification and ignoring the -1 in finding the solution.	Correct simplification leading to correct inequation.
(d)	$M = 5a^2 - 15a + 20 + a^2$ $= 6a^2 - 15a + 20$ $N = 6a^2 - 10a - 12a + 20 + 7a$ $= 6a^2 - 15a + 20$ $M = N$	M or N correctly expanded	M and N correctly expanded and simplified M in terms of N consistent with incorrectly simplified expressions for M and N as long as both expressions are still quadratics.	Correct expression for M in terms of N.

In part (e) of this question, three grades are to be allocated.
Up to 2t grades may be awarded across part (e) of this question for providing:
1. full explanation of the changing of the terminal numbers when the order of the starting numbers are changed.
2. full explanation of the terminal number being divisible by 3 when 4 consecutive numbers are used to form the triangle.
The rearranged triangles may occur on the original triangle.

(e)(i)	Numbers rearranged using 2, 4, 6, 8	Two rearranged triangles set up correctly. OR One rearrangement and correct statement of comparison consistent with their triangles.	At least two different triangles set up resulting in two different terminal numbers and they make a statement as to whether they are the same or different.	
(e)(ii)	Triangle set up using letters for the initial row. $a + 3b + 3c + d$ $a + 2b + c \quad b + 2c + d$ $a + b \quad b + c \quad c + d$ $a \quad b \quad c \quad d$ 1. It stays the same, unless he swaps a with b or c and/or d with b or c. OR 1. If the outside numbers are swapped, or the middle numbers are swapped, the total does not change, but if one or both of the end numbers are swapped with one of the middle numbers, then the total changes. OR If less general example Involving algebraic terms in the first line e.g. x, $x + 2$, $x + 4$ and $x + 6$	One general triangle correct. One triangle established	Incomplete explanation. Two triangles set up where the terminal expressions are different **and** partial explanation.	Explanation given in general terms.
(e)(iii)	$8x + 12$ $4x + 4 \quad 4x + 8$ $2x+1 \quad 2x + 3 \quad 2x + 5$ $x \quad x + 1 \quad x + 2 \quad x + 3$ 2. Three is a factor of 12 hence if x is a multiple of 3 so must the total be. OR 2. Using the triangle from part ii) As the middle two numbers in the last line are multiples of 3, then the last line will be a multiple of 3 if the first and last numbers add to a multiple of 3.	Sets up line 1 of triangle algebraically for 4 consecutive numbers.	Partial explanation.	Full explanation.

THREE	**Evidence**	**Achievement (u)**	**Merit (r)**	**Excellence (t)**
(a)(i)	$n – 5$ accept $(n + 1)(n - 5)$ even if they continue to solve for = 0	Correct factor.		
(ii)	• $n > 5$ • area cannot be negative (or 0) or the length of side(s) must be positive *ignore the missing 0*	Either bullet.	Both bullets.	
(b)	$\frac{4A}{3\pi} = r^2$ $r = \sqrt{\frac{4A}{3\pi}}$ ± not required in front of square root Accept $\sqrt{}(\frac{A}{0.75\pi})$ or $\sqrt{}(\frac{A}{3\pi/4})$	Progress in rearrangement.	One error in the rearranged formula. Square must be rearranged to give square root.	Correct rearrangement.
(c)	$(x – 5)(x + 2) = 0$ $x = 5$ or $–2$	Factorised *(evidence can come from 2 (d)).* OR Correct answers only. OR Consistently solved from $(x + 5)(x - 2) = 0$	Solved correctly.	
(d)	$\frac{(x-5)(x+2)}{(x+5)(x-5)} = \frac{x}{2}$ $\frac{(x+2)}{(x+5)} = \frac{x}{2}$ $2x + 4 = x^2 + 5x$ $x^2 + 3x - 4 = 0$ $(x+4)(x-1) = 0$ $x = -4$ or $x = 1$ OR $2x^2 - 3x - 10 = x^3 - 25x$ $x^3 - 2x^2 – 22x + 10 = 0$ which cannot be solved at NCEA Level 1. *(This method gains highest grade* ***r****)*	Correctly expanded.	Expression simplified. ($x \neq -5$ not required) to second line of evidence Or consistent solution from 2d OR Simplified and = 0.	Solution calculated.

