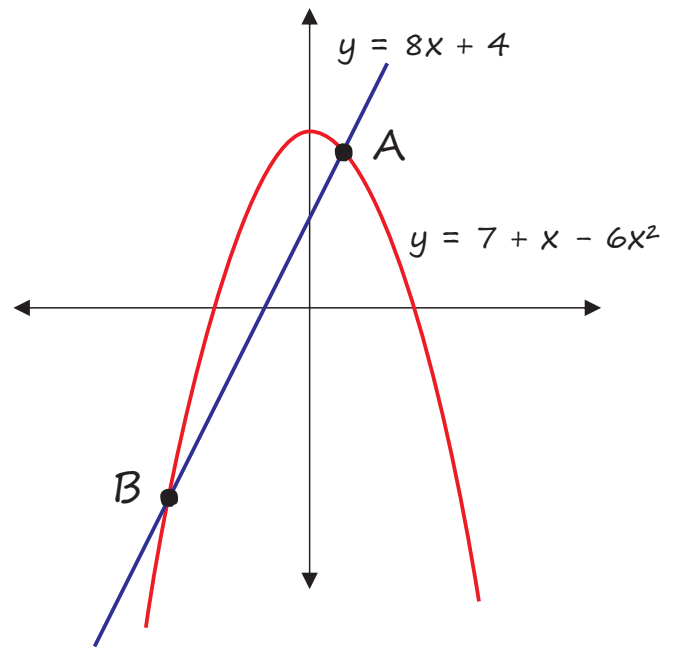
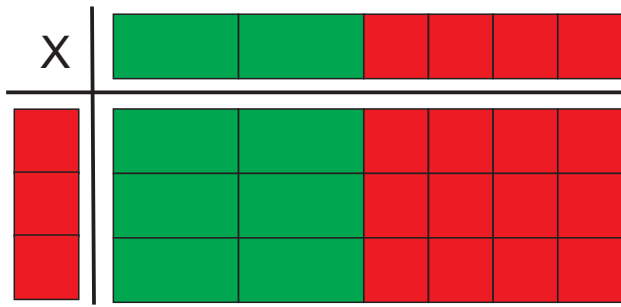


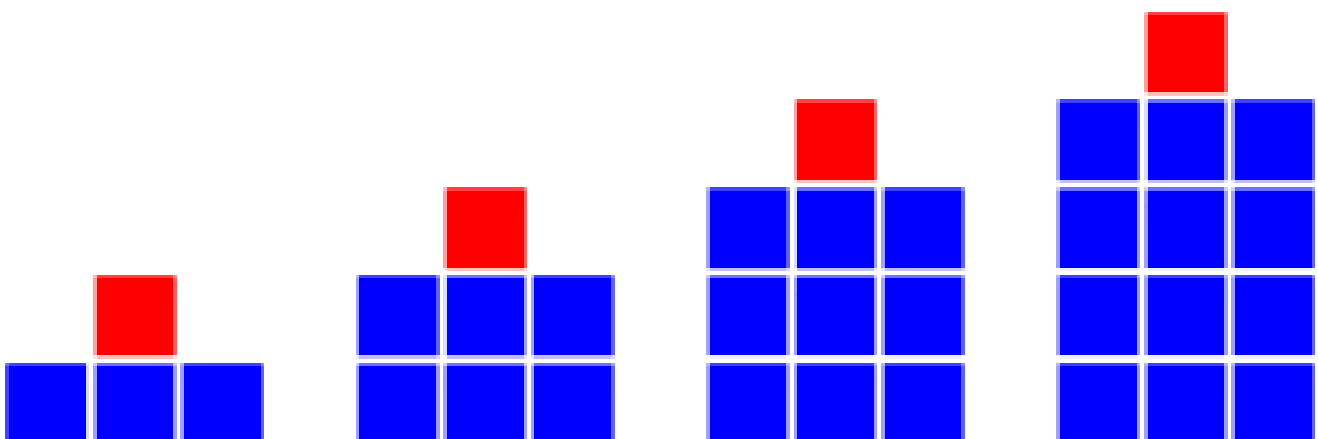
Expand  $3(2x + 4) = 6x + 12$



# THE ALGEBRA PROJECT

Kim Freeman

## THE ANSWERS



**The Algebra Project**  
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# Algebra Like Terms

Simplify all these expressions

- $a + a + a = 3a$
- $x + x = 2x$
- $p + p + p + p + p = 5p$
- $y + y + y = 3y$
- $i + i + i + i + i + i + i = 7i$
- $k + k + k + k = 4k$
- $4x + x = 5x$
- $10x + 2x = 12x$
- $7y + 3y + y = 11y$
- $8j + j + j = 10j$
- $15d + 3d + 2d = 20d$
- $12a + 5a + 2a = 19a$
- $13c - 4c = 9c$
- $17u - 8u = 9u$
- $8g - 10g = -2g$
- $6r + 6r - r = 11r$
- $5t - 3t + 2t = 4t$
- $10h + 8h - 12h = 6h$
- $6x - 8x + x = -x$
- $-8y + 6y - 5y = -7y$
- $3r - 8r + 2r = -3r$
- $-5w - 6w = -11w$
- $3y - 6y + y = -2y$
- $f - 6f - 2f = -7f$
- $4\pi - 7\pi + \pi = -2\pi$
- $3e + 8s - 2s - e = 2e + 6s$
- $9i + 4j - 6i - 2j = 3i - 2j$
- $-3t - 5s + 5t - 2s = 2t - 7s$
- $4d + 2e - 4e - 8d = -4d - 2e$
- $-4f + 3h - 7h + 5f = f - 4h$
- $11 + 3n + 6 + 9n = 17 + 12n$
- $8 + 3d - 6 - 5d = 2 - 2d$
- $r - 5 - 5r + 10 = -4r + 5$
- $g - 12g + 7h = -11g + 7h$
- $7j + 5k - 2j - 5k = 5j$
- $4w - 6x + 8w - 6w = 6w - 12x$
- $-4m + 6n - 10n + 4n = -4m$
- $5j + 5k = 5j + 5k$
- $2s - 7t - 10s - 2t = -8s - 9t$
- $3x^2 + 2x^2 - 3y = 5x^2 - 3y$
- $5x + 6y - 7y - 8x = -3x - y$
- $-2q + 2p - 7p + 8q = 6q - 5p$
- $14ab - 7b - 3ab + 8b = 11ab + b$
- $-2xy + 5x - 8xy - 9x = -10xy - 4x$
- $8y^2 - 2y + 5y - 6y^2 = 2y^2 + 3y$
- $16gh + 8g - 7gh = 9gh + 8g$
- $6bg - 2b - 5bg = bg - 2b$
- $-6mn - 12n + 8n = -6mn - 4n$
- $-5xy + 12yx = 7xy$
- $-9ab + 8ab + 9b = -ab + 9b$

# Algebra Expressions

Simplify all these expressions

- $a \times a = a^2$
- $y \times y \times y \times y = y^4$
- $p \times p \times p = p^3$
- $w \times w \times w \times w \times w = w^5$
- $i \times i \times i \times i \times i \times i \times i = i^7$
- $k \times k \times k \times k = k^4$
- $5a \times b = 5ab$
- $8f \times 2x = 16fx$
- $9y \times 3z = 27yz$
- $5j \times 3k = 15jk$
- $7d \times 3e = 21de$
- $12a \times 5b = 60ab$
- $15c \times 2c = 30c^2$
- $7u \times 8u = 56u^2$
- $8g \times 5g = 40g^2$
- $6r^2 \times r = 6r^3$
- $5p^3q \times 3p = 15p^4q$
- $10h^2j \times 8j = 80h^2j^2$
- $6b^2 \times 9b^3 \times b = 54b^6$
- $-8y \times 6y^2 = -48y^3$
- $4r^2 \times 9r^2 \times r = 36r^5$
- $-5w^3 \times -6w^2 = 30w^5$
- $3y^2 \times 6y^2 \times y = 18y^5$
- $2g^3f \times 6gf^2 = 12g^4f^3$
- $4p^2qr \times 5r \times 2p = 40p^3r^2q$

- $(y^2)^3 = y^6$
- $(5a)^2 = 25a^2$
- $(2y)^5 = 32y^5$
- $(3y)^2 = 9y^2$
- $(5xy)^3 = 125x^3y^3$
- $-(2x)^2 = -4x^2$
- $(-2x)^2 = 4x^2$
- $(x^2)^7 = x^{14}$
- $(k^5h^3)^2 = k^{10}h^6$
- $w^0 = 1$
- $\sqrt{49a^{12}} = 7a^6$
- $\sqrt{x^{100}} = x^{50}$
- $\sqrt{9a^{10}b^6} = 3a^5b^3$
- $\sqrt{100g^2h^{100}} = 10gh^{50}$

If  $a = 2$ ,  $b = 5$ ,  $c = 10$  calculate:

- $(a + b)^2 = 49$
- $4b^2 - (3a)^2 = 64$
- $(a^2)^2 = 16$
- $c^2 - a^2 + b^2 = 121$
- $(2a^2)^3 = 256$
- $2c^2 - 2b^2 = 150$
- $4(a^4 - 10) = 24$
- $b(5c - a^2) = 230$
- $3a + 4c^3 = 4006$
- $\sqrt{c^2} = 10$

# Algebraic Fractions

Simplify all these fractions!

