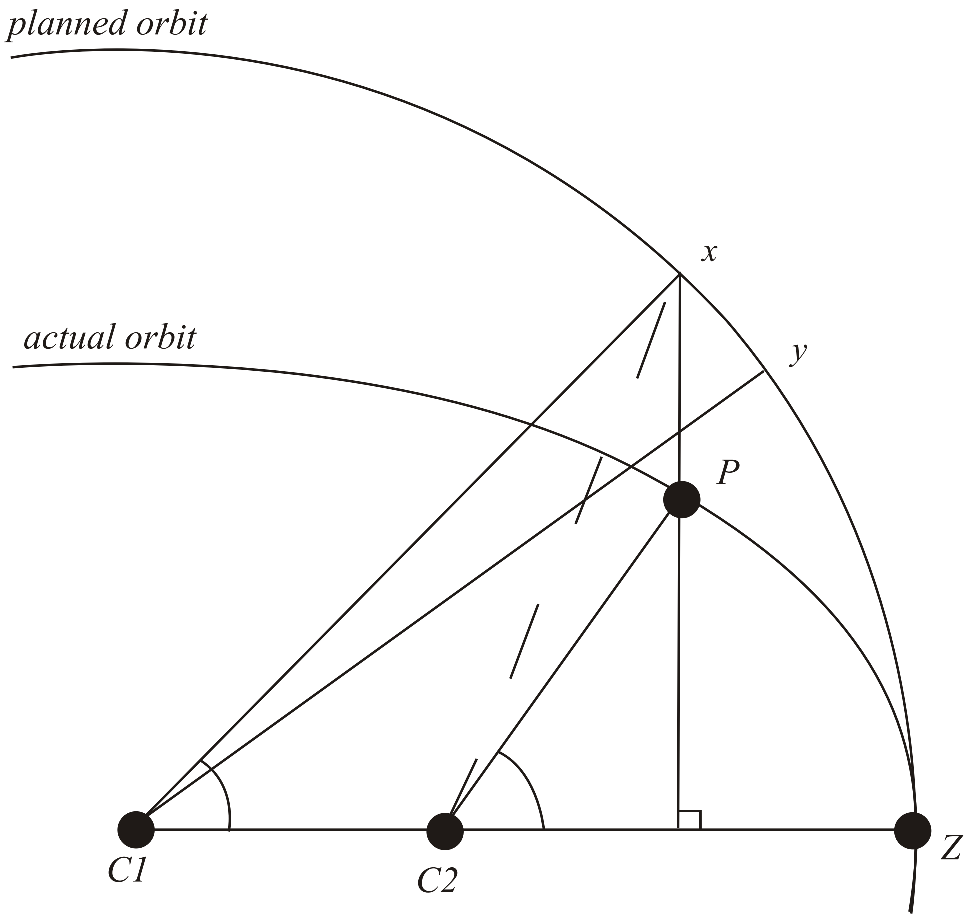
NCEA LEVEL 2 MATHEMATICS

2.14 - AS91269

Apply Systems of Equations in Solving Problems



Questions and Answers

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NCEA Level 2 Mathematics, Questions & Answers
AS91269 Apply Systems of Equations in Solving Problems
Kim Freeman

This edition is part of an eBook series designed to help you study towards NCEA.

Note: This achievement standard is assessed by each individual school and students can use any appropriate technology.

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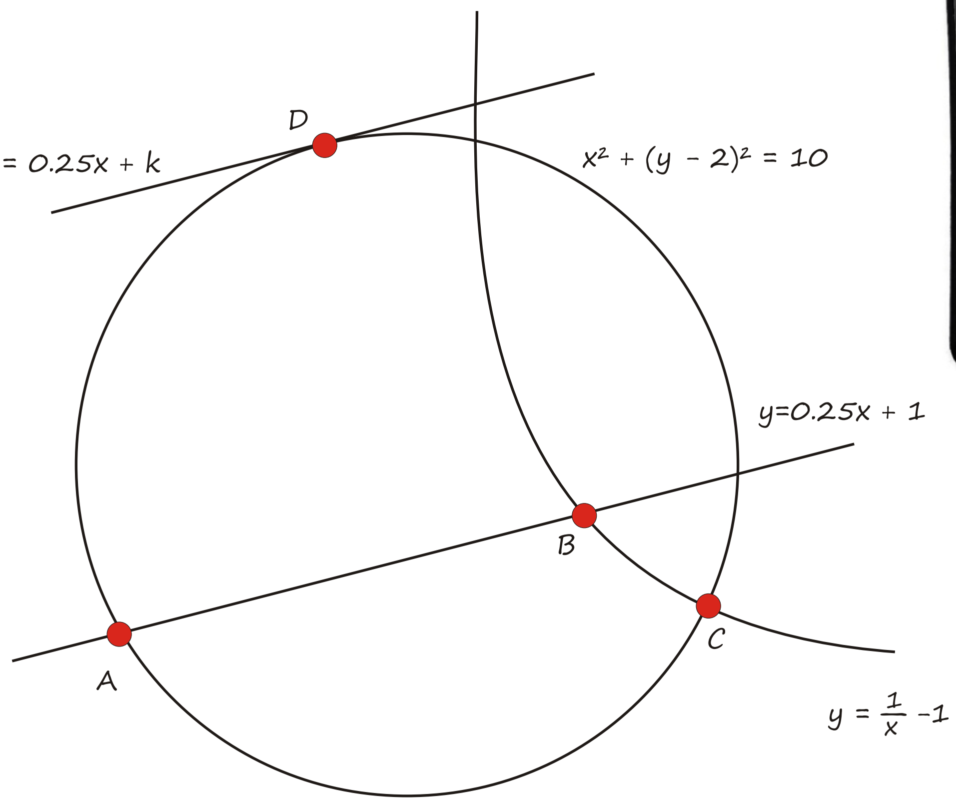
The DS-742ET

Find points **A**, **B**, **C** and **D**.

Find the value of k.



By the end of this booklet we hope you will have learnt to solve this problem.

The exercises were supplied by Kim Freeman. The booklet was funded by sales of the Mahobe DS-742ET calculator.

If you think you can handle a really powerful calculator then go ahead and buy a DS-742ET from Mahobe.

Systems of Equations

A "system" of equations is a collection of equations. Usually you have to find the solutions that they all have in common. Linear equations are ones that graph with straight lines however in this Achievement Standard you have to form and use a pair of equations - one of which is non linear.

A system of equations can have no solutions, 1 solution or lots of solutions. You should also be familiar with the quadratic formula and the "discriminant".
The discriminant tells the number of real roots (the number of x-intercepts) associated with a quadratic equation.

A quadratic equation has the form: $y = ax^2 + bx + c$
a cannot equal 0.

The quadratic formula (to find the roots) is: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Before using this formula make sure that $ax^2 + bx + c = 0$

The discriminant is: $b^2 - 4ac$.

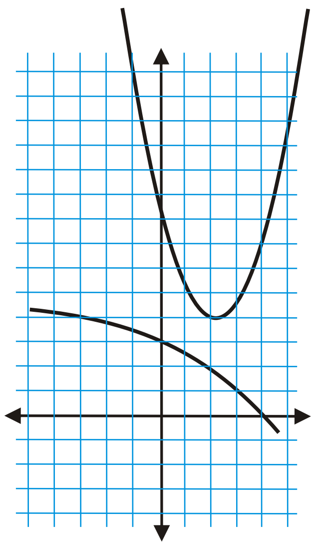
Sometimes you can combine a system of equations to form one quadratic equation. You can then factorise, use the quadratic formula, use a calculator or some graphing software to find the roots.

The roots of a quadratic equation are where the graph crosses the x axis but when you combine equations to form a quadratic, the roots are the common solutions to your equations.

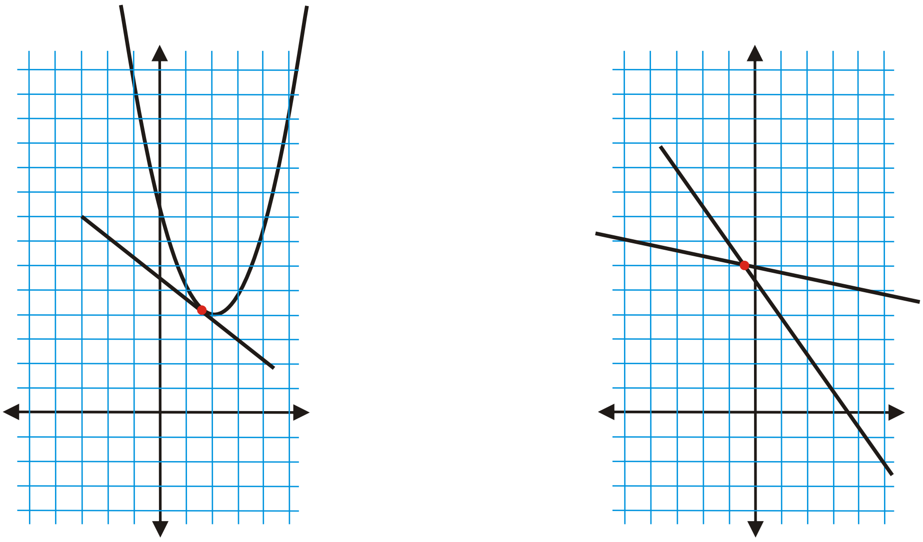
A system of equations can have: **no solutions.**
Note how the lines do not intersect.

If you are able to combine both equations to form a quadratic then the discriminant $b^2 - 4ac < 0$

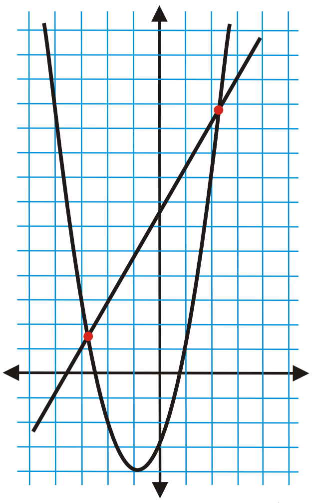
No solution: $b^2 - 4ac < 0$



A system of equations can have: **1 solution.** Note how the lines in the graphs below are either a tangent or only intersect at one point. If you are able to combine both equations to form a quadratic then the discriminant $b^2 - 4ac = 0$



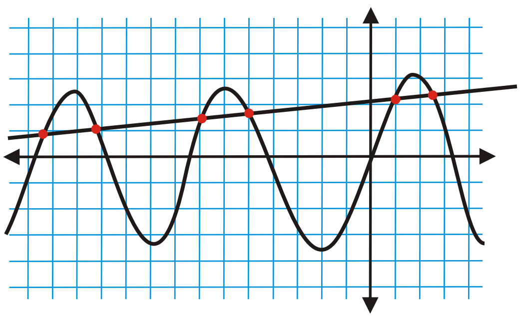
1 solution: $b^2 - 4ac = 0$



A system of equations can have **2 solutions.**
Note how the lines in the graph on the left intersect at two points.

If you are able to combine both equations to form a quadratic then the discriminant $b^2 - 4ac > 0$.

2 solutions: $b^2 - 4ac > 0$



A system of equations can **have more than 1 or 2** solutions but you are unlikely to meet these in this achievement standard.

Examples - Systems of Equations

1. Find the common solution to the system of equations: $y = x^2$
and $y = 8 - x^2$.

The solution is any point (or points) that is the same for both equations – i.e. where both their graphs intersect.

if $y = x^2$ and $y = 8 - x^2$ then $x^2 = 8 - x^2$

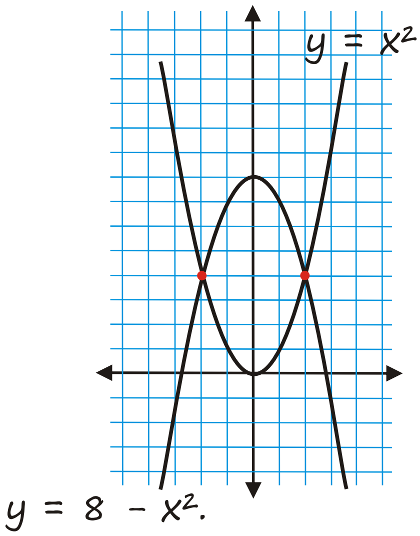
$2x^2 = 8$

$x^2 = 4$

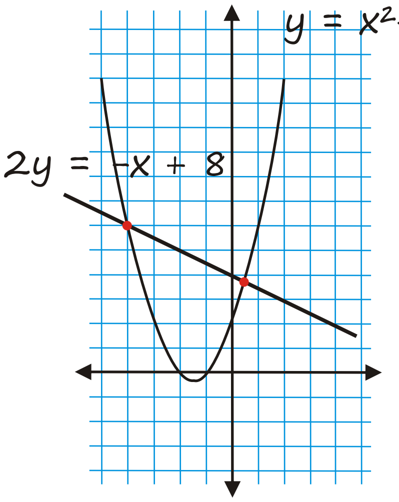
$x = +2$ or -2

If $x = +2$ and $y = x^2$ then $y = 4$
i.e. the first intersection co-ordinate is (2, 2)

If $x = -2$ then $y = x^2$ and $y = 4$
i.e. the other intersection co-ordinate is (–2, 2)



2. Find the common solution to the equations: $y = x^2 + 3x + 2$ and $2y = -x + 8$



After sketching the graph you can see that the solution appears to be (-4, 6) and (0.5, 3.75) but do not assume that this is the answer. Use algebra to prove it.

$$2y = 2x^2 + 6x + 4$$

$$2y = -x + 8$$

$$-x + 8 = 2x^2 + 6x + 4$$

$$2x^2 + 7x - 4 = 0$$

Using the quadratic formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-7 \pm \sqrt{7^2 - (4 \times 2 \times -4)}}{2 \times 2}$$

$$x = \frac{-7 \pm \sqrt{81}}{4}$$

$$x = \frac{-7 + 9}{4} \quad \text{or} \quad x = \frac{-7 - 9}{4}$$

$$x = \tfrac{1}{2} \text{ or } x = -4$$

Using the values in one of the other equations:

$2y = -x + 8$ (using $x = \frac{1}{2}$)	$2y = -x + 8$ (using $x = -4$)
$2y = -\frac{1}{2} + 8$	$2y = 4 + 8$
$2y = 7.5$	$2y = 12$
$y = 3.75$	$y = 6$

Therefore the intersection points are (0.5, 3.75) and (-4, 6).