(e)(i)	The depth of the groove.	Defines y in context.		
(ii)	$-3 = x^2 - 4x$ $x^2 - 4x + 3 = 0$ $(x - 3)(x - 1) = 0$ 3 cm below the maximum width of the groove, $x = 3$ or 1. This may be shown on diagram or written (1, 3) or shown on a table of points Therefore at this depth, the width of the groove = 2 cm. Intercepts are 0 and 4. Maximum width of the groove = 4 cm. At 3 cm deep, the width of the groove is 50% of the maximum width. May be given as a fraction.	Equates relationship to -3. If equates to 3 accept rearranged for **u** ie. $x^2 - 4x - 3 = 0$	Solves equation.	Percentage calculated showing some working. Accept equivalent solution.

To be completed by Candidate and School:

Name: ____________________

NSN No: ____________________

School Code: ____________________

1

SUPERVISOR'S USE ONLY

DAY 1
TUESDAY

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

Level 1 Mathematics and Statistics CAT, 2017

91027 Apply algebraic procedures in solving problems

Tuesday 19 September 2017
Credits: Four

You should attempt ALL the questions in this booklet.

Calculators may NOT be used.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

You are required to show algebraic working in this paper. 'Guess and check' and 'correct answer only' methods do not demonstrate relational thinking and will limit the grade for that part of the question to a maximum of Achievement. Guess and check and correct answer only may only be used a maximum of one time in the paper and will not be used as evidence of solving a problem.

A candidate cannot gain Achievement in this standard without solving at least one problem.

Answers must be given in their simplest algebraic form.

Where a question is given in words you will be expected to write an equation.

Check that this booklet has pages 2–8 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

ASSESSOR'S USE ONLY	**Achievement Criteria**	
Achievement	**Achievement with Merit**	**Achievement with Excellence**
Apply algebraic procedures in solving problems.	Apply algebraic procedures, using relational thinking, in solving problems.	Apply algebraic procedures, using extended abstract thinking, in solving problems.
		Overall level of performance ☐

QUESTION ONE

ASSESSOR'S USE ONLY

(a) The distance, d cm, travelled by an object is given by

$$d = ut + 3t^2$$

If $u = 3$ and $t = 5$, calculate the distance that the object has travelled.

(b) Solve $2x^2 – 3x – 9 = 0$

(c) If $6x - y = 21$ and $-x + 6y = 14$, what is the value of $x - y$?

(d) Solve $9 \times 3^{x-4} > 87$ when x is a whole number.

(e) Jane thinks of a number K.

When Jane's number is cubed, the answer is m times K.

When Jane's number is squared, it is n more than K plus 5.

Give an expression for n in terms of m only.

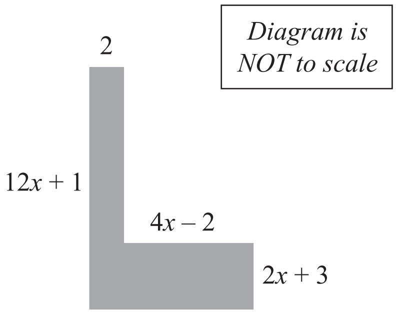
QUESTION TWO

(a) $h = 9 - 4x^2$

Give the equation for x in terms of h.

(b) Simplify $\dfrac{x^2 - 5x + 4}{5x^2 - 20x}$.

(c) An L-shaped model is to be made from the following sketch.



(i) What is the perimeter of the model in terms of x?

(ii) The area of the model is 92 cm^2.

What is the value of x?

(d) Leo is laying square concrete tiles for his deck.

He starts with laying them down to form a square pattern, but his friend thinks it would be better if they were laid out to form a rectangle.

He changes his layout to make the length of the deck 6 tiles longer, and the width of the deck 4 tiles shorter.

He finds he needs 2 extra tiles to complete the rectangular pattern.

How many tiles did he have to start with?

QUESTION THREE

(a) The area of a rectangle can be represented by

$3x^2 + 2x - 40$

(i) State the length and width of this rectangle in terms of x.

(ii) Given that this quadratic expression represents the area of a rectangle, what would be the possible values of x?

Justify your answer.

(b) $2^{3x+4} > 2^{x^2}$

Find the value(s) of x.

(c) Tane and Pete are raising funds for their sports trip.

Between them they need to raise \$1000.

There are only 5 weeks until they need the money.

Tane gets paid \$15 an hour, and Pete gets paid \$16 an hour as he has more experience.

Between them they work for a total of 13 hours each week.

What is the average number of hours that each of them work per week if they are to have exactly the amount of money they need?

(d) A and B are two consecutive odd numbers, where $B > A$.