$$\blacksquare \frac{3}{x} + \frac{5}{x} = \frac{8}{x}$$

$$\blacksquare \frac{7}{w} - \frac{5}{w} = \frac{2}{w}$$

$$\blacksquare \frac{x}{5} + \frac{x}{15} = \frac{4x}{15}$$

$$\blacksquare \frac{b}{3} - \frac{b}{6} = \frac{b}{6}$$

$$\blacksquare \frac{3}{x^2} - \frac{2}{x} = \frac{3 - 2x}{x^2}$$

$$\blacksquare \frac{2}{x^2} + \frac{5}{x^3} = \frac{2x + 5}{x^3}$$

$$\blacksquare \frac{7}{x^3} + \frac{4}{x} = \frac{7 + 4x^2}{x^3}$$

$$\blacksquare \frac{s}{4} + \frac{5}{3} = \frac{3s + 20}{12}$$

$$\blacksquare \frac{x}{3} - \frac{x}{7} = \frac{4x}{21}$$

$$\blacksquare \frac{r}{5} + \frac{r}{8} = \frac{13r}{40}$$

$$\blacksquare \frac{x}{a} + \frac{2}{d} = \frac{xd + 2a}{ad}$$

$$\blacksquare \frac{4}{m} - \frac{x}{n} = \frac{4n - xm}{mn}$$

$$\blacksquare \frac{5}{a} + \frac{c}{c} = \frac{5c + ac}{ac} \text{ or } \frac{5+a}{a} \text{ or } 1\frac{5}{a}$$

$$\blacksquare \frac{7}{xy} + \frac{2}{y} = \frac{7y + 2xy}{xy^2} \text{ or } \frac{7 + 2x}{xy}$$

$$\blacksquare \frac{5}{ab} - \frac{3}{a} = \frac{5a - 3ab}{a^2b} \text{ or } \frac{5 - 3b}{ab}$$

$$\blacksquare \frac{7}{x} + \frac{7}{xy} = \frac{7xy + 7x}{x^2y} \text{ or } \frac{7y - 7}{xy}$$

$$\blacksquare \frac{3h}{gh} = \frac{3}{g}$$

$$\blacksquare \frac{12m}{4n} = \frac{3m}{n}$$

$$\blacksquare \frac{6x}{18y} = \frac{x}{3y}$$

$$\blacksquare \frac{mp}{6m} = \frac{p}{6}$$

$$\blacksquare \frac{3m^4}{6m^2} = \frac{m^2}{2}$$

$$\blacksquare \frac{2w^5}{8w^2} = \frac{w^3}{4}$$

$$\blacksquare \frac{8x^5}{4x^3} = 2x^2$$

$$\blacksquare \frac{50r^4}{5r^2} = 10r^2$$

$$\blacksquare \frac{7e^7}{21e^3} = \frac{e^4}{3}$$

$$\blacksquare \frac{5m^5}{25m^2} = \frac{m^3}{5}$$

$$\blacksquare \frac{12r^9}{3r^3} = 4r^6$$

$$\blacksquare \frac{27m^4}{9m} = 3m^3$$

$$\blacksquare \frac{8a}{15n} \times \frac{5c}{4a} = \frac{2c}{3n}$$

$$\blacksquare \frac{3c}{4e} \times \frac{2e^3}{4c} = \frac{3e^2}{8}$$

$$\blacksquare \frac{5ab}{2a} \times \frac{5a}{10ab} = \frac{5}{4}$$

$$\blacksquare \frac{3}{4b} \times \frac{15}{8b^2} = \frac{45}{32b^3}$$

$$\blacksquare \frac{3ab^2}{5cd} \times \frac{5c}{7a^2} = \frac{3b^2}{7ad}$$

If  $a = \frac{1}{2}$ ,  $b = 4$ ,  $c = 10$  calculate:

$$\blacksquare \frac{2a + b}{c} = \frac{1}{2}$$

$$\blacksquare \frac{4a^2}{b} = \frac{1}{4}$$

$$\blacksquare \frac{ac}{bc} \times \frac{bc}{ac} = 1$$

$$\blacksquare \frac{c}{b} \times \frac{a}{b} = \frac{15}{16}$$

$$\blacksquare \frac{c}{b} \div \frac{a}{b} = 20$$

$$\blacksquare \frac{4a + c}{b} = 3$$

$$\blacksquare a(c^2 + b^2) = 58$$

# Algebra Expansions

Expand all these expressions!

- ▣  $2(6 + 3x) = 12 + 6x$
- ▣  $5(3a - 5) = 15a - 25$
- ▣  $10(2b + 4) = 20b + 40$
- ▣  $4(5c - 7) = 20c - 28$
- ▣  $7(4d + 5e) = 28d + 35e$
- ▣  $4f(f + 2) = 4f^2 + 8f$
- ▣  $5g(2g - 6) = 10g^2 - 30g$
- ▣  $6h(h - 5) = 6h^2 - 30h$
- ▣  $5x(x^2 - x + 2)$   
 $= 5x^3 - 5x^2 + 10x$
- ▣  $3(a^2 - 18a + 4)$   
 $= 3a^2 - 54a + 12$
- ▣  $2(a + 2b) + (a - b)$   
 $= 2a + 4b + a - b$   
 $= 3a + 3b$
- ▣  $4(c - 3d) - 3(c + 2d)$   
 $= 4c - 12d - 3c - 6d$   
 $= c - 18d$
- ▣  $3(2e - f) + 2(e - 2f)$   
 $= 6e - 3f + 2e - 4f$   
 $= 8e - 7f$
- ▣  $3x(x + 8) - x(2x - 5)$   
 $= 3x^2 + 24x - 2x^2 - 5x$   
 $= x^2 + 19x$

- ▣  $(x + 5)(x + 1)$   
 $= x^2 + 6x + 5$
- ▣  $(x - 7)(x - 2)$   
 $= x^2 - 9x + 14$
- ▣  $(a + 6)(a - 4)$   
 $= a^2 + 2a - 24$
- ▣  $(x - 3)(x + 8)$   
 $= x^2 + 5x - 24$
- ▣  $(x - 9)(x + 9)$   
 $= x^2 - 81$
- ▣  $(x + 6)^2$   
 $= x^2 + 12x + 36$
- ▣  $(x + 9)(x + 4)$   
 $= x^2 + 13x + 36$
- ▣  $(x - 12)(x + 1)$   
 $= x^2 - 11x - 12$
- ▣  $(x + 11)(x - 5)$   
 $= x^2 + 6x - 55$
- ▣  $(x - 15)(x - 3)$   
 $= x^2 - 18x + 45$
- ▣  $(7 - x)(7 + x)$   
 $= 49 - x^2$
- ▣  $(9 - x)^2$   
 $= 81 - 18x + x^2$
- ▣  $(3x + 2)(2x - 1)$   
 $= 6x^2 + x - 2$
- ▣  $(4x - 7)(x + 5)$   
 $= 4x^2 + 13x - 35$

# Quadratic Expansions

Want extra practise? Expand all these expressions!

- $$\begin{aligned} \square & (x + 3)(x - 8) \\ & = x^2 - 5x - 24 \end{aligned}$$
- $$\begin{aligned} \square & (x + 12)(x + 10) \\ & = x^2 + 22x + 120 \end{aligned}$$
- $$\begin{aligned} \square & (x - 9)(x - 8) \\ & = x^2 - 17x + 72 \end{aligned}$$
- $$\begin{aligned} \square & (x - 7)(x + 11) \\ & = x^2 + 4x - 77 \end{aligned}$$
- $$\begin{aligned} \square & (x + 8)(x + 1) \\ & = x^2 + 9x + 8 \end{aligned}$$
- $$\begin{aligned} \square & (x - 2)(x + 6) \\ & = x^2 + 4x - 12 \end{aligned}$$
- $$\begin{aligned} \square & (x - 5)(x - 13) \\ & = x^2 - 18x + 65 \end{aligned}$$
- $$\begin{aligned} \square & (x + 15)(x - 4) \\ & = x^2 + 11x - 60 \end{aligned}$$
- $$\begin{aligned} \square & (x + 3)(6 - x) \\ & = -x^2 + 3x + 18 \end{aligned}$$
- $$\begin{aligned} \square & (2 - x)(3 - x) \\ & = 6 - 5x + x^2 \end{aligned}$$
- $$\begin{aligned} \square & (10 - x)(4 + x) \\ & = 40 + 6x - x^2 \end{aligned}$$
- $$\begin{aligned} \square & (x + 8)(5 + x) \\ & = x^2 + 13x + 40 \end{aligned}$$
- $$\begin{aligned} \square & (2x + 3)(3x + 4) \\ & = 6x^2 + 17x + 12 \end{aligned}$$
- $$\begin{aligned} \square & (5x + 1)(x - 2) \\ & = 5x^2 - 9x - 2 \end{aligned}$$
- $$\begin{aligned} \square & (4x - 5)(2x - 3) \\ & = 8x^2 - 22x + 15 \end{aligned}$$
- $$\begin{aligned} \square & (1 - 2x)(3x + 7) \\ & = -6x^2 - 11x + 7 \end{aligned}$$
- $$\begin{aligned} \square & (x - 5)(x + 5) \\ & = x^2 - 25 \end{aligned}$$
- $$\begin{aligned} \square & (1 + 2x)(1 - 2x) \\ & = 1 - 4x^2 \end{aligned}$$
- $$\begin{aligned} \square & (3 - 2x)(2x + 3) \\ & = 9 - 4x^2 \end{aligned}$$
- $$\begin{aligned} \square & (x + 7)^2 \\ & = x^2 + 14x + 49 \end{aligned}$$
- $$\begin{aligned} \square & (x - 5)^2 \\ & = x^2 - 10x + 25 \end{aligned}$$
- $$\begin{aligned} \square & (3x - 1)^2 \\ & = 9x^2 - 6x + 1 \end{aligned}$$
- $$\begin{aligned} \square & (2 + 4x)^2 \\ & = 4 + 16x + 16x^2 \end{aligned}$$
- $$\begin{aligned} \square & (2x + 3y)^2 \\ & = 4x^2 + 12xy + 9y^2 \end{aligned}$$
- $$\begin{aligned} \square & (6x - 2y)^2 \\ & = 36x^2 - 24xy + 4y^2 \end{aligned}$$
- $$\begin{aligned} \square & 2(3x - 5)^2 \\ & = 18x^2 - 60x + 50 \end{aligned}$$