With some questions you could just use a calculator such as the Mahobe DS-742ET to find the answer. Often questions will be written in such a way that you have to use algebra.

3. Show that $y = 2x^2 + 3x + 6$ and $y = x^2 + 2x + 2$ is an inconsistant system of equations.

$$2x^2 + 3x + 6 = x^2 + 2x + 2$$
$$x^2 + x + 4 = 0$$

Using $b^2 - 4ac$ »»» $= 1^2 - 4 \times 1 \times 4$

$= 1 - 16$

$= -15$

Because $b^2 - 4ac < 0$ there are no solutions.

If you did use the quadratic formula to calculate the answer you would get the following:

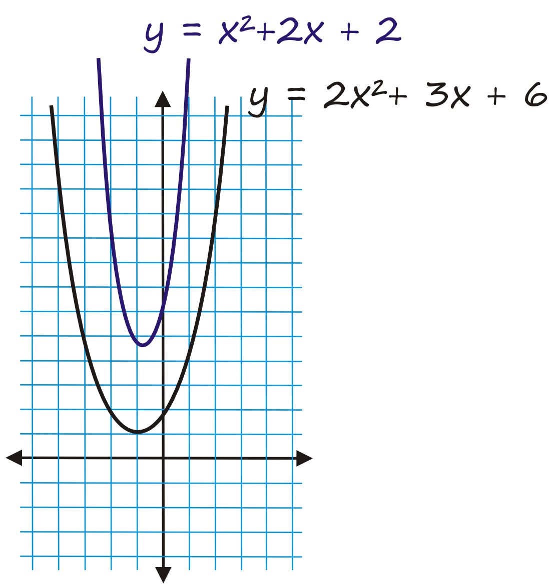
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-1 \pm \sqrt{1^2 - (4 \times 1 \times 4)}}{2 \times 1}$$

$$x = \frac{-1 \pm \sqrt{-15}}{2}$$

Notice the negative inside the square root. Using a calculator will give you a solution but it is a theoretical one using a value called i (an imaginary number).

On the right we have sketched the graphs of the equations.

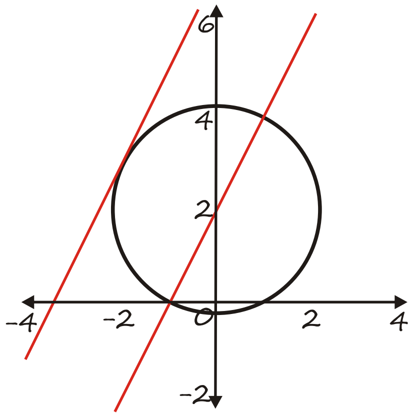


4. The graph below shows the following system of equations.

$y = x^2 + (y - 2)^2 = 5$

$y = 2x + 2$

and $y = 2x + 7$



a. At what points does the line $y = 2x + 2$ intersect with the circle $x^2 + (y - 2)^2 = 5$?

If $y = x^2 + (y - 2)^2 = 5$ and $y = 2x + 2$

then $x^2 + (2x + 2 - 2)^2 = 5$

$x^2 + (2x)^2 = 5$

$5x^2 = 5$

$x = \pm 1$

Using $y = 2x + 2$ and $x = \pm 1$, coordinates are $(-1, 0)$ and $(1, 4)$

b. The line $y = 2x + 7$ appears to be a tangent to the circle $x^2 + (y - 2)^2 = 5$. Confirm whether it is a tangent or not.

$x^2 + (2x + 7 - 2)^2 = 5$

$x^2 + (2x + 5)^2 = 5$

$x^2 + 4x^2 + 20x + 25 = 5$

$5x^2 + 20x + 20 = 0$

Using $b^2 - 4ac = 0$ for a tangent (1 solution)

$20^2 - (4 \times 5 \times 20)$

$400 - 400 = 0$

Therefore **yes** it is a tangent – the actual point is $(-2, 3)$

5. Solve the system of equations: $y^2 = 13 - x^2$ and $x^2 + 4y^2 = 25$

Rewrite the first equation: $y^2 + x^2 = 13$

Subtract the second equation: $\underline{4y^2 + x^2 = 25}$

$-3y^2 = -12$

$y = \pm 2$

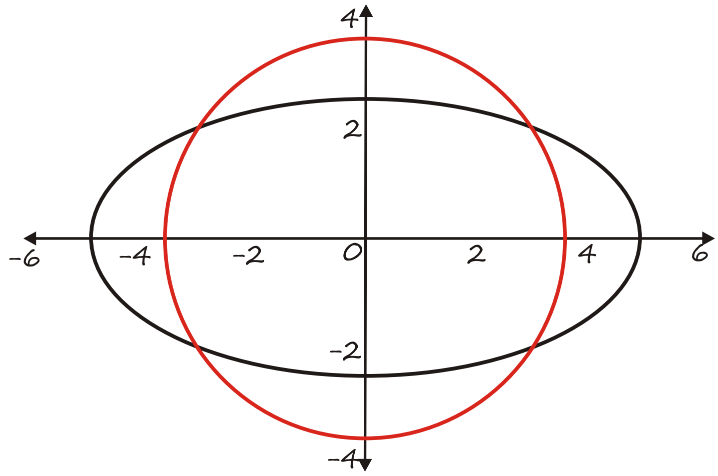
Substitute 2 and -2 into one of the original equations:

$y^2 = 13 - x^2$

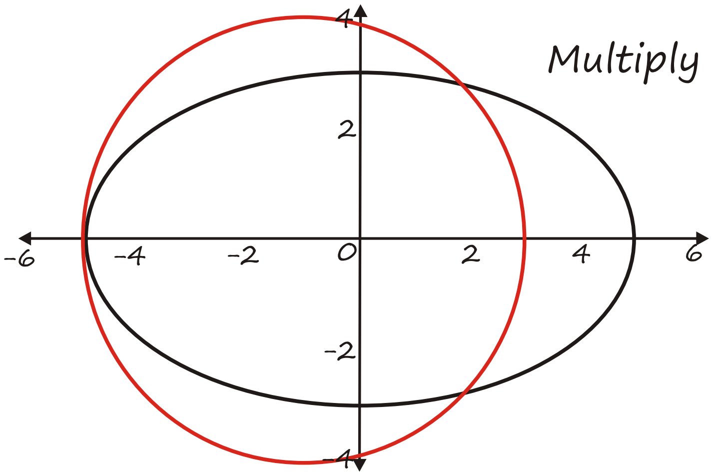
$(2)^2 = 13 - x^2 \qquad (-2)^2 = 13 - x^2$

$x = \pm 3 \qquad x = \pm 3$

The solutions are (3, 2),(-3, 2), (-3, -2) and (3, -2)



6. Solve the following system of equations: $9x^2 + 25y^2 = 225$
and $x^2 + y^2 + 2x = 15$



Multiply equation 2 by -25 and add

$9x^2 + 25y^2 = 225$

$\underline{-25x^2 - 25y^2 - 50x = -375}$

$-16x^2 - 50x = -150$

Using the quadratic formula or a Mahobe DS-742ET

$x = \frac{15}{8}$ or -5

Substituting these values back into one of the original equations the solutions are $(-5, 0)$, $(\frac{15}{8}, 2.781)$, $(\frac{15}{8}, -2.781)$

Exercises

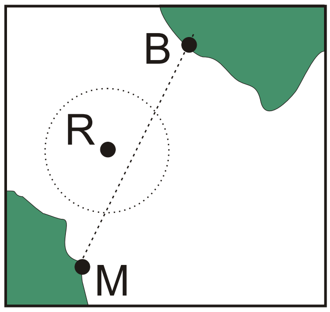
1. Sketch the graph of a circle $x^2 + y^2 = 17$ and a line $y = x + 3$
Calculate the intersection points.

2. Sketch the graph of the circle $x^2 + y^2 = 20$ and the parabola $y = x^2 + 3.5x$.
Calculate the intersection points.

3. Investigate the relationship between the straight line $x + y = 2$ and the hyperbola $xy = 1$

4. Find all the intersection points of the circle $x^2 + (y - 3)^2 = 9$ and the parabola $y = x^2$

5. Sketch the graph of $x^2 + y^2 = 22$ and $x^2 - 2y^2 = 4$.
Calculate any intersection points.



6. A ferry is sailing from Sandy Beach (B) to Port Mahobe (M). At point R there are rocks which have a marker and a locator beacon attached.

Point R has co-ordinates (0, 0) and the ferry follows the path represented by the equation $y = 2x - 8$. The locator beacon can be detected by points with the area described by the equation $x^2 + y^2 = 85$. The ferry uses its radar to pick up the signal of the locator beacon on the rocks. At what position is the ferry first able to detect the locator beacon?

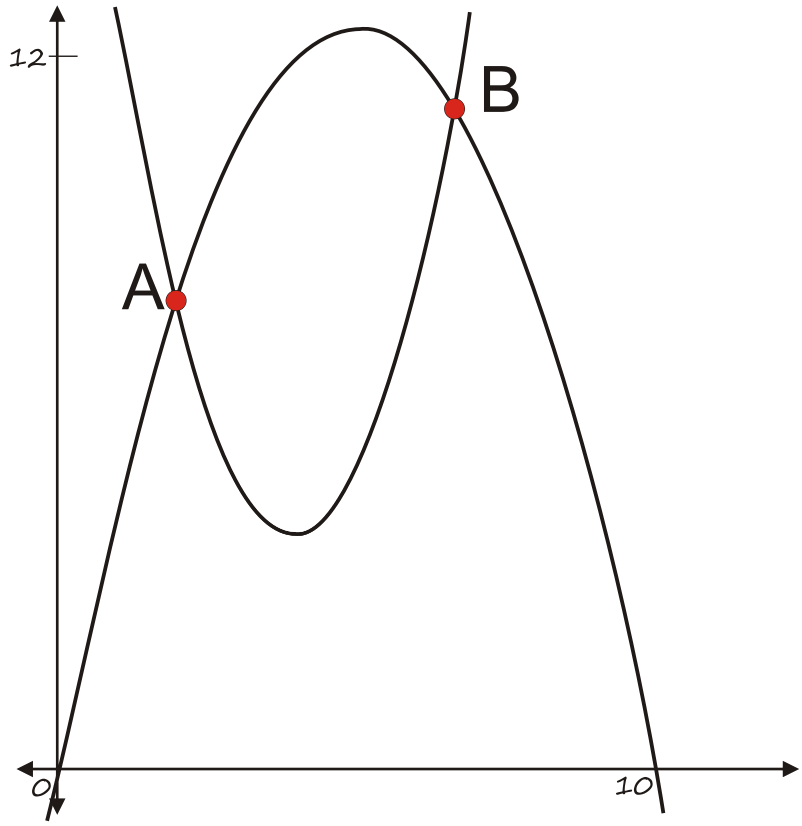
7. The cross section of a tunnel that holds power cables is a semicircle. This shape is represented by the top half of the circle modeled by $x^2 + y^2 = 12$.
Each unit in the x and y values represents 1 metre.
Maintenance men have to periodically change light bulbs near the top of the tunnel. They reach these by climbing a ladder. The equation that best models the ladder position is $y = 3x - 4$.

a. How wide is the tunnel at ground level?

b. If the ladder is placed against the tunnel wall what will be the shortest distance of the ladder from the tunnel wall?

c. Find the solution to the system of equations $x^2 + y^2 = 12$ and $y = 3x - 4$ and explain what the solutions tell you.

8. The graph below shows the parabolas formed by: $y = \frac{1}{2}x(10 - x)$ and $y = (x - 4)^2 + 4$. Calculate the intersection points A and B.



9. Find the solution to the system of equations $x(x + 2) + y(y - 3) = 3$ and $2x(x + 2) - y(y - 18) = 54$. What does the solution tell you about the relationship between the two equations?

10. **a.** Calculate where the line $y = 2x - 2$ intersects the circle $x^2 + y^2 = 25$.

A new line that is parallel to $y = 2x - 2$ is drawn. The new line is a tangent to the circle $x^2 + y^2 = 25$ and has the equation $y = 2x + c$

b. Calculate the value of c and therefore the equation of the new line.

c. Use your equation to find the tangent point(s).

Systems of Equations - Excellence Examples

a. Calculate where the line $y = x + 1$ intersects the circle $x^2 + y^2 = 5$.

$x^2 + y^2 = 5$ and $y = x + 1$ therefore $x^2 + (x + 1)^2 = 5$

$x^2 + x^2 + 2x + 1 = 5$

$2x^2 + 2x - 4 = 0$

$2(x^2 + x - 2) = 0$

$2(x + 2)(x - 1) = 0$

$x = -2, x = 1$

Using the equation $y = x + 1$ then $x = -2, y = -1$

and $x = 1, y = 2$

Intersection points are $(-2, -1)$ and $(1, 2)$.

A new line that is parallel to $y = x + 1$ is drawn. The new line is a tangent to the circle $x^2 + y^2 = 5$ and has the equation $y = x + k$.

b. Calculate the value of k and therefore the equation of the new line.