If $C = \frac{B}{A} - \frac{A}{B}$, give the value of C in terms of A,

and explain why this will always be $\frac{\text{an even number}}{\text{an odd number}}$.

To be completed by Candidate and School:

Name: ____________________

NSN No: ____________________

School Code: ____________________

1

SUPERVISOR'S USE ONLY

DAY 2
THURSDAY

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

Level 1 Mathematics and Statistics CAT, 2017

91027 Apply algebraic procedures in solving problems

Thursday 21 September 2017
Credits: Four

You should attempt ALL the questions in this booklet.

Calculators may NOT be used.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

You are required to show algebraic working in this paper. 'Guess and check' and 'correct answer only' methods do not demonstrate relational thinking and will limit the grade for that part of the question to a maximum of Achievement. Guess and check and correct answer only may only be used a maximum of one time in the paper and will not be used as evidence of solving a problem.

A candidate cannot gain Achievement in this standard without solving at least one problem.

Answers must be given in their simplest algebraic form.

Where a question is given in words you will be expected to write an equation.

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YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

ASSESSOR'S USE ONLY

Achievement Criteria

Achievement	**Achievement with Merit**	**Achievement with Excellence**
Apply algebraic procedures in solving problems.	Apply algebraic procedures, using relational thinking, in solving problems.	Apply algebraic procedures, using extended abstract thinking, in solving problems.

Overall level of performance ☐

QUESTION ONE

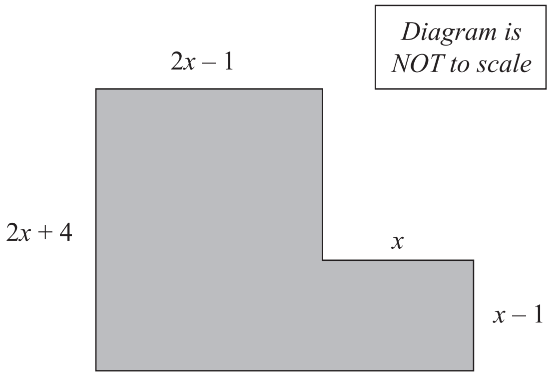
(a) The area, A m^2, to be concreted for a pathway and barbecue area is given by

$$A = xy + 5y^2$$

If $x = 2$, and $y = 4$, calculate the area to be concreted.

(b) Solve $3x^2 + 8x - 16 = 0$.

(c) A plan is made by joining two rectangles.



(i) What is the perimeter of the plan in terms of x?

(ii) The area of the plan is 146 cm^2.

What is the value of x?

(d) Riki thinks of a number N.

When Riki's number is squared, he gets k less than N plus 4 .

When Riki's number is cubed, the answer is m times N .

Give an expression for k in terms of m only.

QUESTION TWO

(a) The area of a rectangle can be represented by:

$3x^2 - 4x - 32$

(i) State the length and width of this rectangle in terms of x.

(ii) Given that this quadratic expression represents the area of a rectangle, what would be the possible values of x?

Justify your answer.

(b) If $x - 5y + 15 = 0$ and $-5x + y + 21 = 0$, what is the value of $x + y$?

(c) Jane is planning to fence an area for her pet lamb .

Jane's father tells her that he had planned to make it square with the sides of length *x*.

Jane decides to make it a rectangle with the length 5 metres longer than *x*, and the width 2 metres wider than *x*.

Jane's father says the area of Jane's pen is 24 m^2 larger than what he had planned to make.

What was the area of the pen that Jane's father had planned to make?

(d) Pita is going on holiday for 5 weeks.

He looks after pet cats and dogs when their owners go away.

While Pita goes on holiday, his neighbour is going to feed the 13 pets he is looking after.

Pita spends a total of $445 on the food for the pets before he leaves.

On average the cost for food for a week is $5 to feed one cat, and $9 to feed one dog.

How many cats and how many dogs did Pita have for the neighbour to feed?

QUESTION THREE

ASSESSOR'S USE ONLY

(a) $n = 9m^2 - 16$

Give the equation for m in terms of n.

(b) Simplify $\dfrac{6x^2 - 18x}{2x^2 - 7x + 3}$.

(c) $5^{x^2-6} > 5^x$

Find the value(s) of x.

(d) Solve $16 \times 4^{x-5} > 65$ when x is a whole number.

(e) A and B are two consecutive even numbers where $B > A$.

If $C = \frac{B}{A} - \frac{A}{B}$, give the value of C in terms of A,

and explain that this will always be $\frac{\text{an even number}}{\text{an even number}}$.