# Factorising

Factorise all these expressions!

$$\blacktriangleright 10a - 15b = 5(2a - 3b)$$

$$\blacktriangleright 3xy - 12xz = 3x(y - 4z)$$

$$\blacktriangleright 4\pi x - 4\pi y = 4\pi(x - y)$$

$$\blacktriangleright 4a^2 + 3a = a(4a + 3)$$

$$\blacktriangleright x^3 + x^2y = x^2(x + y)$$

$$\blacktriangleright a^2 - 6y = a^2 - 6y$$

$$\blacktriangleright 2x^3 + 2xy^2 = 2x(x^2 + y^2)$$

$$\blacktriangleright y^2 + y^3 = y^2(1 + y)$$

$$\blacktriangleright x^2 + 2xy - x = x(x + 2y - 1)$$

$$\blacktriangleright 36 + 6x^3 = 6(6 + x^3)$$

$$\blacktriangleright 2\pi r^2 + \pi rh = \pi r(2r + h)$$

$$\blacktriangleright x^2y - xy^2 = xy(x - y)$$

$$\blacktriangleright 6a^2 - 12a = 6a(a - 2)$$

$$\blacktriangleright 5x^2 - 10xy = 5x(x - 2y)$$

$$\blacktriangleright 4a^2b + 8ab^2 = 4ab(a + 2b)$$

$$\blacktriangleright 21x^3 - 14x^2 = 7x^2(3x - 2)$$

$$\blacktriangleright t^2 + 4t = t(t + 4)$$

$$\blacktriangleright 20x + 10x - 5y + 25y$$

$$= 10(3x + 2y)$$

$$\blacktriangleright 14a^2b + 21ab^2$$

$$= 7ab(2a + 3b)$$

$$\blacktriangleright 3c + 6d + 12e$$

$$= 3(c + 2d + 4e)$$

$$\blacktriangleright 14a - 21b = 7(2a - 3b)$$

$$\blacktriangleright 3xy + 9yz = 3y(x + 3z)$$

$$\blacktriangleright 2\pi d - 2\pi r = 2\pi(d - r)$$

$$\blacktriangleright 15x^2 - 20x = 5x(3x^2 - 4)$$

$$\blacktriangleright r^3 + r^2 = r^2(r + 1)$$

$$\blacktriangleright 9 + 3n^2 = 3(3 + n^2)$$

$$\blacktriangleright 2x^2 + 2xy = 2x(x + y)$$

$$\blacktriangleright 5x^2 + 5y^2 = 5(x^2 + y^2)$$

$$\blacktriangleright 3x^3 + xy = x(3x^2 + y)$$

$$\blacktriangleright 4ab - 8bc = 4b(a - 2c)$$

$$\begin{aligned} \blacktriangleright ax + bx + ay + by \\ &= x(a + b) + y(a + b) \\ &= (x + y)(a + b) \end{aligned}$$

$$\begin{aligned} \blacktriangleright 1 + x + y + xy \\ &= 1(1 + x) + y(1 + x) \\ &= (1 + y)(1 + x) \end{aligned}$$

$$\begin{aligned} \blacktriangleright 2xy + 6y - x - 3 \\ &= 2y(x + 3) - 1(x + 3) \\ &= (2y - 1)(x + 3) \end{aligned}$$

$$\begin{aligned} \blacktriangleright a^2 + ab - ac - bc \\ &= a(a + b) - c(a + b) \\ &= (a - c)(a + b) \end{aligned}$$

$$\begin{aligned} \blacktriangleright 2ax - 4ay - 3bx + 6by \\ &= 2a(x - 2y) - 3b(x - 2y) \\ &= (2a - 3b)(x - 2y) \end{aligned}$$



# Quadratic Factoring 1

Factorise all these quadratic expressions!

- $x^2 + 11x + 24$   
=  $(x + 8)(x + 3)$
  - $x^2 + 11x + 18$   
=  $(x + 9)(x + 2)$
  - $x^2 - 100$   
=  $(x + 10)(x - 10)$
  - $x^2 + 9x - 36$   
=  $(x + 12)(x - 3)$
  - $x^2 + 5x - 84$   
=  $(x + 12)(x - 7)$
  - $x^2 + 6x - 16$   
=  $(x + 8)(x - 2)$
  - $x^2 - 3x - 40$   
=  $(x - 8)(x + 5)$
  - $x^2 - 11x + 18$   
=  $(x - 9)(x + 2)$
  - $2x^2 - 20x + 32$   
=  $2(x^2 - 10x + 16)$   
=  $2(x - 8)(x - 2)$
  - $3x^2 - 24x + 48$   
=  $3(x^2 - 8x + 16)$   
=  $3(x - 4)^2$
  - $2x^2 - 2x - 84$   
=  $2(x^2 - x - 42)$   
=  $2(x - 7)(x - 6)$
  - $36 - x^2$   
=  $(6 - x)(6 + x)$
- $4x^2 + 21x + 5$        $4 \times 5 = 20$  (20, 1)  
=  $4x^2 + 20x + x + 5$   
=  $4x(x + 5) + 1(x + 5)$   
=  $(4x + 1)(x + 5)$
  - $3x^2 + 16x + 13$        $3 \times 13 = 39$  (3, 13)  
=  $3x^2 + 3x + 13x + 13$   
=  $3x(x + 1) + 13(x + 1)$   
=  $(3x + 13)(x + 1)$
  - $2x^2 - 2x - 12$        $2 \times 12 = 24$  (4, 6)  
=  $2x^2 + 4x - 6x - 12$   
=  $2x(x + 2) - 6(x + 2)$   
=  $(2x - 6)(x + 2)$  or  $2(x - 3)(x + 2)$
  - $8x^2 - 18x + 9$        $8 \times 9 = 72$  (12, 6)  
=  $8x^2 - 12x - 6x + 9$   
=  $4x(2x - 3) - 3(2x - 3)$   
=  $(4x - 3)(2x - 3)$
  - $5x^2 - 13x - 6$        $5 \times 6 = 30$  (10, 3)  
=  $5x^2 - 10x - 3x - 6$   
=  $5x(x - 2) + 3(x - 2)$   
=  $(5x + 3)(x - 2)$
  - $6x^2 + 16x + 10$        $6 \times 10 = 60$  (10, 6)  
=  $6x^2 + 6x + 10x + 10$   
=  $6x(x + 1) + 10(x + 1)$   
=  $(6x + 10)(x + 1)$  or  $2(3x + 5)(x + 1)$
  - $6x^2 + 5x - 6$        $6 \times 6 = 36$  (9, 4)  
=  $6x^2 + 9x - 4x - 6$   
=  $3x(2x + 3) - 2(2x + 3)$   
=  $(3x - 2)(2x + 3)$
  - $10x^2 + 6x - 4$        $10 \times 4 = 40$  (10, 4)  
=  $10x^2 + 10x - 4x - 4$   
=  $10x(x + 1) - 4(x + 1)$   
=  $(10x - 4)(x + 1)$  or  $2(5x - 2)(x + 1)$
  - $3x^2 - 6x - 24$        $3 \times 24 = 82$  (12, 6)  
=  $3x^2 - 12x + 6x - 24$   
=  $3x(x - 4) + 6(x - 4)$   
=  $(3x + 6)(x - 4)$  or  $3(x + 2)(x - 4)$

# Quadratic Factoring 2

Factorise all these quadratic expressions!