If the new line is $y = x + c$ then $x^2 + (x + k)^2 = 5$

$x^2 + x^2 + 2xk + k^2 = 5$

$2x^2 + 2xk + k^2 - 5 = 0$

Using $b^2 - 4ac = 0$ for a tangent and $a = 2$, $b = 2k$, $c = k^2 - 5$

$4k^2 - [4 \times 2 \times (k^2 - 5)] = 0$

$4k^2 - 8k^2 + 40 = 0$

$-4k^2 = -40$

$k^2 = 10$

$k = \pm\sqrt{10}$

This means the equation of the tangent is either:

$y = x + \sqrt{10}$ or $y = x - \sqrt{10}$

Exercises

1. During November SIM Traders collected \$2700 from the sale of telephone memory SIM cards. During December they lowered the price by \$15 per SIM card. They managed to sell 30 more SIM cards and took in a total of \$3375 for the month.

a. Write a system of equations to model this situation.

b. Find the cost of the SIM cards during each month.

2. Find the value of k so that the line $x + 3y = k$ is a tangent to $x = 2y^2$.

3. A person using a cell phone can be located in relation to three towers. Using a coordinate grid system where each unit represents 1 km a caller is determined to be 50 km from a tower at the origin, 40 km from a tower located at (0, 30) and 13 km from a tower at (35, 18).
Give the co-ordinates of the caller. Hint: the system of equations are:
$x^2 + y^2 = 50^2$, $x^2 + (y - 30)^2 = 40^2$ and $(x - 35)^2 + (y - 18)^2 = 13^2$

4. The orbit of the planet Pluto can be modeled approximately by the equation:

$$\frac{x^2}{40^2} + \frac{y^2}{39^2} = 1$$

Astronomers discover a comet that they calculate is following a path modeled by the equation $x = 0.01y^2 + 35$. The values of x and y can be measured in astronomical units. Is it possible that the two will collide?

5. Mahobe releases two weather satellites into space. The equations of the orbits of the satellites are: $\frac{x^2}{300^2} + \frac{y^2}{900^2} = 1$ and $\frac{x^2}{600^2} + \frac{y^2}{690^2} = 1$

The distances are measured in kilometres and the centre of the earth is the centre of each curve. Compare the orbits of each satellite by graphing the equations and calculating any intersection points.

6. Investigate the following system of equations:

$$x^2 + y^2 = k^2 \quad \text{and} \quad \frac{x^2}{9} + \frac{y^2}{4} = 1$$

Find the values of k for which the system has no solutions, one solution and two or more solutions.

7. Find the value of k so that the graphs of $x = 2y^2$ and $-x + 3y = k$ are tangent to each other.

8. Solve the following systems of equations:

a. $x^2 + y = 0$
$2x - y = 3$

b. $x + y = 9$
$xy = 20$

c. $x^2 + y^2 = 25$
$x^2 - y = 5$

d. $(3 \div x) + (2 \div y) = 10$
$(1 \div x) - (1 \div y) = -10$

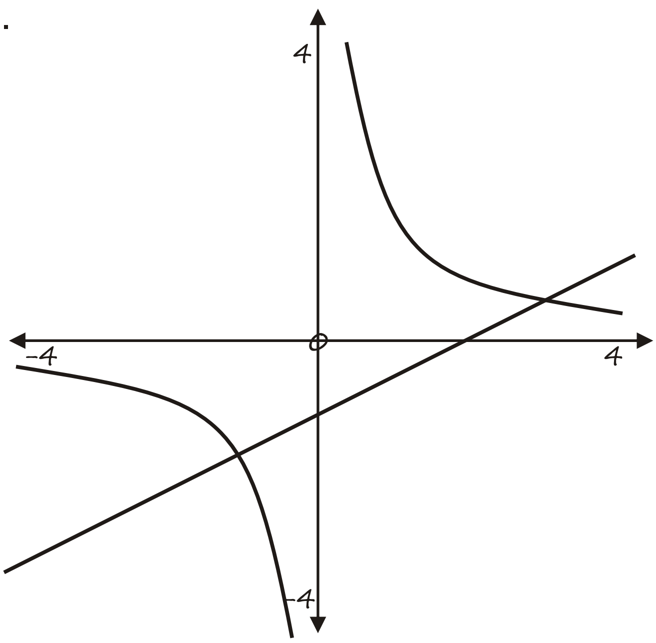
e. $x^2 + y^2 = 8$
$xy = 4$

f. $(y - 1)^2 = 4 + x$
$x + y = -1$

9. The equation $y = (x + 4)^2$ represents a parabola.
The straight line $y = 5x + 14$ intersects the parabola.
Find the point(s) of intersection of the line and the parabola.

10. The graph below shows the hyperbola: $y = \frac{4}{x}$
and the straight line: $y - 0.5x + 2 = 0$

a. Find the points of intersection where the straight line meets the hyperbola.



b. Consider the function $y = \frac{4}{x}$ and $y = mx + 2$.

Investigate the values of m that provide 0, 1 or 2 points of intersection. Illustrate your investigation results with diagrams.

Systems of Equations - Excellence Example

A circle has the equation $x^2 + y^2 = 2500$.

A straight line with the equation $y = kx - 150$ makes a tangent to the circle.

Find the value of k and the coordinates of the tangent.

$x^2 + y^2 = 2500$ and $y = kx - 150$

therefore $x^2 + (kx - 150)^2 = 2500$

$$x^2 + k^2x^2 - 300kx + 22\ 500 = 2500$$

$$(1 + k^2)x^2 - 300kx + 20\ 000 = 0$$

Using $b^2 - 4ac = 0$, $a = (1 + k^2)$, $b = -300k$, $c = 20\ 000$

$$(-300k)^2 - [4 \times (1 + k^2) \times 20\ 000) = 0$$

$$90\ 000k^2 - [80\ 000 \times (1 + k^2)] = 0$$

$$90\ 000k^2 - 80\ 000 - 80\ 000k^2 = 0$$

$$10\ 000k^2 - 80\ 000 = 0$$

$$10\ 000k^2 = 80\ 000$$

$$k^2 = 8$$

$$k = \pm\sqrt{8}$$

This means the equation of the tangent is either:

$y = +\sqrt{8}x - 150$ or $y = -\sqrt{8}x - 150$

Using a DS-742ET calculator (and confirming it on DESMOS) we further calculated the intersection points as:

(47.14, -16.67) for the positive gradient

and (-47.14, -16.67) for the negative gradient.

The DS-742ET

Mahobe have added some amazing technology into their new eTool advanced scientific calculator.

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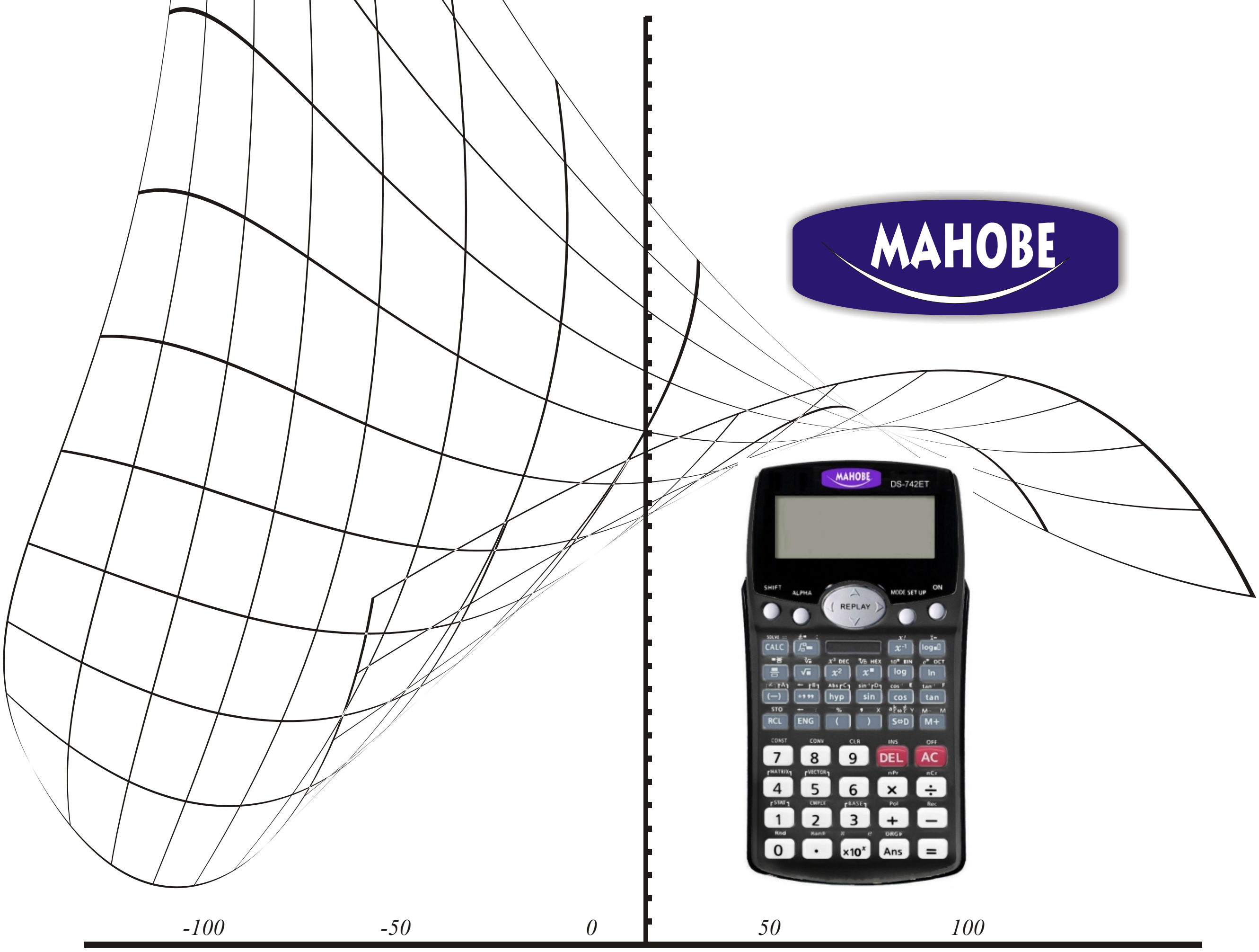

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Exercises

1. **a.** A half circle can be modeled by $y = -\sqrt{25 - x^2}$. The line $y = 2x - 2$ intersects the half circle. Find any points of intersection.

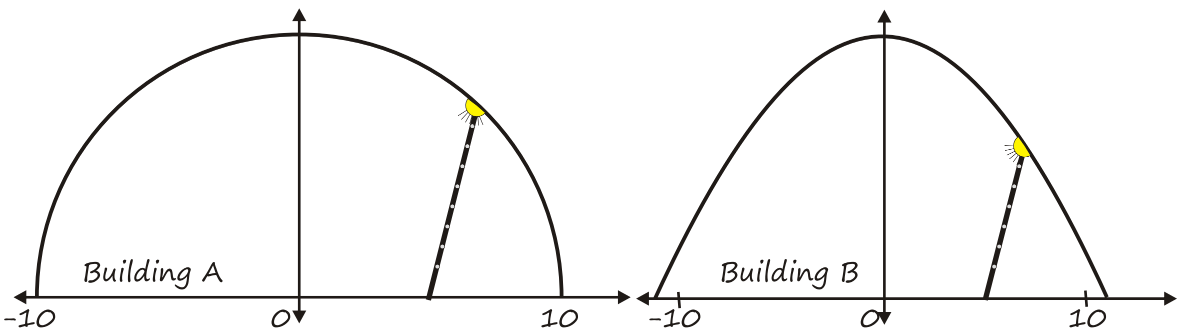
b. Find the equation of the line parallel to $y = 2x - 2$ that makes a tangent to the half circle $y = -\sqrt{25 - x^2}$

2. **a.** A circle can be modeled by $x^2 + (y - 2)^2 = 4$
The line $y = x + 2$ intersects the circle. Find the points of intersection.

b. Find the equation of the line parallel to $y = x + 2$ that makes a tangent to the circle $x^2 + (y - 2)^2 = 4$.

3. The equation $y = \frac{1}{4}x(12 - x)$ has a tangent $y = -x + k$.
Find the value of k.

4. This question involves an electrician changing light bulbs in two buildings.
Building A has a semicircular shape that can be modeled by $x^2 + y^2 = 100$
Building B has a parabolic shape that can be modeled by $x^2 + 12y - 120 = 0$
In both cases $y > 0$.
Both interior and exterior lights are positioned at the same height.
When changing the interior light bulbs the top of each ladder just reaches the roof when positioned at $y = 4x - 20$.
The diagram below shows the two scenarios.

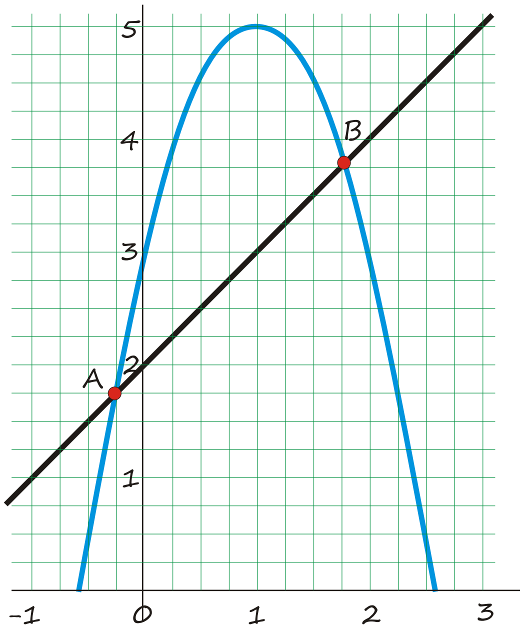


a. How much higher are the lights in Building A compared to the lights in Building B?

b. When changing the outside lights of the buildings, the safest gradient for the ladder is determined to be -1.5. Calculate the co-ordinates of the base of the ladder (assume that ground level is at $y = 0$) for both Building A and Building B and comment.