- $3x^2 + 33x + 84$   
=  $3(x + 4)(x + 7)$

- $2x^2 + 20x + 32$   
=  $2(x + 2)(x + 8)$

- $4x^2 + 12x - 72$   
=  $4(x - 3)(x + 6)$

- $2x^2 + 14x - 36$   
=  $2(x - 2)(x + 9)$

- $2x^2 - 6x - 80$   
=  $(x + 5)(x - 8)$

- $3x^2 - 12x - 15$   
=  $3(x + 1)(x - 5)$

- $2x^2 - 36x + 160$   
=  $2(x - 8)(x - 10)$

- $5x^2 - 20x + 15$   
=  $5(x - 1)(x - 3)$

- $4x^2 - 88x + 484$   
=  $4(x - 11)^2$

- $2x^2 + 20x + 50$   
=  $2(x + 5)^2$

- $4x^2 - 400$   
=  $4(x - 10)(x + 10)$

- $6x^2 - 6$   
=  $6(x + 1)(x - 1)$

- $2x^2 + 5x + 3$   
=  $(2x + 3)(x + 1)$

- $3x^2 + 8x + 4$   
=  $(3x + 2)(x + 2)$

- $8x^2 + 14x + 3$   
=  $(4x + 1)(2x + 3)$

- $3x^2 + 5x - 2$   
=  $(3x - 1)(x + 2)$

- $2x^2 + 3x - 5$   
=  $(2x + 5)(x - 1)$

- $5x^2 - 8x + 3$   
=  $(5x - 3)(x - 1)$

- $3x^2 - 17x + 10$   
=  $(3x - 2)(x - 5)$

- $2x^2 - 3x - 9$   
=  $(2x + 3)(x - 3)$

- $2x^2 + 9x - 18$   
=  $(2x - 3)(x + 6)$

- $15x^2 + x - 6$   
=  $(5x - 3)(3x + 2)$

- $4x^2 + 2x - 20$   
=  $2(x - 2)(2x + 5)$

- $8x^2 + 14x + 6$   
=  $2(4x + 3)(x + 1)$

# Quadratic Factoring 3

Factorise all these quadratic expressions!

$$\begin{aligned} \bullet \quad 6x^2 + 14x + 4 \\ &= (2x + 4)(3x + 1) \end{aligned}$$

$$\begin{aligned} \bullet \quad 4x^2 + 6x + 2 \\ &= (2x + 1)(2x + 2) \end{aligned}$$

$$\begin{aligned} \bullet \quad 8x^2 - 10x + 3 \\ &= (4x - 3)(2x - 1) \end{aligned}$$

$$\begin{aligned} \bullet \quad 3x^2 + x - 2 \\ &= (3x - 2)(x + 1) \end{aligned}$$

$$\begin{aligned} \bullet \quad 3x^2 + 16x + 5 \\ &= (3x + 1)(x + 5) \end{aligned}$$

$$\begin{aligned} \bullet \quad 3x^2 + 11x + 10 \\ &= (3x + 5)(x + 2) \end{aligned}$$

$$\begin{aligned} \bullet \quad 14x^2 + x - 3 \\ &= (7x - 3)(2x + 1) \end{aligned}$$

$$\begin{aligned} \bullet \quad 4x^2 - 8x - 5 \\ &= (2x - 5)(2x + 1) \end{aligned}$$

$$\begin{aligned} \bullet \quad 6x^2 + 13x - 28 \\ &= (2x + 7)(3x - 4) \end{aligned}$$

$$\begin{aligned} \bullet \quad 2x^2 + 5x + 3 \\ &= (2x + 3)(x + 1) \end{aligned}$$

$$\begin{aligned} \bullet \quad 5x^2 + 12x + 4 \\ &= (5x + 2)(x + 2) \end{aligned}$$

$$\begin{aligned} \bullet \quad 4x^2 - 12x - 7 \\ &= (2x - 7)(2x + 1) \end{aligned}$$

$$\begin{aligned} \bullet \quad 2x^2 - x - 10 \\ &= (2x - 5)(x + 2) \end{aligned}$$

$$\begin{aligned} \bullet \quad 6x^2 + x - 2 \\ &= (2x - 1)(3x + 2) \end{aligned}$$

$$\begin{aligned} \bullet \quad 8x^2 + 14x + 3 \\ &= (4x + 1)(2x + 3) \end{aligned}$$

$$\begin{aligned} \bullet \quad 2x^2 + 9x - 5 \\ &= (2x - 1)(x + 5) \end{aligned}$$

$$\begin{aligned} \bullet \quad 2x^2 - 24x + 40 \\ &= (2x - 4)(x - 10) \end{aligned}$$

$$\begin{aligned} \bullet \quad 16x^2 - 49 \\ &= (4x + 7)(4x - 7) \end{aligned}$$

$$\begin{aligned} \bullet \quad 2x^2 - 50 \\ &= 2(x + 5)(x - 5) \end{aligned}$$

$$\begin{aligned} \bullet \quad 3x^2 - 147 &= 3(x^2 - 49) \\ &= 3(x - 7)(x + 7) \end{aligned}$$

$$\begin{aligned} \bullet \quad 4 - 36x^2 \\ &= 4(1 - 3x)(1 + 3x) \end{aligned}$$

$$\bullet \quad 2x^2 + 4x - 48 = 0$$

$$(2x - 8)(x + 6) = 0$$

$$x = 4 \text{ or } x = -6$$

$$2x^2 + 2x - 12 = 0$$

$$(2x + 6)(x - 2) = 0$$

$$x = -3 \text{ or } x = 2$$

$$2 \times -3 = -6 \text{ and } 2 \times 2 = 4$$

The solutions are 2x bigger

# Algebra (1)

Simplify all these expressions

- $3a + 8a = 11a$
- $b + b + 5b = 7b$
- $4c + 6c - 8c = 2c$
- $5d - d - 3e - 2e = 4d - 5e$
- $5e + 2f + 3e - 4f = 8e - 2f$
- $3g - 3h + 4g - 5h = 7g - 8h$
- $a \times a \times 5 = 5a^2$
- $9b \times b \times b \times b = 9b^4$
- $14y \times 2 = 28y$
- $c \div c = 1$
- $12d \div 4 = 3d$
- $5e \times 6 = 30e$
- $4f \times f = 4f^2$
- $2g \times 8h = 16gh$
- $20m \div 10m = 2$
- $15xy \div 3 = 5xy$
- $6r \times 7r = 42r^2$
- $5g^2 \times 4g = 20g^3$

Expand and simplify all these expressions

- $2(a + 4b) = 2a + 8b$
  - $5(3c - 8) = 15c - 40$
  - $-10(2d - 3) = -20d + 30$
  - $-2(4f + 3e) = -8f - 6e$
  - $2(a + 2) + (a - 1)$   
 $= 3a + 3$
  - $4(b - 1) + 2(b + 2)$   
 $= 6b + 4$
  - $(c - d) + 6(c + 2d)$   
 $= 7c + 11d$
  - $3(e + 9) - 2e$   
 $= e + 27$
  - $f(2f - 4) + 4f$   
 $= 2f^2$
- 
- $x = 3$  and  $y = 4$   
Find the values:
- 
- $2x + 3y = 18$
  - $2(x + y) = 14$
  - $5x - 2y = 7$
  - $x^2 - y^2 = -7$
  - $3x^2 = 27$
  - $\frac{6x + y}{2} = 4$
  - $xy^2 = 48$
  - $\sqrt{(4x + y)} = 4$