5. Astronomers are tracking a large asteroid and predict that it may strike the earth with disastrous results. They draw a grid pattern and plot the gravitational field around the earth. If the asteroid's path falls within this field there is a major possibility it will be then pulled towards earth.
On their grid, the gravitational field is modeled by the equation $x^2 + y^2 = 40$.
The asteroid is modeled by the equation $x = (0.5y - 4)^2 + 4$.
Will the asteroid enter the gravitational field?

6. Satellite pictures show that part of the Grand Canyon river flows with an approximate model of $y = -2x^2 + 4x + 3$. Visitors can view the river at two different locations: A and B. The path that these viewing locations are on can be mapped by the equation $y = x + 2$.
A diagram of the situation is below. Each grid square is 1 km².
What is the distance between points A and B?



Pictures taken of the Grand Canyon. Arizona USA.

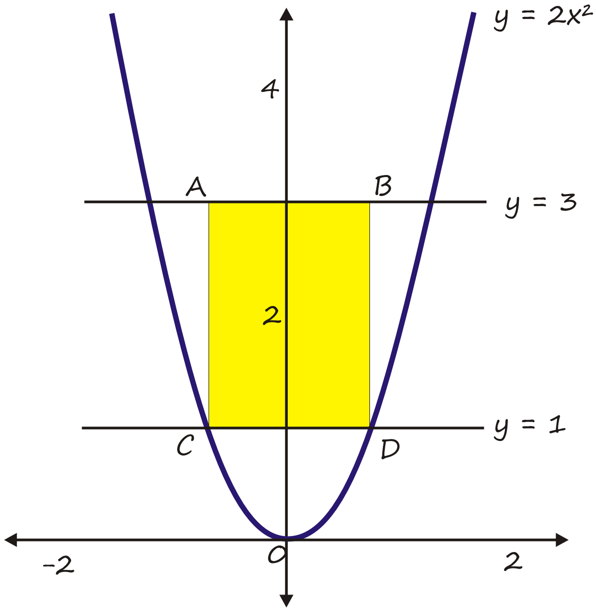
7. The shape of the famous "Gateway to the West" arch found in St Louis, Missouri USA can be modeled by the equation:

$$y = -\frac{1}{150}x^2 + \frac{21}{5}x$$

What is the distance from 1 side of the arch to the other?

Examples

1. The graph below shows the lines y = 2x², y = 3 and y = 1.
What is the area of ABCD?



Find where $y = 1$ meets $y = 2x^2$

$1 = 2x^2$

$x = \pm\sqrt{½}$

Therefore Point $C = (-\sqrt{½}, 1)$

Point $D = (+\sqrt{½}, 1)$

Point $A = (-\sqrt{½}, 3)$

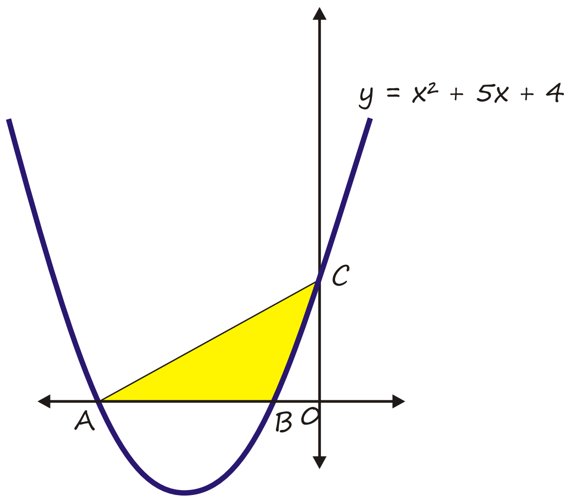
Point $B = (+\sqrt{½}, 3)$

The left and right sides are 2 (units).

The top and bottom sides are $2\sqrt{½}$ (units)

Area is $2 \times 2\sqrt{½} = 2\sqrt{2}$ or 2.828 units²

2. Calculate the area of the triangle ABC.



Find the roots of $y = x^2 + 5x + 4$

$y = 0$ therefore $(x + 4)(x + 1) = 0$

$x = -4$ or $x = -1$

Therefore AB = 3 units

At $x = 0$, point $C = (0, 4)$

Area ABC = ½ Base × height

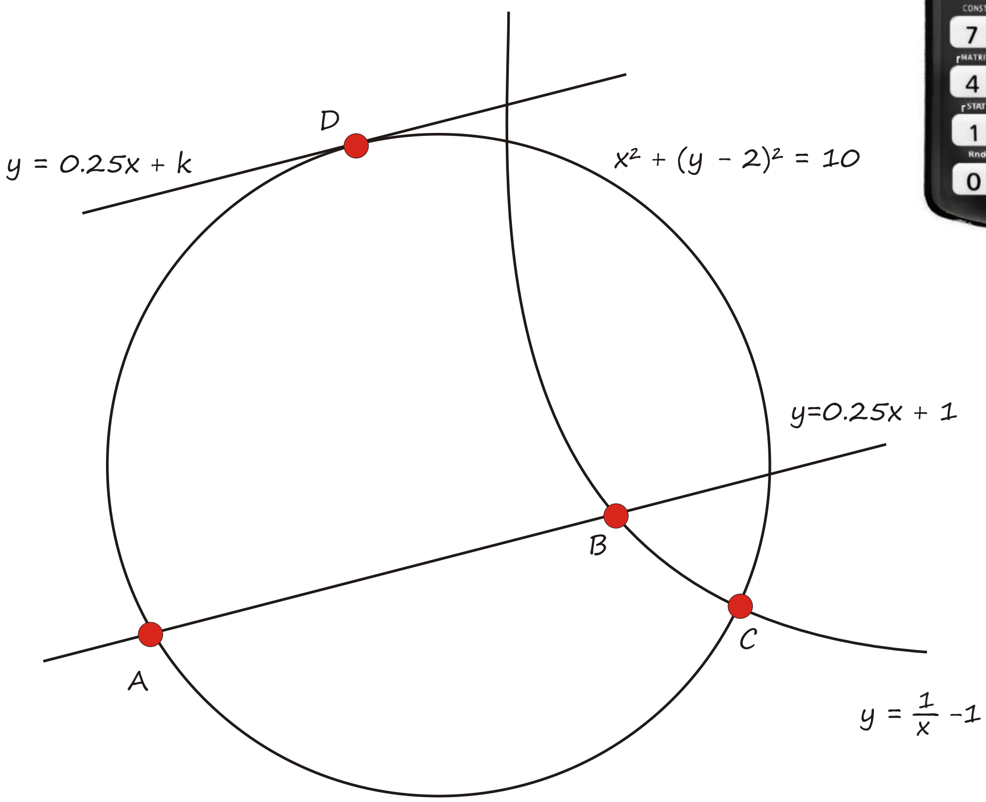
$= ½ \times 3 \times 4$

$= 6$ units²

The DS-742ET

Find points **A**, **B**, **C** and **D**.

Find the value of k.



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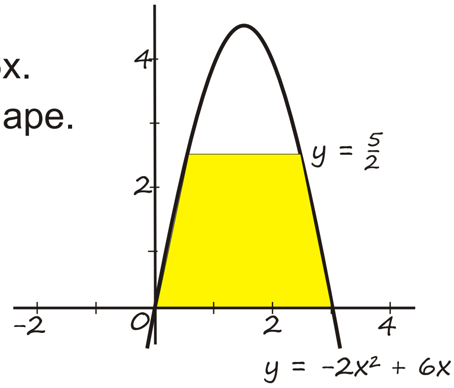
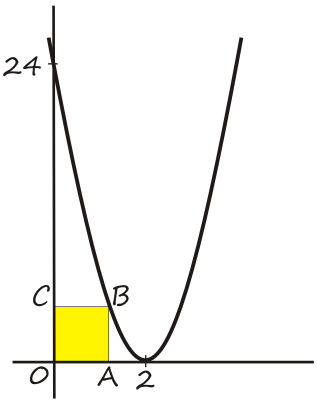
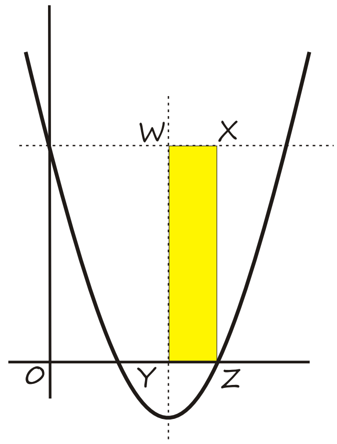
Exercises

1. The diagram shows the graph $y = -2x^2 + 6x$.
Inside the graph is a shaded trapezium shape.
Find the area of the trapezium.





2. The diagram shows the graph of $y = 6(x-2)^2$.
There is a square labelled OABC.
Calculate the area of the square.



3. This graph shows the parabola modeled by $x^2 - 6x + 5$.
Calculate the area of the rectangle WXYZ.

4. Claris thinks that the graph of the hyperbola $y = \frac{1}{x}$ and the line $y = -x$ will not intersect. Is she right? Show your proof mathematically.

5. Three seismographic stations have detected an earthquake in their region. Readings indicate that the epicentre is 50 km from Station A, 50 km from Station B and 13 km from Station C. On a map in which each square unit represents 1 square kilometre the first station is located at the origin, the second at (10, 30) and the third is at (-35, 18).
The system of equations that models this situation is:

$$x^2 + y^2 = 50^2$$

$$(x - 10)^2 + (y - 30)^2 = 50^2$$

and $(x + 35)^2 + (y - 18)^2 = 13^2$

Solve the system of equations and determine the location of the epicentre.

6. Solve the system of equations: $x(x + 2) + y(y - 3) = 3$
$2x(x + 2) - y(y - 18) = 54$

7. Draw graphs to illustrate the following equations:

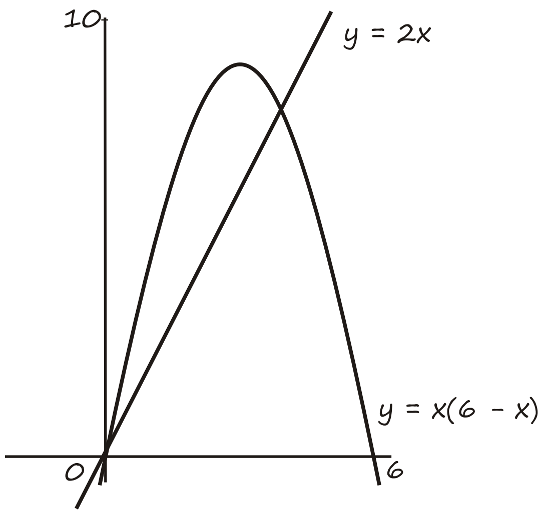
a. $x^2 + y^2 < 16$
$x - 2y > -4$

b. $x^2 + y^2 > 16$
$x^2 + (y - 4)^2 > 16$

c. $x^2 - y > 2$
$x - y < 4$

8. The graph below shows the graph of the equations $y = x(6 - x)$ and $y = 2x$

a. Find the intersection points of the two lines.



b. The graph is a representation of two power cables running down a no-exit street. The cables are split at (0,0) and then follow their projected paths. However engineers have found that when two electrical cables cross there can be problems with the magnetic fields that surround the casings and therefore power supply can be interrupted.

For this reason the engineers decide that the path of cable $y = 2x$ has to be altered. It can either be moved so that it is parallel to the current position or the gradient can be altered.

Calculate the new equations so that the cable for both cases makes a tangent to the parabolic path.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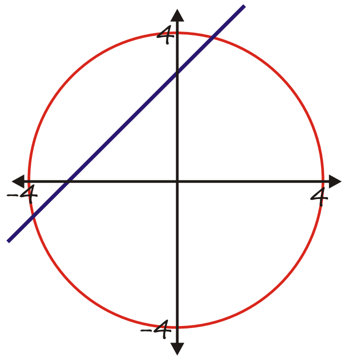
The Answers

Page 11

1. Below is a sketch of the circle and line.



$x^2 + (x + 3)^2 = 17$

$x^2 + x^2 + 6x + 9 = 17$

$2x^2 + 6x - 8 = 0$

$2(x^2 + 3x - 4) = 0$

$x(x + 4)(x - 1) = 0$

$x = -4$ or $x = 1$

Put the x values into an equation to solve for y.

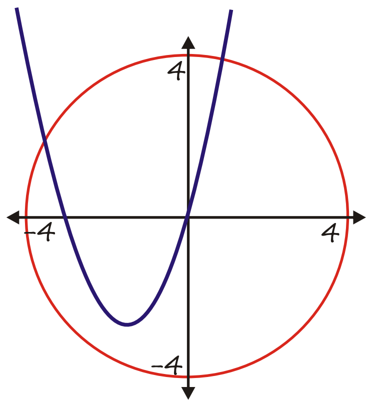
$y = x + 3$

$y = -4 + 3$ $\quad$ $y = 1 + 3$

$y = -1$ $\quad$ $y = 4$

Intersection points: $(-4, -1)$ and $(1, 4)$

2.



$x^2 + (x^2 + 3.5x)^2 = 20$

$x^2 + (x^2 + 3.5)(x^2 + 3.5)^2 = 20$

$x^2 + x^4 + 7x^3 - 20 = 0$

$x^4 + 7x^3 + 13.25x^2 - 20 = 0$

Using Mahobe DS-742ET calculator.