# Algebra (2)

Simplify all these expressions

$$3a + 2a - 4a = 2a$$

$$6b - 4b - 3b = -b$$

$$2c - 3d + c - d = 3c - 4d$$

$$ab + ba = 2ab$$

$$a^2 \times a \times 5 = 5a^3$$

$$9b \times 6b = 54b^2$$

$$14y \times 2 = 28y$$

$$c^2 \div c = c$$

$$d^3 \div d^2 = d$$

$$5e \times 8e = 40e^2$$

$$4f \times 9g = 36fg$$

$$2g \times 8g^2 = 16g^3$$

$$8m^3 \div 2m^2 = 4m$$

$$15x^3 \div 3x^3 = 45$$

$$4d^2 + 4d^2 = 8d^2$$

$$ef + 5ef - 4ef = 2ef$$

$$4ab \times 4ac \div 4bc = 4a^2$$

$$(3x^2y^3)^2 = 9x^4y^6$$

$$\sqrt{(25r^2s^6)} = 5rs^3$$

$$a^3 \times a^5 = a^8$$

$$(b^5)^2 = b^{10}$$

$$c^5 \div c^3 = c^2$$

$$d^7 \times d^7 = d^{14}$$

$$(3e^4)^3 = 27e^{12}$$

$$f \times f^3 \times f^3 = f^7$$

$$3g^5 \times 2g^3 = 6g^8$$

$$3h^6 \div h^2 = 3h^4$$

$$2(3a - 5) = 6a - 10$$

$$4(6b + 8c) = 24b + 32c$$

$$5d(d^2 - d + 2) = 5d^3 - 5d^2 + 10d$$

$$5a + (-2a) = 3a$$

$$b - (-2b) = 3b$$

$$(-5c) - (-8c) = 3c$$

$$(-2d) + (-2d) = 0$$

$$-8y \times 4y = -32y^2$$

$$6 \times -4r^2 = -24r^2$$

$$-3g^2 \times -4g^3 = 12g^5$$

$$-6x \div -6x = 1$$

# Algebra (3)

Simplify all these expressions

$$\bullet 13a + (2a - 8a) = 7a$$

$$\bullet 2b - 3c + b - 2c = 3b - 5c$$

$$\bullet 6a^2 \times 7 \times a = 42a^3$$

$$\bullet 9b \times 6b^3 = 54b^4$$

$$\bullet 14y \times 2y = 28y^2$$

$$\bullet c^8 \div c^2 = c^6$$

$$\bullet (8d)^2 = 64d^2$$

$$\bullet e^3 + e^3 = 2e^3$$

$$\bullet 4f^2 + 4f^2 = 8f^2$$

$$\bullet 5ab \times 5ac \div 5bc = 5a^2$$

$$\bullet (8e^2f^3)^2 = 64e^4f^6$$

$$\bullet \sqrt{(16g^2h^8)} = 4gh^4$$

$$\bullet (-2x)^2 = 4x^2$$

$$\bullet -(3x)^3 = -27x^3$$

$$\bullet 2(a + 2b) + (a - b)$$

$$= 3a + 5b$$

$$\bullet x(2x + 3) - 4(3x - 1)$$

$$= 2x^2 - 9x + 4$$

$$\bullet x(x^2 + 1) - x^2(x + 1)$$

$$= x - x^2$$

$$\bullet \frac{12a^2b}{3ab} = 4a$$

$$\bullet \frac{6cd^2}{2c^2d} = \frac{3d}{c}$$

$$\bullet \frac{10e}{3} - \frac{2e}{9} = \frac{28e}{9}$$

$$\bullet \frac{d}{21} + \frac{4d}{7} = \frac{13d}{21}$$

$$\bullet \frac{3x}{8} - \frac{x}{6} = \frac{10x}{48}$$

$$\bullet \frac{5a}{2} + \frac{a}{10} = \frac{26a}{10}$$

$$\bullet \frac{4c}{15d} \times \frac{5e}{8} = \frac{ce}{6d}$$

$$\bullet \frac{3}{4e} \div \frac{15}{8e^2} = \frac{2e}{5}$$

$$\bullet \frac{3ab}{10cd} \times \frac{2c^3}{9b^2} = \frac{ac^2}{15db}$$

$$\bullet \frac{8}{9x} \div \frac{2}{3x} = \frac{4}{3}$$

$$\bullet \frac{a+1}{2} + \frac{a-1}{3} = \frac{5a+1}{6}$$

$$\bullet \frac{b+c}{2} - \frac{b-2}{5} = \frac{3a+5c+4}{10}$$

$$\bullet \frac{3x-1}{3} + \frac{2-x}{4} = \frac{9x+2}{12}$$

$$\bullet \frac{4a+1}{2} - \frac{a+3}{5} = \frac{18a-1}{10}$$

---


$$x = 5 \quad y = 3 \quad z = 1$$


---

$$\bullet 6x + 2y = \underline{\quad 36 \quad} \quad \bullet x^2 + y^2 = \underline{\quad 34 \quad}$$

$$\bullet 2x^2 + 3z = \underline{\quad 53 \quad} \quad \bullet (4x - 2z)^2 = \underline{\quad 324 \quad}$$

---


$$a = \frac{1}{2} \quad b = \frac{1}{8} \quad c = \frac{3}{4}$$


---

$$\bullet a + b = \underline{\quad \frac{5}{8} \quad} \quad \bullet 24a^2 + 4c = \underline{\quad 9 \quad}$$

$$\bullet \frac{a}{b} = \underline{\quad 4 \quad} \quad \bullet 4c(12a^2b) = \underline{\quad \frac{9}{8}(1\frac{1}{8}) \quad}$$

# Equations (1)

Solve all these equations!

▶  $4x = 44$

$x = 11$

▶  $x - 8 = 9$

$x = 17$

▶  $\frac{1}{4}x = 8$

$x = 32$

▶  $x + 7 = 19$

$x = 12$

▶  $6x = 48$

$x = 8$

▶  $\frac{1}{2}x = 20$

$x = 40$

▶  $x - 5 = 25$

$x = 30$

▶  $5x = 0$

$x = 0$

▶  $x - 14 = 5$

$x = 19$

▶  $x + 8 = 24$

$x = 16$

▶  $3x + 2 = 14$

$x = 4$

▶  $14 + 4x = 22$

$x = 2$

▶  $12x - 5 = 31$

$x = 3$

▶  $8x - 21 = 11$

$x = 4$

▶  $3x = x + 10$

$x = 5$

▶  $7x + 3 = 33 + x$

$x = 5$

▶  $15x - 4 = 3x + 8$

$x = 1$

▶  $4 + 6x = 12 + 2x$

$x = 2$

▶  $3(x + 2) - x = 30$

$x = 12$

▶  $4(1 + 2x) = 52$

$x = 6$

▶  $6(x + 1) - 6 = 30$

$x = 5$

▶  $2(x - 9) = x - 4$

$x = 14$

▶  $8x = 2(3x + 6)$

$x = 6$

▶  $5(x + 8) = 50$

$x = 2$

▶  $(x + 1) + 2(x + 2) = 29$

$x = 8$

▶  $4(3x - 1) + 10 = 18$

$x = 1$

▶  $x + (x + 2) + (x + 4) = 21$

$x = 5$

▶  $6x - 3 = 2x + 25$

$x = 7$

▶  $4(x + 5) = 3(16 - x)$

$x = 4$

▶  $8x - 5 = 2x + 13$

$x = 3$

▶  $6x - 4 = 2(x + 14)$

$x = 8$

# Equations (2)

Solve all these equations!

- |                               |           |                               |           |
|-------------------------------|-----------|-------------------------------|-----------|
| ■ $a - 5 = 19$                | $a = 24$  | ■ $2(2a - 5) + 4(a + 7) = 82$ |           |
| ■ $b + 19 = 38$               | $b = 19$  |                               | $b = 8$   |
| ■ $7c = 91$                   | $c = 13$  | ■ $5(c + 3) + (c - 6) = 27$   |           |
| ■ $\frac{d}{5} = 20$          | $d = 100$ |                               | $c = 3$   |
| ■ $11e - 5 = 72$              | $e = 7$   | ■ $2(e + 5) - 3(6 - e) = 42$  |           |
| ■ $2f + 4 = 80$               | $f = 38$  |                               | $e = 10$  |
| ■ $g + 5g = 72$               | $g = 12$  | ■ $5(f - 1) + 3(f - 4) = 23$  |           |
| ■ $3h = 90 - 2h$              | $h = 18$  |                               | $f = 5$   |
| ■ $3j + 2 = 27 - 2j$          | $j = 5$   | ■ $22 - 5k - (k + 10) = 0$    |           |
| ■ $36 - 4k = 0$               | $k = 9$   |                               | $k = 2$   |
| ■ $12m - 5 = 15 + 8m$         |           | ■ $2(3m - 4) - 2(m + 7) = 0$  |           |
|                               | $m = 5$   |                               | $m = 5.5$ |
| ■ $2n + 7 = 31 - 4n$          |           | ■ $5n - 2 = n - 3(n + 3)$     |           |
|                               | $n = 4$   |                               | $n = -1$  |
| ■ $3(p + 2) - p = 26$         |           |                               |           |
|                               | $p = 10$  |                               |           |
| ■ $4(2r - 6) + 3(r + 5) = 35$ |           |                               |           |
|                               | $r = 4$   |                               |           |
| ■ $\frac{1}{2}(t - 4) = 12$   |           |                               |           |
|                               | $t = 28$  |                               |           |
| ■ $\frac{w + 7}{5} = 5$       |           |                               |           |
|                               | $w = 18$  |                               |           |



# Equations (3)

Solve all these equations!

$$\blacksquare 3a + 5 = 17 - a$$

$$a = 3$$

$$\blacksquare 25 - 7c = -10$$

$$c = 5$$

$$\blacksquare 12d - 5 = 15 + 10d$$

$$d = 10$$

$$\blacksquare 2e + 7 = 31 - 4e$$

$$e = 4$$

$$\blacksquare 3(f + 2) - f = 22$$

$$f = 8$$

$$\blacksquare \frac{1}{4}(g - 12) = 9$$

$$g = 48$$

$$\blacksquare \frac{h+7}{4} = 10$$

$$h = 33$$

$$\blacksquare \sqrt[3]{j} - 2 = 10$$

$$j = 16$$

$$\blacksquare 4k - 5 = 2k + 43$$

$$k = 24$$

$$\blacksquare 5(m + 3) + (m - 5) = 100$$

$$m = 15$$

$$\blacksquare 3(2n - 5) - 4(n + 7) = 13$$

$$n = 28$$

$$\blacksquare \frac{2a-5}{3} = \frac{a-2}{2}$$

$$a = 4$$

$$\blacksquare \frac{c-1}{3} = c - \frac{3(c+2)}{5}$$

$$c = 13$$

$$\blacksquare \sqrt[3]{4}(5e - 12) - \frac{1}{4}(4 + e) = 32$$

$$e = 12$$

$$\blacksquare x^2 = 81$$

$$x = \pm 9$$

$$\blacksquare x^2 = 0.36$$

$$x = \pm 0.6$$

$$\blacksquare 3^x = 81$$

$$x = 4$$

$$\blacksquare x^3 = 64$$

$$x = 4$$

$$\blacksquare \sqrt[3]{x} = 2$$

$$x = 8$$

$$\blacksquare 4^x = \frac{1}{16}$$

$$x = -2$$

$$\blacksquare \text{Let } x = 2y.$$

$$\text{Find the value of } \frac{x^2 - 2y^2}{x^2 + 2y^2}$$

$$= \frac{1}{3}$$

±

# Equations (4)

Solve all these equations

- $3^{x+1} = 81$

$$x = 3$$

- $2^{x-1} = 64$

$$x = 7$$

- $5 \times 5^{3x} = 5^{-2x^2}$

$$x = -1 \text{ or } x = -\frac{1}{2}$$

- $7 \times 7^{1-x} = 7^{3x^2}$

$$x = -1 \text{ or } x = -\frac{1}{2}$$

- $2^{3x+4} > 2^x$

$$x = \frac{2}{3} \text{ or } x = -1$$

- $5^{x+6} = 5^{x^2}$

$$x = 3 \text{ or } x = 2$$

- $9 \times 3^{x-4} < 27$

$$x < 5$$

- $10 \times 2^{p-1} < 1280$

$$p < 8$$

- $25 \times 5^{(2y+13)} = 5^{y^2}$

$$y = 5 \text{ or } y = -3$$

- $9 \times 3^{(x-4)} > 81$

$$x > 6$$

- $216 \times 6^{(2x+5)} = 6^{x^2}$

$$x = 4, x = -2$$

- $8 \times 2^{(2y+13)} = 2^y$

$$y = -16$$

- $64 \times 4^{x-5} > 512$

$$x > \frac{13}{2} \text{ (6.5)}$$

- $16 \times 2^x = 2^{6x+3}$

$$x = \frac{1}{5}$$

- $6 \times 2^{2n+1} > 96$

$$n > \frac{3}{2}$$

- $9 \times 3^x = 3^{5x+4}$

$$x = -\frac{1}{2}$$

# Simultaneous Equations

Solve all these equations on your own paper!

1.  $y = 2x + 4$   
 $y = 5x - 5$   
 $x=3, y=10$

2.  $x = -y + 12$   
 $x = 2y$   
 $x=8, y=4$

3.  $y = 7$   
 $y = 3x - 20$   
 $x=9, y=7$

4.  $x = 1 - 2y$   
 $x = y - 5$   
 $x=-3, y=2$

5.  $x = 2$   
 $x = 3y - 1$   
 $x=2, y=1$

6.  $y = 6x - 1$   
 $y = 10x - 1$   
 $x=0, y=-1$

7.  $y = 3x + 1$   
 $y = 2x - 2$   
 $x=-3, y=-8$

8.  $x = 3y$   
 $x = 5 - 2y$   
 $x=3, y=1$

9.  $y = 10$   
 $y = 2 - 4x$   
 $x=-2, y=10$

10.  $x = y - 4$   
 $x = 3 - 6y$   
 $x=-3, y=1$

11.  $y = 5x - 6$   
 $2x + y = 8$   
 $x=2, y=4$

12.  $x = 3y + 6$   
 $2y + x = 1$   
 $x=3, y=-1$

13.  $x = 5 - 3y$   
 $13 = x + 2y$   
 $x=29, y=-8$

14.  $x = 13 - 2y$   
 $3y = x + 27$   
 $x=-3, y=8$

15.  $y = 6x - 5$   
 $y + 2x = -5$   
 $x=0, y=-5$

16.  $x + y = 3$   
 $2x - 3 = y$   
 $x=8.3, y=2.3$

17.  $2x + y = 7$   
 $y = 4x - 5$   
 $x=2, y=3$

18.  $5y - x = 2$   
 $x = 4y - 1$   
 $x=3, y=1$

19.  $5 = x + 8y$   
 $x = 7 - 10y$   
 $x=-3, y=1$

20.  $x = 3 - 3y$   
 $x = 2y + 18$   
 $x=12, y=-3$

21.  $x + y = 14$   
 $x - y = 10$   
 $x=12, y=2$

22.  $-x + y = 2$   
 $x + y = 28$   
 $x=13, y=15$

23.  $x - 2y = 4$   
 $x + 2y = 14$   
 $x=9, y=2.5$

24.  $-3x + y = -5$   
 $3x + y = 19$   
 $x=4, y=7$

25.  $y + 2x = 1$   
 $4x - y = -25$   
 $x=-4, y=9$

26.  $4y - x = -5$   
 $x + 3y = 5$   
 $x=5/4, y=-15/16$

27.  $x + 2y = 1$   
 $x - 2y = -11$   
 $x=-5, y=3$

28.  $5x + y = 17$   
 $-5x + y = -3$   
 $x=2, y=7$

29.  $-y + 3x = 8$   
 $3x + y = 16$   
 $x=4, y=4$

30.  $7x + y = 11$   
 $y + 3x = 3$   
 $x=2, y=-3$

# Simultaneous Equations

1.  $x = \text{apples}; y = \text{oranges}$

$$x + y = 8$$

$$3x + 2y = 20$$

Solution: 4 apples and 4 oranges

2.  $x = \text{muffins}; y = \text{cupcakes}$

$$x + y = 27$$

$$6x + 4.5y = 117$$

Solution: 12 muffins and 10 cupcakes.

3.  $x = \text{paperback}; y = \text{hardcover books.}$

$$x + y = 12$$

$$30x + 45y = 960$$

Solution: 20 paperbacks and 8 hardcovers.

4.  $x = \text{pens}; y = \text{pencils}$

$$x + y = 18$$

$$12x + 8y = 224$$

Solution: 12 pens and 10 pencils.

5.  $x = \text{sopranos}; y = \text{altos}$

$$x + y = 28$$

$$4x + 3y = 101$$

Solution: 13 sopranos and 15 altos.

6.  $x = \text{red marbles}; y = \text{blue marbles.}$

$$x + y = 40$$

$$8x + 3y = 225$$

Solution: 20 red marbles and 20 blue marbles.

7.  $x = \text{boys}; y = \text{girls}$

$$x + y = 25$$

$$20x + 25y = 550$$

Solution: 10 girls, 15 boys.

# Simultaneous Equations

8. Solution: 700 Adults, 300 Children
9. Solution:  $x = 5\% \$10,000$ ,  $y = 8\% \$20,000$
10.  $x =$  present age of the woman;  $y =$  the present age of her son  
 Present age of the mother in relation to her son:  $x = y + 30$   
 15 years ago:  $x - 15 = 2(y - 15)$ .  
 Solution:  $y$  (son) = 45 years,  $x$ , (mother), is 75 years.
11.  $x =$  the number of adult tickets;  $y =$  the number of children's tickets.  
 Solution:  $x = 500$ ,  $y = 225$
12.  $x =$  amount invested at 6%;  $y =$  amount invested at 7%  
 Total investment:  $x + y = \$20,000$   
 Total interest:  $0.06x + 0.07y = \$1,280$   
 Solution:  $x = \$12,000$ .  $y = \$8,000$ .
13. 9 litres of 30% alcohol solution  $\rightarrow 0.3 \times 9 = 2.7$  litres of pure alcohol  
 $x =$  number of litres 20% solution;  $y =$  number of litres 50% solution.  
 Total number of litres:  $x + y = 9$   
 Total litres of pure alcohol:  $0.2x + 0.5y = 2.7$   
 Solution:  $x = 6$  litres  $y = 3$  litres.
14. Let  $x =$  speed of the boat in still water;  $y =$  the speed of the current.  
 Downstream speed:  $x + y = 24$   
 Upstream speed:  $x - y = 18$   
 Solution:  $x = 9$  kmph.  $y = 3$  kmph.
15. Let  $x =$  the amount of money Andre has,  $y =$  amount Bob has.  
 Andre gives Bob \$20, they have the same amount  $x - 20 = y + 20$   
 Bob gives Andre \$22, Andre has twice as much as Bob  $x + 22 = 2(y - 22)$   
 Solution: Bob has \$106. Andre has \$146.

# Changing the Subject

Make  $x$  the subject of the equations

$$\blacktriangleright 2x + 5y = 15$$

$$x = \frac{15 - 5y}{2}$$

$$\blacktriangleright 4y = 2x + 8$$

$$x = 2y - 4$$

$$\blacktriangleright 3x - y = 12$$

$$x = 4 + \frac{y}{3}$$

$$\blacktriangleright xy = 19$$

$$x = \frac{19}{y}$$

$$\blacktriangleright ax = \frac{b}{10}$$

$$x = \frac{b}{10a}$$

$$\blacktriangleright \frac{3}{x} = \frac{y}{2}$$

$$x = \frac{6}{y}$$

$$\blacktriangleright \sqrt{\frac{x}{5}} = y$$

$$x = 5y^2$$

$$\blacktriangleright 2x^2 = 3y$$

$$x = \sqrt{\frac{3y}{2}}$$

$$\blacktriangleright 12y = \frac{x}{6}$$

$$x = 72y$$

$$\blacktriangleright x^2 = 5y$$

$$x = \sqrt{5y}$$

$$\blacktriangleright y^2 = x^2 + 2a$$

$$x = \sqrt{y^2 - 2a}$$

$$\blacktriangleright T = \frac{1}{4}\pi x^2$$

$$x = \sqrt{\frac{4T}{\pi}}$$

$$\blacktriangleright \frac{3}{8}kx = y$$

$$x = \frac{8y}{3k}$$

$$\blacktriangleright \frac{x}{11} = \frac{y}{5}$$

$$x = \frac{11y}{5}$$

$$\blacktriangleright x^2 + y^2 = r^2$$

$$x = \sqrt{r^2 - y^2}$$

$$\blacktriangleright s = 4.9x^2$$

$$x = \sqrt{\frac{s}{4.9}}$$

# Simultaneous Equations

Solve all these simultaneous equations!