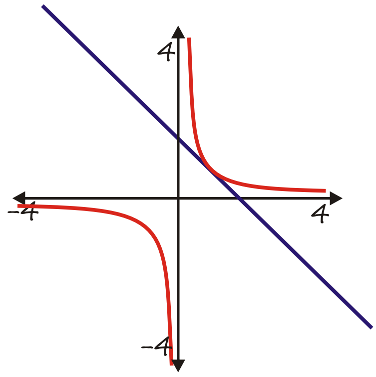
$x = 0.9753$ or $x = -4$

Using $y = x^2 + 3.5x$

if $x = 0.9753$ $y = 4.3648$

if $x = -4$, $y = -2$

3.



If $x + y = 2$

then $y = 2 - x$

Using $xy = 1$ then $x(2 - x) = 1$

$2x - x^2 = 1$

Rewrite $x^2 - 2x + 1 = 0$

$(x - 1)(x - 1) = 0$

$x = 1$

Using $x + y = 2$, $x = 1$, $y = 1$

There is only one solution $(1, 1)$

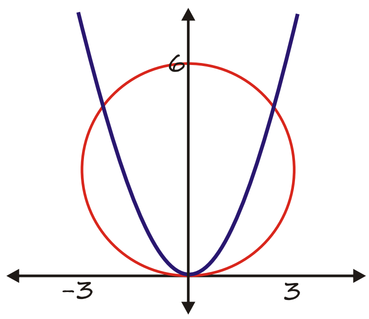
Using the discriminant $b^2 - 4ac$

and the equation $x^2 - 2x + 1 = 0$

$(-2)^2 - 4 \times 1 \times 1 = 0$

Therefore the line $x + y = 2$ is a tangent to the hyperbola $xy = 1$

4. From the sketch below there appears to be 3 solutions.



See over the page for the calculations

Page 11 (continued)

4. Substituting $y = x^2$ into $x^2 + (y-3)^2 = 9$

$y + (y - 3)^2 = 9$

$y + y^2 - 6y + 9 = 9$

$y^2 - 5y = 0$

$y(y - 5) = 0$

therefore $y = 0$ or $y = 5$

Using the equation $y = x^2$

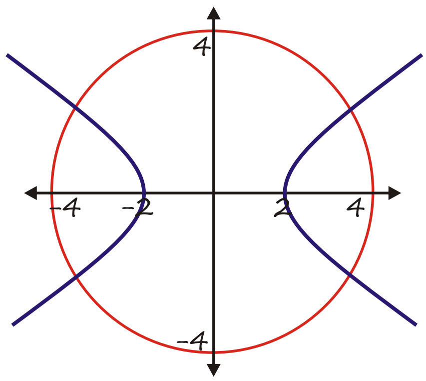
If $y = 0$, $x = 0$

If $y = 5$, $x = \pm\sqrt{5}$

You can keep the answer as above or further write the co-ordinates.

(0, 0), (2.2361, 5), (-2.2361, 5)

5. Sketch of the graph is below.



If $x^2 - 2y^2 = 4$

$x^2 = 4 + 2y^2$

Substituting this into $x^2 + y^2 = 22$

$4 + 2y^2 + y^2 = 22$

$3y^2 = 18$

$y^2 = 6$

$y = \pm\sqrt{6}$ or 2.45, -2.45

Using $x^2 + y^2 = 22$

$x^2 + 6 = 22$

$x^2 = 16$

$x = \pm 4$

The points are (4, 2.45), (4, -2.45), (-4, 2.45) and, (-4, -2.45)

6. Substitute $y = 2x - 8$ into $x^2 + y^2 = 85$

$x^2 + (2x - 8)^2 = 85$

$x^2 + 4x^2 - 32x + 64 = 85$

$5x^2 - 32x - 21 = 0$

$(5x + 3)(x - 7) = 0$

$x = -0.6$ or $x = 7$

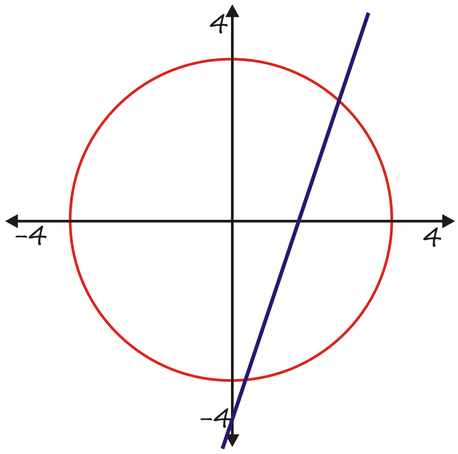
Using the equation $y = 2x - 8$

$y = 2(-0.6) - 8$, co-ordinate (-0.6, 9.2)

$y = 2(7) - 8$, co-ordinate (7, 6)

The rocks are at (0, 0) and the boat is approaching from positive co-ordinates. This means the ferry will first be able to detect the beacon when 7 km east and 6 km north of the rocks.

7.



A sketch of the graph of the situation is above. However only values where $y > 0$ will apply as the tunnel is a semi circle.

7a. Width of the tunnel

$x^2 + y^2 = 12$

at $y = 0$, $x^2 = 12$

$x = \pm 2\sqrt{3}$ or ± 3.464

This means the width is $4\sqrt{3}$ or 6.92 m

7b. Ladder equation is $y = 3x - 4$

At $y = 0$, $3x = 4$, $x = 1.333$ m

From the tunnel wall

$3.464 - 1.333 = 2.131$ metres

7c. See over the page.

Page 11 (continued)

7.c. $x^2 + y^2 = 12$ and $y = 3x - 4$

$x^2 + (3x - 4)^2 = 12$

$x^2 + 9x^2 - 24x + 16 = 12$

$10x^2 - 24x + 4 = 0$

At this point you can use the quadratic formula or an equation solving calculator such as the Mahobe **DS-742ET**.

$x = 2.2198$ or $x = 0.1802$

Using $y = 3x - 4$

$y = 3(2.2198) - 4$, $y = 2.6594$

$y = 3(0.1802) - 4$, $y = -3.4594$

The solution is the positive x and y points (2.22, 2.66). The graph on the previous page shows how the other value is not valid as it is a point underground ($y < 0$).

Page 12

8. $\frac{1}{2}x(10 - x) = (x - 4)^2 + 4$

$5x - \frac{1}{2}x^2 = x^2 - 8x + 16 + 4$

$5x - \frac{1}{2}x^2 = x^2 - 8x + 20$

$1\frac{1}{2}x^2 - 13x + 20 = 0$

$3x^2 - 26x + 40 = 0$

Using the Mahobe DS-742

$x = 2$ or $x = 6.67$

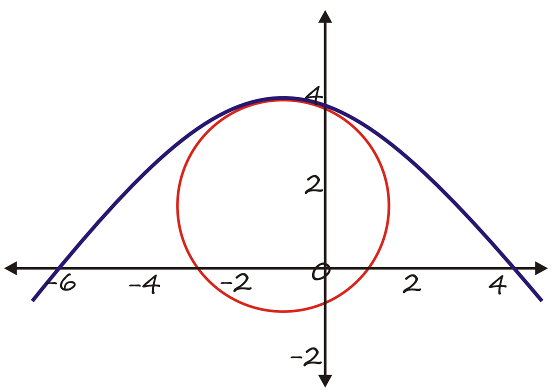
Using $y = \frac{1}{2}x(10 - x)$

$x = 2$, $y = 8$

$x = 6.67$, 11.11

$A = (2, 8)$ $B = (6.67, 11.11)$

9. Below is the graph of the two lines. From first glance it appears that they only intersect at one point (a tangent).



Page 12 (continued).

9. Firstly expand out the equations.

Equation 1: $x^2 + 2x + y^2 - 3y = 3$

Equation 2: $2x^2 + 4x - y^2 + 18y = 54$

multiply equation 1 by 2 and subtract eq 2

$2x^2 + 4x + 2y^2 - 6y = 6$

$\underline{2x^2 + 4x - y^2 + 18y = 54}$

$3y^2 - 24y = -48$

$3y^2 - 24y + 48 = 0$

$3(y^2 - 8y + 16) = 0$

$3(y - 4)(y - 4) = 0$

$y = 4$

Using $y = 4$ and $x^2 + 2x + y^2 - 3y = 3$

$x^2 + 2x + 16 - 12 = 3$

$x^2 + 2x + 1 = 0$

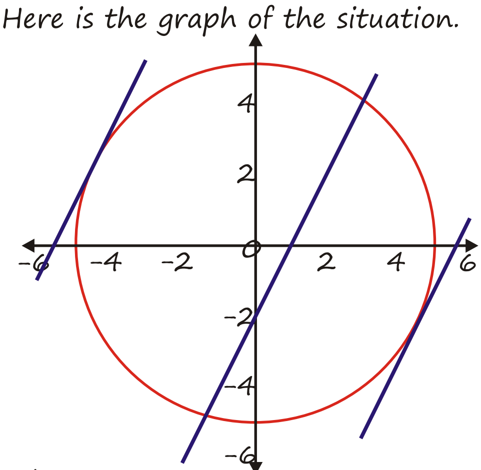
$(x + 1)^2 = 0$

$x = -1.$

Point of intersection is $(-1, 4)$

The point is a tangent.

10. Here is the graph of the situation.



substituting 1 equation into the other

$x^2 + (2x - 2)^2 = 25$

$x^2 + 4x^2 - 8x + 4 = 25$

$5x^2 - 8x - 21 = 0$

Using the Mahobe DS-742ET

$x = 3$ or $x = -1.4$

Substituting these values into $y = 2x - 2$

$x = 3$, $y = 4$ and $x = -1.4$, $y = -4.8$

Page 12 (continued)

10b. Using $y = 2x + c$ and $x^2 + y^2 = 25$

$x^2 + (2x + c)^2 = 25$

$x^2 + 4x^2 + 4xc + c^2 = 25$

$5x^2 + 4xc + c^2 - 25 = 0$

A quadratic equation is $ax^2 + bx + c$

A tangent means $b^2 - 4ac = 0$

i.e. 1 root. This means in the equation above that values of a, b and c are:

$a = 5,\ b = 4c,\ c = c^2 - 25$

$(4c)^2 - [4 \times 5 \times (c^2 - 25)] = 0$

$16c^2 = 20c^2 - 500$

$-4c^2 = -500$

$c^2 = 125$

$c = \pm\sqrt{125}$

The two possible equations are

$y = 2x - \sqrt{125}$

$y = 2x + \sqrt{125}$

Note this problem is best completed by using surds. This is more accurate and is where using the Mahobe DS-742ET is probably the best calculator.

Finding the intersection points.

$x^2 + (2x - 125\ )^2 = 25$

$x^2 + 4x^2 - 20\sqrt{5}\,x + 125 = 25$

$5x^2 - 20\sqrt{5}\,x + 100 = 0$

Using the Mahobe DS-742ET

$x = 4.4721,\ y = -2.2358$

Doing a similar calculation for the other point: $x = -4.4721\ y = 2.2358$

Page 15

1a. Let x = Total SIM cards sold.

Let y = price per SIM card in November.

November sales $xy = 2700$

or $y = \frac{2700}{x}$

1b. December sales $(x + 30)(y - 15) = 3375$

$(x + 30)(y - 15) - 3375 = 0$

$xy - 15x + 30y - 450 - 3375 = 0$

$xy - 15x + 30y - 3825 = 0$

substitute in the xy and y values (above)

$2700 - 15x + 30\left(\frac{2700}{x}\right) - 3825 = 0$

$-15x - 1125 + \left(\frac{81\,000}{x}\right) = 0$

1b. (continued)

$x^2 + 75x - 5400 = 0$

$(x - 45)(x + 120) = 0$

$x = 45$ or $x = -120$

In this example, sales of SIM cards cannot be negative. Therefore during November there were 45 SIM cards sold.

The price of each SIM card was \$60 (as $xy = 2700$) and the price of the cards in December was \$45 (\$60 – \$15).

2. $x = 2y^2$ and $x + 3y = k$

Therefore $2y^2 + 3y = k$

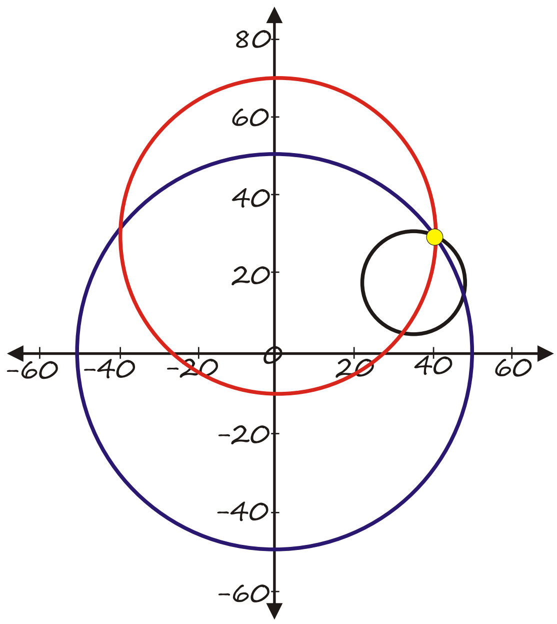
and $2y^2 + 3y - k = 0$

Using $b^2 - 4ac = 0$ for tangent.