$$\blacktriangleright x + y = 10$$

$$x - y = 12$$

$$x = 11, y = -1$$

$$\blacktriangleright 5x + y = 14$$

$$4x - y = 22$$

$$x = 4, y = -6$$

$$\blacktriangleright 2x + 3y = -2$$

$$3x - y = 13$$

$$x = -69, y = 47$$

$$\blacktriangleright -4x + 5y = 20$$

$$4x - 3y = -4$$

$$x = 5, y = 8$$

$$\blacktriangleright 5x + 8y = 71$$

$$7x - 3y = 71$$

$$x = 11, y = 8$$

$$\blacktriangleright 3x + 5y = 5x - 3y$$

$$3x + 5y = 19 - 2y$$

$$x = 4, y = 1$$

$$\blacktriangleright 2x + y = 13$$

$$x = 8 - y$$

$$x = 5, y = 3$$

$$\blacktriangleright x = 4y - 1$$

$$3x + 2y = 11$$

$$x = 3, y = 1$$

$$\blacktriangleright 2x = 20 + 2y$$

$$4x + y = 20$$

$$x = 6, y = -4$$

$$\blacktriangleright y = 6x - 5$$

$$y + 2x = -5$$

$$x = 0, y = -5$$

$$\blacktriangleright y = 5x - 2$$

$$y = 4x + 4$$

$$x = 6, y = 28$$

$$\blacktriangleright -2x + 3y = 3$$

$$7x + 2y = 27$$

$$x = 3, y = 3$$

# Sequences

Use  $n = 1, 2, 3$  and  $4$  to find the first 4 terms of each sequence

- $3n - 1$

$n$	1	2	3	4
	2	5	8	11

- $100 - 10n$

$n$	1	2	3	4
	90	80	70	60

- $5n + 4$

$n$	1	2	3	4
	9	14	19	24

- $-2n + 3$

$n$	1	2	3	4
	1	-1	-3	-5

- $10^n$

$n$	1	2	3	4
	10	100	1000	10000

- $2^n$

$n$	1	2	3	4
	2	4	8	16

- $n(n + 3)$

$n$	1	2	3	4
	4	10	18	28

- $n^2 + n$

$n$	1	2	3	4
	2	6	12	20

- $6n + 4$

$n$	1	2	3	4
	10	16	22	28

- $25 - 3n$

$n$	1	2	3	4
	22	19	16	13

- $2n^2 - n$

$n$	1	2	3	4
	1	6	15	28

- $n + 2^n$

$n$	1	2	3	4
	3	6	11	20

- $n^3 - 1$

$n$	1	2	3	4
	0	7	26	63

- $n^2(n - 2)$

$n$	1	2	3	4
	-1	0	9	32

- $\frac{n}{n+2}$

$n$	1	2	3	4
	0.33	0.5	0.6	0.67

- $\frac{n}{2n+1}$

$n$	1	2	3	4
	0.33	0.4	0.43	0.44



# Sequences

Find the next 2 terms and the rule for all these sequences!

■ 3, 7, 11, 15, 19, 23, 27

$$t_n = 4n - 1$$

■ 6, 9, 12, 15, 18, 21, 24

$$t_n = 3n + 3$$

■ -3, -1, 1, 3, 5, 7, 9

$$t_n = 2n - 5$$

■ 6, 11, 16, 21, 26, 31, 36

$$t_n = 5n + 1$$

■ 4, 2, 0, -2, -4, -6, -8

$$t_n = -2n + 6$$

■ 95, 90, 85, 80, 75, 70, 65

$$t_n = -5n + 100$$

■ 7, 4, 1, -2, -5, -8, -11

$$t_n = -3n + 10$$

■ 6.5, 7, 7.5, 8, 8.5, 9, 9.5

$$t_n = 0.5n + 6$$

■ 1, 4, 9, 16, 25, 36, 49

$$t_n = n^2$$

■ 3, 6, 11, 18, 27, 38, 51

$$t_n = n^2 + 2$$

■ 0, 3, 8, 15, 24, 35, 48

$$t_n = n^2 - 1$$

■ 3, 10, 21, 36, 55, 78, 105

$$t_n = 2n^2 + n$$

■ 3, 8, 15, 24, 35, 48, 63

$$t_n = n^2 + 2n$$

■ 2, 10, 24, 44, 70, 102, 140

$$t_n = 3n^2 - n$$

■ 6, 12, 20, 30, 42, 56, 72

$$t_n = n^2 + 3n + 2$$

■ 8, 14, 24, 38, 56, 78, 104

$$t_n = 2n^2 + 6$$

# Sequences

Can you answer these sequence questions?

- The first five terms in a number sequence are: 7, 10, 13, 16, 19
  - nth term of the sequence is  $t_n = 3n + 4$
  - 100th term in this number sequence  $3 \times 100 + 4 = 304$ .
- A number sequence has nth term  $6n + 5$ 
  - First four terms of this sequence are 11, 17, 23, 29
  - Not correct  $(1008 - 5) / 6 = 167.2$  (needs to be an integer)
- The first 5 terms of a number sequence are: 1, 5, 9, 13, 17.
  - nth term of the sequence is  $t_n = 4n - 3$
  - Is 97 a term in this sequence?  $4 \times 25 - 3 = 97$  yes 25th term.
- The nth term of a number sequence is  $n^2 + 5$ .
  - First three terms of this sequence are 6, 9, 14
  - $t_{10} = 105$ ,  $t_5 = 30$ , difference = 75
- The first 5 terms in a number sequence are: 13, 10, 7, 4, 1.
  - nth term of the sequence is  $t_n = -3n + 16$
  - 50th term of the sequence is  $-3 \times 50 + 16 = -134$
- Kim calculates first 50 terms of the sequence  $150 - 4n$ .  
First negative term  $150 - 4n = 0$ ,  $n = 37.5$ , term 38 = -2
- The nth term of a sequence is  $(n + 2)(n + 5)$   
First 5 terms of the sequence: 18, 28, 40, 54, 70.
- The nth term of a sequence is  $5n - 9$ 
  - First 5 terms are: -4, 1, 6, 11, 16
  - Term 99 = 486, term 100th = 491, difference = 5  
The last term of this sequence is 741.
  - How many terms in the sequence?  $5 \times 150 - 9 = 150$  terms

# Sequences

Can you answer these harder sequence questions?

1. A linear sequence starts 300, 296, 292, 288 ...

Which term will be the first to have a negative value?

$$t_n = -4n + 304 \quad 4n = 304 \quad \text{first negative term } n = 76\text{th term}$$

2. The  $n$ th term of a sequence is  $4n - 10$

Write an expression for the term  $n + 1$

$$\text{first 5 terms for } 4n - 10 = -6, -2, 2, 6, 10,$$

$$\text{first 5 terms for } n+1 = -2, 2, 6, 10, 14, t_n = 4n - 6$$

3. A linear sequence starts off 4, 11, 18, 25, ...

Write an expression for the  $n$ th term.

How many terms in the sequence will be less than 150?

$$T_n = 7n - 3, T_{22} = 151 \text{ so } 21 \text{ terms } < 150$$

4.  $n$ th term of Sequence A: 4, 7, 10, 13,  $t_n = 3n + 1$

$$n\text{th term of Sequence B: } 16, 49, 100, 169, T_n = 9n^2 + 6n + 1$$

Show that the 30th term of sequence B equals  $49 \times 169$ .

$$49 \times 169 = 8281 \quad T_{30} = 9(30)^2 + 6(30) + 1 \\ = 8281$$

5. A linear sequence has the terms  $a+b$ ,  $a+3b$ ,  $a+5b$ ,  $a+7b$ ,  $a+9b$ , ...

Find the  $n$ th term of the sequence.

$$\text{Term 8: } a + 15b = 59$$

$$\text{Term 5: } a + 9b = 35$$

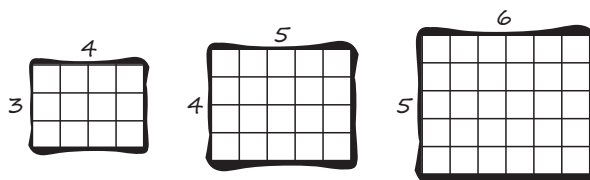
$$6b = 24, \quad b = 4, \quad a = -1$$

Sequence is therefore: 3, 11, 19, 27, 35,  $T_n = 8n - 5$

6. A pattern of rectangles continues as below. The areas of each rectangle form a sequence: 12, 20, 30, 42, ...

Find the  $n$ th term of the sequence of areas.