$3^2 - (4 \times 2 \times k) = 0$

$9 - 8k = 0$ therefore $k = \frac{9}{8}$

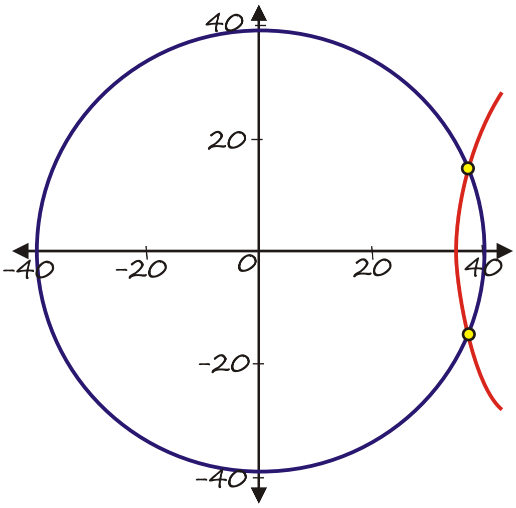
3. The best approach to this problem is to use graphing software such as DESMOS found at **www.desmos.com.**



The intersection point is (40, 30)

Page 15 (continued)

4. The situation is modeled in the graph below.



$$\frac{x^2}{40^2} + \frac{y^2}{39^2} = 1$$

and $x = 0.01y^2 + 35$

therefore $0.01y^2 = x - 35$

$$y^2 = 100x - 3500$$

$39^2(x^2) + 40^2(100x - 3500) = 40^2 \times 39^2$

$1521x^2 + 160\,000x - 8\,033\,600 = 0$

Using the Mahobe DS-742ET calculator and the quadratic equation solver

$x = 37.115$ (ignore the -142.309)

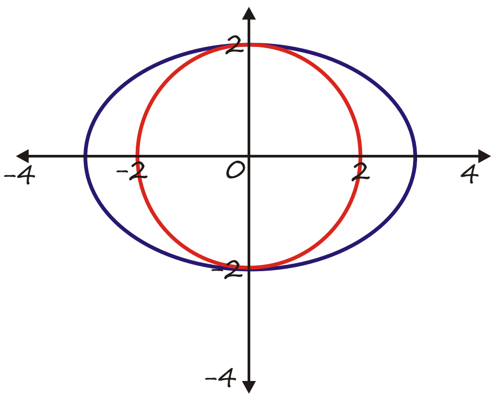
at $x = 37.115$ and $y^2 = 100x - 3500$

$y^2 = 211.5$ and $y = \pm 14.543$

Note that it is the two paths that we see. We do not have information on where each is on that path. While the paths collide, it doesn't mean that they will collide.

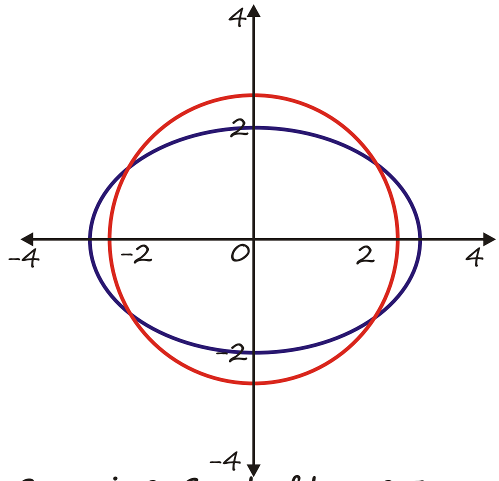
5. Intersection points are $(-209, 647)$, $(209, 647)$, $(-209, -647)$, $(209, -647)$

6.

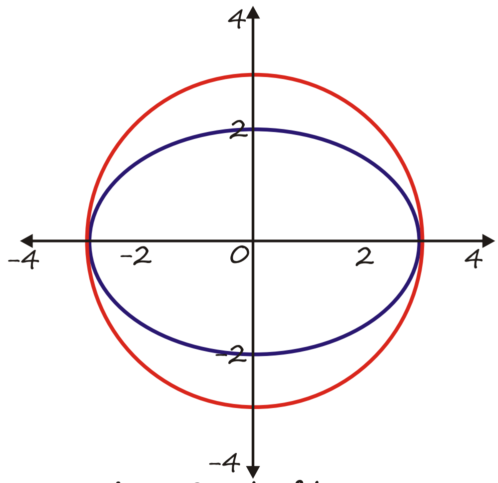


Scenario 1. K = ±2 (inside circle)

6.



Scenario 2. Graph of k = ±2.5



Scenario 3. Graph of k = ±3

The graphs of 3 scenarios have been drawn. Both equations are centred around the point (0, 0). If $x = 0$ then $y^2 = k^2$ and $k^2/4 = 1$. Therefore $k = 2$ or -2

If $2 > k > -2$ then there are no solutions.

Scenario 1

The two equations intersect at (0, 2) and (0, −2) i.e. $k = 2$ or $k = -2$ two solutions.

Scenario 2

If $2 < k < 3$ and $-2 > k > -3$ there are 4 solutions.

Scenario 3

If $k = 3$ or $k = -3$ there are two solutions (3, 0) and (−3, 0)

If $k > 3$ or $k < -3$ there are no solutions.

There are **no** scenarios with only 1 solution.

Page 16

7. Rearrange the equations

$x = 2y^2$ and $x = 3y - k$

Therefore $3y - k = 2y^2$

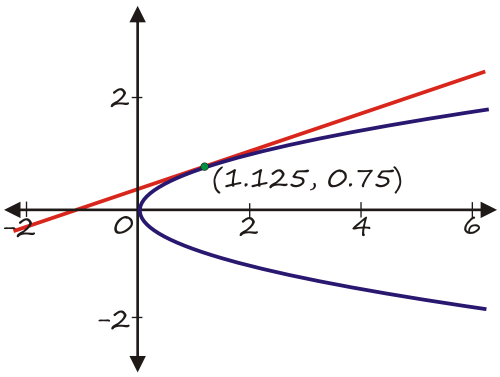
$2y^2 - 3y + k = 0$

Using $b^2 - 4ac = 0$

$(-3)^2 - (4 \times 2 \times k) = 0$

$9 - 8k = 0$

$k = \frac{9}{8}$ therefore the equation of the tangent is : $x = 3y - \frac{9}{8}$



8a. $x^2 + y = 0$

$\underline{2x - y = 3}$

add $x^2 + 2x = 3$

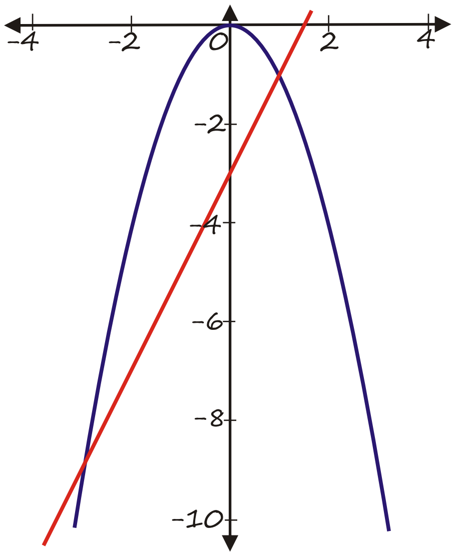
$x^2 + 2x - 3 = 0$

$(x + 3)(x - 1) = 0$

$x = -3$ or $x = 1$

Using one of the equations intersection points are:

$(-3, -9)$ and $(1, -1)$



Desmos is a free interactive calculator that graphs equations as you write them.

desmos

www.desmos.com

8b. $x + y = 9$

or $y = 9 - x$

and $xy = 20$

therefore $x(9 - x) = 20$

$9x - x^2 = 20$

rewrite $x^2 - 9x + 20 = 0$

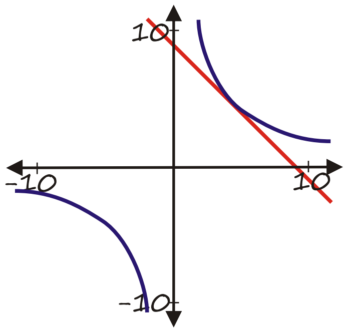
$(x - 5)(x - 4)$

$x = 5$ or 4

Using $xy = 20$

intersection points are:

$(5, 4)$ and $(4, 5)$



At first glance the graph above looks like a tangent but it actually goes through two points.

8c. $x^2 + y^2 = 25$ or $x^2 = 25 - y^2$

$x^2 - y = 5$

therefore $25 - y^2 - y = 5$

rewrite $y^2 + y - 20 = 0$

$(y - 5)(y - 4)$

$y = 5, y = 4$

using $x^2 = 25 - y^2$

$x^2 = 25 - 25,$

intersection $(0, 5)$

$x^2 = 25 - 16$

intersection $(3, 4)(-3, 4)$

Page 16 (continued)

8d.

$$\frac{3}{x} + \frac{2}{y} = 10$$

$$\frac{1}{x} - \frac{1}{y} = -10 \quad \text{multiply this by 2}$$

$$\frac{2}{x} - \frac{2}{y} = -20 \quad \text{add to equation 1}$$

$$\frac{5}{x} = -10$$

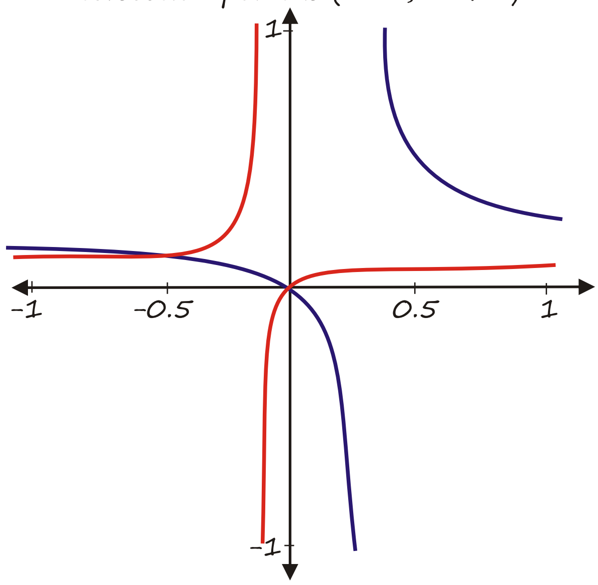
$$x = -0.5$$

Put this into one of the equations

$$\frac{3}{-0.5} + \frac{2}{y} = 10$$

$$\frac{2}{y} = 16 \quad , y = 0.125$$

Intersection point is (−0.5, 0.125)



8e.

$$x^2 + y^2 = 8$$

$$xy = 4 \qquad \text{or } y = \frac{4}{x}$$

$$x^2 + \left(\frac{4}{x}\right)^2 = 8$$

$$x^2 + \frac{16}{x^2} = 8$$

$$x^4 + 16 = 8x^2$$

$$x^4 - 8x^2 + 16 = 0$$

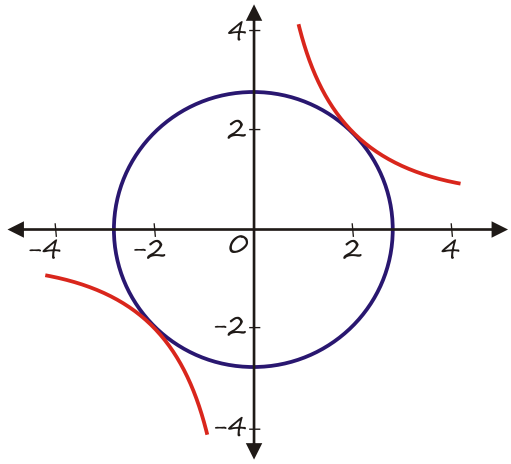
Using the equation solver on the

Mahobe DS-742ET

$x = \pm 2$

Therefore intersection points are:

(2. 2) and (−2, −2)



8f.

$$(y - 1)^2 = 4 + x$$

$$x + y = -1$$

Rewrite equation 2 and add

$$(y - 1)^2 = 4 + x$$

$$y = -1 - x$$

$$(y - 1)^2 + y = 3$$

$$y^2 - 2y + 1 + y = 3$$

$$y^2 - y - 2 = 0$$

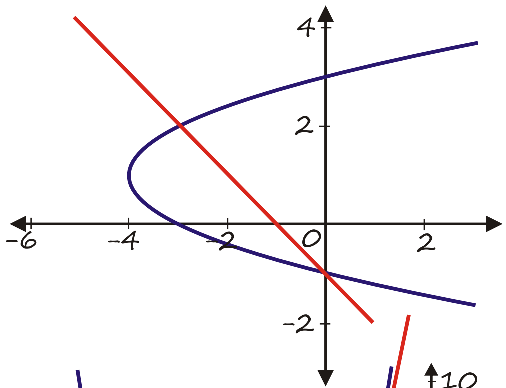
$$(y - 2)(y + 1) = 0$$

$$y = 2 \text{ or } y = -1$$

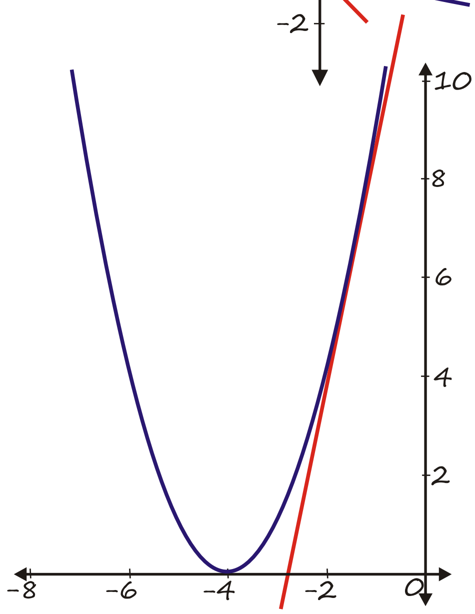
Using $x + y = -1$

$x + 2 = -1$ point is (−3, 2)

$x - 1 = -1$ point is (0, −1)



9.



Page 16

9. See the graph on the previous page.

$y = 5x + 14$ and $y = (x + 4)^2$

$5x + 14 = (x + 4)^2$

$5x + 14 = x^2 + 8x + 16$

$x^2 + 3x + 2 = 0$

$(x + 2)(x + 1) = 0$

therefore $x = -2$ or $x = -1$

When $x = -2$, and $y = 5x + 14$, $y = 4$

When $x = -1$, and $y = 5x + 14$, $y = 9$

Therefore intersection points are

$(-2,4)$ and $(-1, 9)$

Note: At first glance of the graph there seemed to be only one intersection point but algebra showed us something different.

10a. $y = \frac{4}{x}$, $y - 0.5x + 2 = 0$

rewrite equation 2: $y = 0.5x - 2$

$0.5x - 2 = \frac{4}{x}$

$0.5x^2 - 2x = 4$

$0.5x^2 - 2x - 4 = 0$

Use the DS-742ET equation solver!

$x = 5.4641$ or $x = -1.4641$

using $y = 0.5x - 2$

$x = 5.4641$, $y = 0.73205$

$x = -1.4641$, $y = -2.73205$

10b. $y = \frac{4}{x}$ and $y = mx + 2$

therefore $\frac{4}{x} = mx + 2$

$4 = mx^2 + 2x$

$mx^2 + 2x - 4 = 0$

Using $b^2 - 4ac = 0$

$2^2 - (4 \times m \times -4) = 0$

$4 + 16m = 0$

$16m = -4$

$m = \frac{-1}{4}$

No solutions $b^2 - 4ac < 0$

1 solution $b^2 - 4ac = 0$

2 solutions $b^2 - 4ac > 0$

10b. (continued)

$m < -¼$ for 0 points

$m = -¼$ for 1 point

$m > -¼$ for 2 points

Page 19

1a. Equation 1 $y^2 = 25 - x^2$

Equation 2: $y^2 = (2x - 2)^2$

$(2x - 2)^2 = 25 - x^2$

$4x^2 - 8x + 4 = 25 - x^2$

$5x^2 - 8x - 21 = 0$

Using the Mahobe DS-742ET

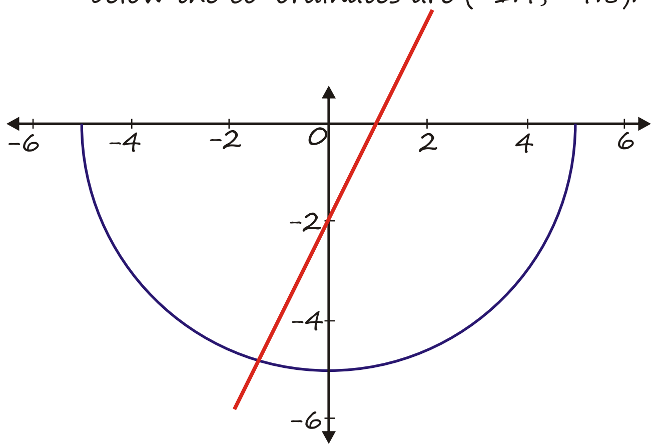
$x = 3$ or $x = \frac{-7}{5}$

Putting the two values back into the original $y = 2x - 2$

$x = 3$, $y = 4$

$x = -1.4$ (or $\frac{-7}{5}$), $y = -4.8$ (or $\frac{-24}{5}$)

Looking at the graph of the situation below the co-ordinates are $(-1.4, -4.8)$.



1b. Using the equations $y^2 + x^2 = 25$ and $y = 2x + c$

$x^2 + (2x + c)^2 = 25$

$x^2 + 4x^2 + 4xc + c^2 = 25$

$5x^2 + 4xc + c^2 - 25 = 0$

$b^2 - 4ac = 0$ for 1 root

Therefore $a = 5$, $b = 4c$ $c = c^2 - 25$

$(4c)^2 = 4 \times 5 \times (c^2 - 25)$

$16c^2 = 20c^2 - 500$

$4c^2 = 500$

$c^2 = 125$

$c = \pm 11.18$

continued on the next page »»»

Page 19 (continued)

1b. cont

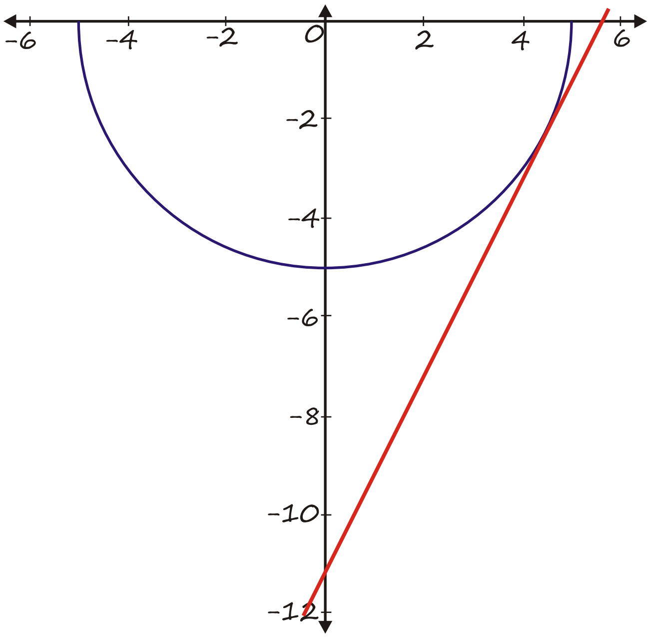
Therefore the equations (if it was a complete circle) are:

$y = 2x - 11.18$

$y = 2x + 11.18$

The answer is $y = 2x - 11.18$

See the graph below.



2a. $x^2 + (y - 2)^2 = 4$ and $y = x + 2$

therefore

$x^2 + (x + 2 - 2)^2 = 4$

$x^2 + x^2 = 4$

$2x^2 = 4$

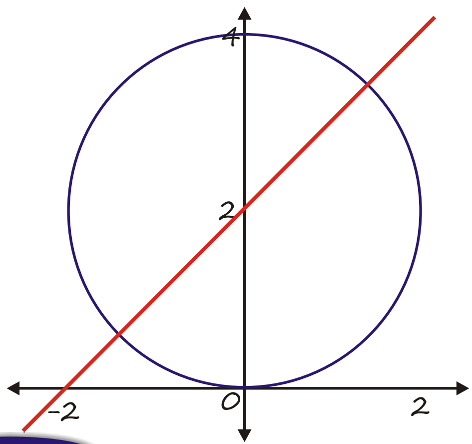
$x^2 = 2$

$x = \pm 1.4142$

Using $y = x + 2$

$x = 1.4142, y = 3.4142$

$x = -1.4142, y = 0.5859$



2b. The parallel line is $y = x + k$.

The circle is $x^2 + (y - 2)^2 = 4$

Therefore $x^2 + (x + k - 2)^2 = 4$

$x^2 + (x + k - 2)(x + k - 2) = 4$

Expanding $(x + k - 2)(x + k - 2)$

$x^2 + xk - 2x + xk + k^2 - 2k - 2x - 2k + 4$

$= x^2 + 2xk - 4x + k^2 - 4k + 4$

Therefore

$x^2 + (x + k - 2)(x + k - 2) = -4$

$2x^2 + 2xk - 4x + k^2 - 4k = 0$

$2x^2 + (2k - 4)x + k^2 - 4k = 0$

Note how this is in a quadratic form:

$ax^2 + bx + c = 0$

where $a = 2$, $b = (2k - 4)$, $c = k^2 - 4k$

Using $b^2 - 4ac = 0$ (for a tangent)

$(2k - 4)^2 - (4 \times 2 \times (k^2 - 4k)) = 0$

$(4k^2 - 16k + 16) - (8k^2 - 32k) = 0$

$-4k^2 + 16k + 16 = 0$

Using the Mahobe DS-742ET

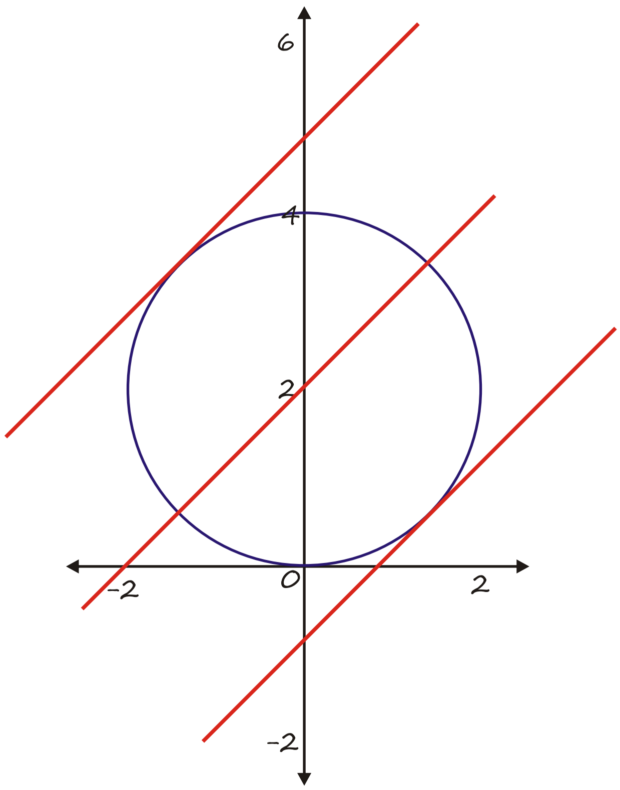
$k = 4.8284$ or $k = -0.8284$

There are two possible equations:

$y = x + 4.8284$

$y = x - 0.8284$

Here is the graph of the 2 scenarios.



Page 19 (continued)

3. $y = 0.25x(12 - x)$, tangent $y = -x + k$.

$3x - 0.25x^2 = -x + k$

Rewrite the equation

$-0.25x^2 + 4x - k = 0$

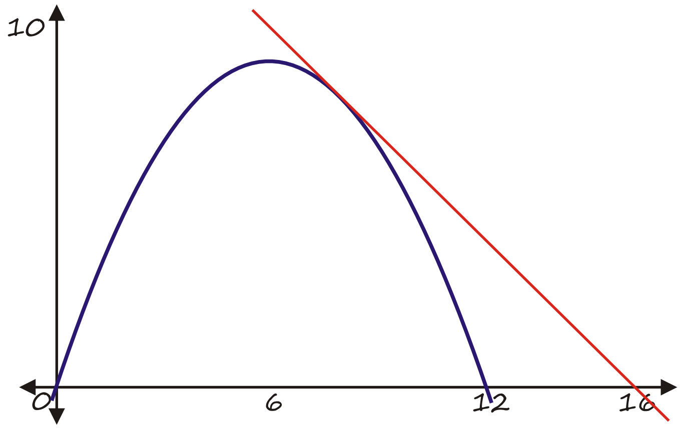
Using $b^2 - 4ac = 0$ for a tangent

$4^2 - [4 \times (-0.25) \times (-k)] = 0$

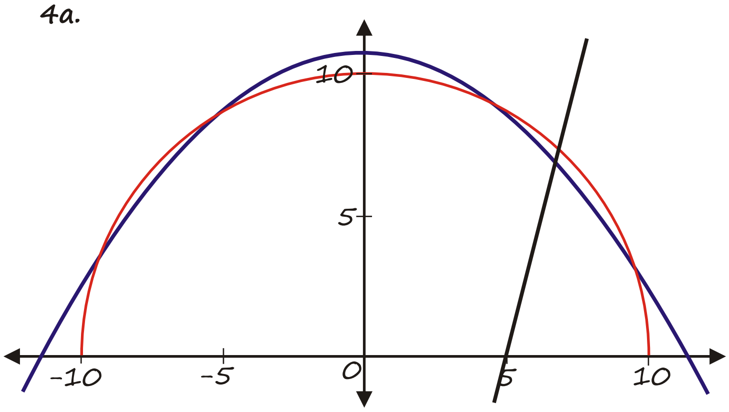
$16 - k = 0$

$k = 16$

Therefore the tangent is $y = -x + 16$



4a.



Building A

$x^2 + y^2 = 100$, $y = 4x - 20$

$x^2 + (4x - 20)^2 = 100$

$x^2 + 16x^2 - 160x + 400 = 100$

$17x^2 - 160x + 300 = 0$

Using the Mahobe DS-742ET equation solver $x = 6.8268$ or $x = 2.5850$

We are only concerned with the value at $x = 6.8286$

Using $y = 4x - 20$, $y = 7.3144$

Therefore Building A intersection is: $(6.8286, 7.3144)$

Building B

$x^2 + 12y - 120 = 0$, $y = 4x - 20$

$x^2 + 12(4x - 20) - 120 = 0$

$x^2 + 48x - 240 - 120 = 0$

$x^2 + 48x - 360 = 0$

$x = 6.5941$ or $x = -54.5941$

Look at the diagram of the situation.

We are only concerned with the value at $x = 6.5941$. Using $y = 4x - 20$, $y = 6.3764$. Therefore Building B intersection is: $(6.5941, 6.3764)$

The question asks about the difference between the light heights of Building A and B: $7.3144 - 6.3764 = 0.9388$

Note we were not given any units.

4b. **Tangent to Building A**

$x^2 + y^2 = 100$, $y = -1.5x + k$

$x^2 + (-1.5x + k)^2 = 100$

$x^2 + 2.25x^2 - 3kx + k^2 = 100$

$3.25x^2 - 3xk + k^2 - 100 = 0$

Compare this to $b^2 - 4ac = 0$

$a = 3.25$, $b = 3k$, $c = k^2 - 100$

$(3k)^2 - [4 \times 3.25 \times (k^2 - 100)] = 0$

$9k^2 - [13k^2 - 1300] = 0$

$-4k^2 + 1300 = 0$

$-4k^2 = -1300$

$k^2 = 325$, therefore $k = \pm 18.0278$

The equation is $y = -1.5x + 18.0$

Therefore the coordinates of the ladder at ground level ($y = 0$) is $(12, 0)$

Note: The equation $y = -1.5x - 18$ would not be applicable as the y intercept (-18) implies it is underground.

Tangent to Building B

Use $x^2 + 12y - 120 = 0$ and $y = -1.5x + k$

$x^2 + 12(-1.5x + k) - 120 = 0$

continued over the page.

Page 19 (continued) – tangent at Building B

4b. $x^2 - 18x + 12k - 120 = 0$

Using the quadratic form $ax^2 + bx + c$

and $b^2 - 4ac = 0$ (for a tangent).

$a = 1$, $b = -18$, $c = 12k - 120$

$(-18)^2 - [\,4 \times (12k - 120)] = 0$

$324 - 48k + 480 = 0$

$804 - 48k = 0$

$k = 16.75$

Therefore the equation of the ladder to building B will be $y = -1.5x + 16.75$

When $y = 0$, the co-ordinates of the base of the ladder will be (11.167, 0).

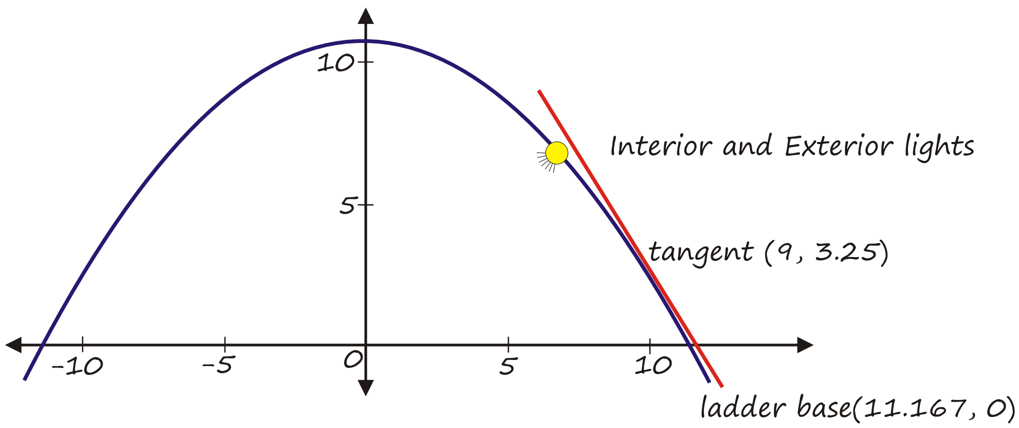
Also note that the ladder would only reach the point (9, 3.25).

Considering that the exterior light is over 6 m high the electrician may have to re-evaluate the ladder's gradient for Building B

Desmos is a free interactive calculator that graphs equations as you write them.

Most of our answers were checked by using this website.

www.desmos.com

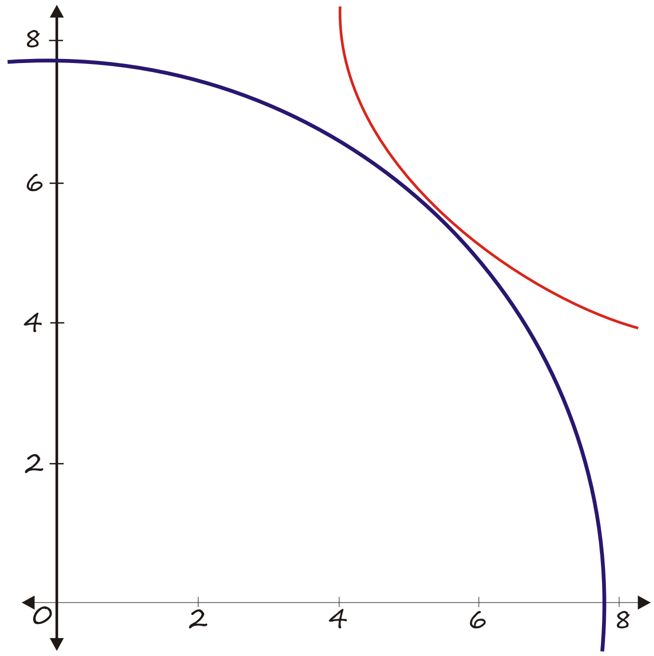


For our calculations we only use the Mahobe DS-742ET calculator.
It is powerful and fast.

If you use anything else then good luck.

Page 20

5. While it can be proved algebraically that there are no solutions why not just use technology. Below is the graph of the two paths. The asteroid just comes outside the gravitational field.



Algebraically

$x = (0.5y - 4)^2 + 4$

$= (0.5y - 4)((0.5y - 4) + 4$

$= 0.25y^2 - 4y + 20$

Substituting into $x^2 + y^2 = 40$

$x^2 = (0.25y^2 - 4y + 20)(0.25y^2 - 4y + 20)$

$0.0625y4 - 2y^3 + 26y^2 - 160 + 400 + y^2 = 40$

$0.0625y^4 - 2y^3 + 27y^2 - 160 + 360 = 0$

Using the equation solver in the DS-742ET there is no solution. This confirms what our graph above shows.

6. $y = -2x^2 + 4x + 3$ and $y = x + 2$

$-2x^2 + 4x + 3 = x + 2$

$-2x^2 + 3x + 1 = 0$

Therefore using the Mahobe DS-742ET equation solver: $x = -0.2808$ or 1.7801

Using $y = x + 2$, $x = -0.2808, y = 1.7192$

$x = 1.7801, y = 3.7801$

6. (continued)

Using Pythagoras

$\text{Dist} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(1.78+0.28)^2 + (3.78-1.72)^2}$

$= 2.91$ km

7. This can easily be solved by using the equation solver in the Mahobe DS-742ET

The roots are 0 and 630. Therefore the width of the arch base is 630m.

Page 23

1. $y = -2x^2 + 6x$

$2.5 = -2x^2 + 6x$

$-2x^2 + 6x - 2.5 = 0$

$x = 0.5, 2.5$

Area of a trapezim

= ½(top + bottom) × height.

$= \frac{1}{2}(2 + 3) \times 2.5$

$= 6.25$ units^2

2. For the square OABC the side lengths are $y = x$.

$y = 6(x - 2)^2$

$x = 6(x^2 - 4x + 4)$

$x = 6x^2 - 24x + 24$

$0 = 6x^2 - 25x + 24$

$x = \frac{8}{3}$ or $\frac{3}{2}$ (2.67 or 1.5)

Looking at the diagram we are only interested in the $x = 1.5$ value. Therefore both points C and A must be 1.5. The area will be $1.5^2 = 2.25$ units^2.

3. $x^2 - 6x + 5 = 0$

$(x - 5)(x - 1) = 0$

$x = 5, x = 1$

Therefore the graph crosses the x axis at (1, 0) and (5, 0). The minimum of the parabola is mid way between these two points i.e. (3, –4). **Continued on next page**

Page 23 (continued)

3. (continued) This means y is at (3, 0)

The y intercept is at x = 0, i.e. (0, 5)

This means the width of the rectangle is 2 units and the height is 5 units.

The area is 10 units².

4. $\frac{1}{x} = -x$

$1 = -x^2$

$x^2 + 1 = 0$

Using $b^2 - 4ac$

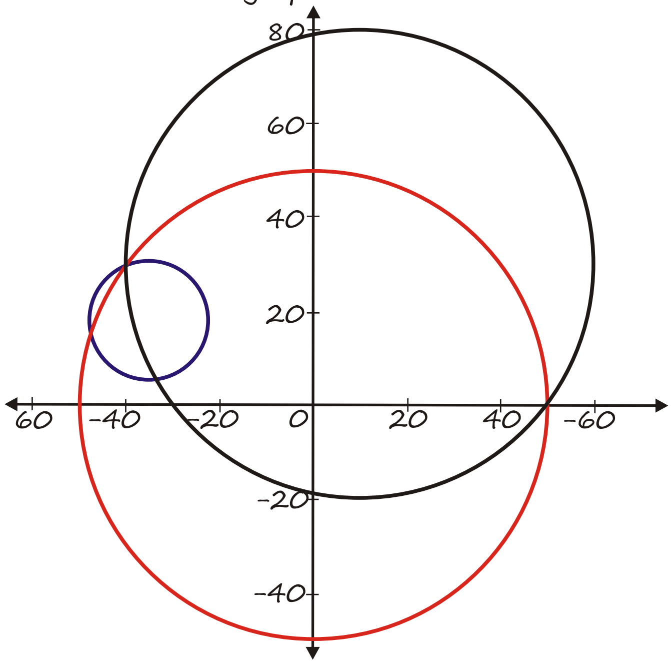
and a = 1, b = 0, c = 1

$0 - (4 \times 1 \times 1) = -4$

As this is < 0 then there are no solutions.

This means that they will not intersect.

5. Below is a graph of the situation



The graph intersects at (-40, 30) but you may want to see it calculated algebraically.

$y^2 = 50^2 - x^2$ and $y = \sqrt{50^2 - x^2}$

$(x - 10)^2 + (y - 30)^2 = 50^2$

$x^2 - 20x + 100 + y^2 - 60y + 900 = 50^2$

Substitute y^2 and y

$x^2 - 20x + 1000 + 50^2 - x^2 - 60\sqrt{50^2 - x^2} = 50^2$

$-20x + 1000 - 60\sqrt{50^2 - x^2} = 0$

At this stage you could continue by squaring everything to cancel the square root or just use the equation solver in the Mahobe DS-742ET to get x = -40 or x = 0

We are only interested in the x = -40 value

5. (continued)

Putting x = -40 into one of the formulas gets y = 30

To confirm this point (-40, 30) put it into the third equation

$(-40+35)^2 + (30-18)^2 = 13^2$

Therefore the epicentre is (-40, 30).

Page 24

6. $x(x + 2) + y(y - 3) = 3$

$2x(x + 2) - y(y - 18) = 54$

Expand out all the brackets

$x^2 + 2x + y^2 - 3y = 3$

$2x^2 + 4x - y^2 + 18y = 54$

Multiply equation 1 by 2 then subtract equation 2.

$2x^2 + 4x + 2y^2 - 6y = 6$

$\underline{2x^2 + 4x - y^2 + 18y = 54}$

$3y^2 - 24y = -48$

$3y^2 - 24y + 48 = 0$

$3(y^2 - 8y + 16) = 0$

$3(y - 4)^2 = 0$

Therefore y = 4

Putting this into equation 1

$x(x + 2) + 4(4 - 3) = 3$

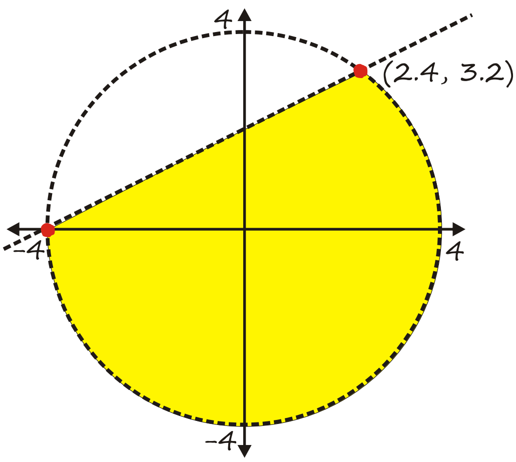
$x^2 + 2x + 1 = 0$

$(x + 1)^2 = 0$

Therefore x = -1

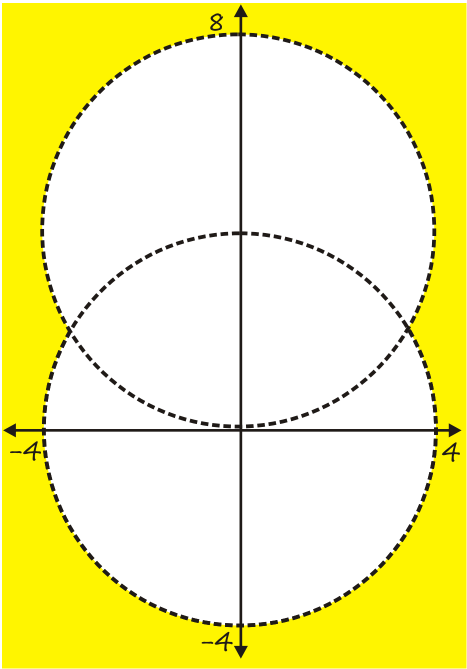
Intersection point is (-1, 4)

7a.

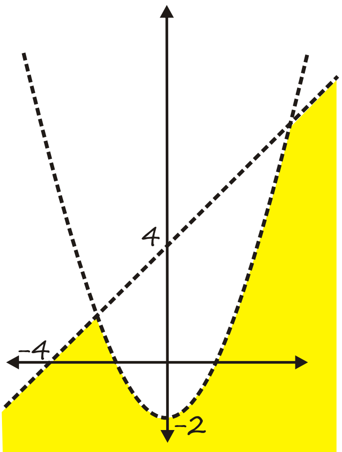


Page 24 (continued)

7b.



7c.



8a. $y = 6x - x^2$, $y = 2x$

to find intersection point $y = y$

$6x - x^2 = 2x$

$x^2 - 4x = 0$

$x(x - 4) = 0$

$x = 0$, $x = 4$

Using $y = 2x$ at $x = 0$ point is $(0, 0)$

at $x = 4$, point is $(4, 8)$.

8b. $y = 6x - x^2$,

$y = 2x + k$ (a parallel line)

$6x - x^2 = 2x + k$

8b. (continued)

$x^2 - 4x + k = 0$

$b^2 - 4ac = 0$ (for 1 root)

$(-4)^2 - 4 \times 1 \times k = 0$

$16 - 4k = 0$

$k = 4$

Therefore a parallel line which hits the parabola at a tangent is $y = 2x + 4$.

The point of intersection will be $(2, 8)$

The second part of the question involves changing the gradient.

Therefore $y = 6x - x^2$ and $y = kx$

$kx = 6x - x^2$

$x^2 + (k - 6)x = 0$

Using $b^2 - 4ac = 0$

$(k - 6)^2 - 4 \times 1 \times 0 = 0$

$(k - 6)^2 = 0$

$k = 6$

Therefore the other line must be $y = 6x$. It only meets the parabola at $(0, 0)$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When they collide, the DS-742ET will be there calculating it for you.