$$t_n = n^2 + 5n + 6$$



# Algebra Practise

## Basic Skill Questions - Set A

1.  $(x + 2)(4x - 5) = 4x^2 + 3x - 10$

2.  $2x^2 - 15x + 18 = (2x - 3)(x - 6)$

3. Coordinates cut x axis at  $y = 0$   
 $= (0, 0)(-3, 0)$

4. If  $x = 2$  and  $h = x(x - 1) + 2$   
 $2(2 - 1) + 2, h = 4$

5.  $(3x + 7)(x - 2) = 3x^2 + x - 14$

6.  $3x^2 - 11x + 6 = (3x - 2)(x - 3)$

7. If  $x = 2$  and  $h = 2x(x - 1.5) + 1$   
 $2(2)(2 - 1.5) + 1, h = 3$

8. Coordinates =  $(0, 0), (-9, 0)$

9.  $4(x + 5) = 24, x = 1$

10.  $(x + 7) \div 4 = 5, x = 13$

11. Area =  $w(w + 2)$   
 $= w^2 + 2w$

12. Area =  $L(L - 3)$   
 $= L^2 - 3L$

▲ 13.  $x^2 - x - 2$

▼  $= (x + 1)(x - 2)$

▲ 14.  $x^2 + 5x - 36.$

▼ Length of each side =  $(x - 4)(x + 9)$

▼ 15.  $y = 3x^2 - 2x + 5$

▲  $y = 3(4)^2 - 2(4) + 5$

▼  $y = 45$

▼ 16.  $y = 3x^2 - 5x + 7$

▲  $y = 3(2)^2 - 5(2) + 7$

▼  $y = 9$

▲ 17.  $n^2 - 4n - 5.$

▼  $= (n + 1)(n - 5)$

▼ 18.  $x^2 + 4x - 12$

▲  $= (x + 6)(x - 2)$

▼ length =  $x+2$  width =  $x-2$

▼ 19.  $y = -(x^2 - 4x)$

▼  $x$  measures the horizontal distance travelled

▼ 20.  $d = ut + 3t^2$

▼  $d = (3 \times 5) + 3(5)^2$

▲  $d = 90$

# Algebra Practise

## Basic Skill Questions - Set B

1.  $A = xy + 5y^2$

$$A = 2 \times 4 + 5 \times 4^2$$

$$A = 88$$

2.  $3x^2 - 4x - 32$

$$(3x + 8)(x - 4)$$

3.  $h = 9 - 4x^2$      $4x^2 = 9 - h$

$$x^2 = \frac{9 - h}{4}$$

$$x = \sqrt{\frac{9 - h}{4}}$$

4.  $n = 9m^2 - 16$

$$9m^2 = n + 16$$

$$m^2 = \frac{n + 16}{9}$$

$$m = \sqrt{\frac{n + 16}{9}}$$

5.  $3x^2 + 2x - 40$

$$= (3x - 10)(x + 4)$$

6.  $(2x + 3)(x - 2)$

$$= 2x^2 - x - 6$$

7.  $3^4 = 81, x = 3$

8.  $2^6 = 64, x = 7$

9.  $r^2 = \frac{9A}{25}$      $r = \sqrt{\frac{9A}{25}}$

10.  $r^2 = \frac{4A}{9}$      $r = \sqrt{\frac{4A}{9}}$

▲ 11.  $A = 3(2x^2y - 2x)$

▼  $A = 3(2 \times 3^2 \times 5 - 2 \times 3)$

▲  $A = 252$

▼ 12.  $A = 2(x + 3xy^2)$

▼  $A = 2(5 + 3 \times 5 \times 2^2)$

▲  $A = 130$

▼ 13.  $(3x - 2)(x + 2)$

▼  $= 3x^2 + 4x - 4$

▲ 14.  $2x^4 - 3x + 5$

▼  $= 2(-2)^4 - 3(-2) + 5$

▲  $= 43$

▼ 15.  $60 \div x + 12 = 14$

▼  $x = 30$

▲ 16.  $w = pq^2 + r$

▲  $p = \frac{w - r}{q^2}$

▼ 17.  $5m^2n \times m^3n^2 = 5m^5n^3$

▼ 18. a.  $4a^3 = 32$      $a = 2$

▲ b.  $(b^3)^2 = 64$      $b = 2$

▼ c.  $5^c = 125$      $c = 3$

▲ d.  $(2d^2)^2 = 324$      $d = 3$

▼ 19.  $12a^2b + 6ab^2 - 7a^2b$

▲  $= 5a^2b + 6ab^2$

▼ 20.  $3(2x + 7) = 9$

▲  $x = -2$

# Algebra Practise

## Proficiency Skill Questions - Set A

1.  $h = \text{hours}$       $8 + 3(h - 2) = 23$

$$8 + 3h - 6 = 23$$

$$3h + 2 = 23$$

$$h = 7$$

James hires the bike for 7 hours.

2.  $10 \times 2^{n-1} \geq 160$

$$2^{n-1} \geq 16$$

$$2^4 = 16$$

$$\text{therefore } n - 1 \geq 4$$

$$n \geq 5$$

3.  $5r^3 = 320$

$$r^3 = 64$$

$$r = 4$$

4.  $\text{area} = x^2 - x - 2.$

Factorise  $(x + 1)(x - 2)$

Width =  $(x - 2)$

5. Show that  $\frac{2}{x} + \frac{3+x}{5} = \frac{x^2 + 3x + 10}{5x}$

$$\frac{2}{x} + \frac{3+x}{5} = \frac{5 \times 2 + 3x + x^2}{5x}$$

$$= \frac{x^2 + 3x + 10}{5x}$$

6.  $= \frac{3ab^2}{4ab^2} - \frac{4a^3b}{4ab^2} + \frac{ab^2}{4ab^2}$

$$= \frac{3}{4} - \frac{a^2}{b} + \frac{1}{4}$$

$$= 1 - \frac{a^2}{b}$$

7.  $x^2 + 3x - 10 = (x + 5)(x - 2)$

$$y = 0 \text{ when } x = -5 \text{ or } x = 2$$

calculating when  $y < 0$

test between  $x = -5$  and  $x = 2$

$$x = 0, y = 0^2 + 3 \times 0 - 10$$

$y = -10$ , therefore  $y$  is negative

when  $-5 < x < 2$

8.  $x^2 + 4x - 12 = (x + 6)(x - 2)$

$$y = 0 \text{ when } x = -6 \text{ or } x = 2$$

test  $x = 3, y = 3^2 + 4 \times 3 - 12$

$y = 9$ , therefore  $y$  is positive when

$x < -6$  or  $x > 2$

9.  $D = 7 \times 2^{n-1}, 112 = 7 \times 2^{n-1}$

$$16 = 2^{n-1}, 2^4 = 16$$

$$n = 5 \text{ (5 weeks)}$$

10.  $77 = 38 + 13h$

$$39 = 13h, h = 3 \text{ hours}$$

11.  $= \frac{5xy^2}{4xy^2} - \frac{2a^3y}{4xy^2} + \frac{xy^2}{4xy^2}$

$$= \frac{5}{4} - \frac{x^2}{2y} + \frac{1}{4} \qquad \frac{5}{4} + \frac{1}{4} = \frac{3}{2}$$

$$= \frac{3}{2} - \frac{x^2}{2y}$$

$$= \frac{3y - x^2}{2y}$$

12.  $486 = 6r^4$

$$81 = r^4$$

$$r = 3$$

rate of spread is  $3m^2$  per week

# Algebra Practise

## Proficiency Skill Questions - Set B

1.  $x^2 + 2x - 8 = 0$

$(x + 4)(x - 2) = 0, x = -4 \text{ or } x = 2$

2.  $x^2 + 4x - 12$

$= (x + 6)(x - 2) \rightarrow \text{side lengths}$

$x^2 + 4x - 12 = 128$

$x^2 + 4x - 140 = 0$

$(x + 14)(x - 10)$

$x = -14 \text{ (discard) or } x = 10$

$\text{side lengths} = (10+6)(10-2)$

$= 16 \text{ and } 8$

3.  $y^2 - 3y - 10 = 0$

$y = 5, y = -2$

4.  $3x^2 + 8x - 16 = 0,$

$3x^2 + 12x - 4x - 16 = 0$

$3x(x + 4) - 4(x + 4) = 0$

$(3x - 4)(x + 4) = 0$

$x = 4/3, -4$

5.  $2a^2 - 3a - 9 = 0$

$2a^2 - 6a + 3a - 9 = 0$

$2a(a - 3) + 3(a - 3) = 0$

$(2a + 3)(a - 3) = 0$

$a = 3/2, 3$

6.  $10x^2 - 27x - 9 = 0$

$10x^2 - 30x + 3x - 9 = 0$

$10x(x - 3) + 3(x - 3) = 0$

$(10x + 3)(x - 3) = 0$

$x = -3/10, 3$

7.  $15y^2 - 10y + 6y - 4 = 0$

$5y(3y - 2) + 2(3y - 2) = 0$

$(5y + 2)(3y - 2) = 0, y = -2/5, 2/3$

8.  $3x^2 + 12x - 10x - 40$

$3x(x + 4) - 10(x + 4)$

$(3x - 10)(x + 4)$

$\text{possible values } x = 10/3, -4$

9.  $\frac{6x}{2x - 1}$

10.  $\frac{x - 1}{5x}$

11.  $\frac{5x - 1}{(x + 1)(x - 2)}$

12.  $\frac{36x - 23}{12}$

13.  $\frac{-4}{3}$

14.  $x < -2$

15.  $y > -3$

16.  $6x+5 + 8x+3 + 4x-2 + 10x+1 + AB$

$28x + 7 + AB = 32x - 7$

$AB = 4x - 14$

17.  $2(8x + 3) + 2(6x + 2) = 290$

$16x + 6 + 12x + 4 = 290$

$28x + 10 = 290, x = 10$

18.  $10 \times -4 \times 3 \div (-4 + 3)^2 = -120$

19.  $(2a)^3 \times 2a^k = 16a^8$

$8a^3 \times 2a^k = 16a^8, k = 5$

20.  $36 = \frac{1}{2} \times (15 + 9) h.$

$36 = 12 \times h,$

$h = 12$

# Algebra Practise

## Proficiency Skill Questions - Set C

1.  $4x^2 + 8x + 3 = 0, x = -1.5.$

Possible factorised equations

$$(2x + 1)(2x + 3) \times$$

$$(4x + 2)(x + 1.5) \checkmark$$

Other solution =  $-0.5$

2.  $V = 50 \times (w + 10) \times w$

$$V = 50w^2 + 500w$$

$$60 \text{ litres} = 60,000\text{cm}^3$$

$$50w^2 + 500w = 60000$$

$$50w^2 + 500w - 60000 = 0$$

$$50(w^2 + 10w - 1200) = 0$$

$$50(w + 40)(w - 30) = 0$$

$$w = 30, w+10 = 40, \text{height} = 50 \text{ (cm)}$$

3. Perimeter = 30 cm the length of each side = 10 cm (equilateral triangle)

$$2x - y = 10$$

$$2y + x = 10 \text{ or } x = 10 - 2y$$

Substitute  $2(10 - 2y) - y = 10$

$$20 - 4y - y = 10$$

$$20 - 5y = 10$$

$$-5y = -10, y = 2$$

If  $y = 2$  then  $2x - 2 = 10$

$$2x = 12, x = 6$$

4. 
$$\frac{(x - 2y)(x + 2y)}{x(x - 2y)} = \frac{x + 2y}{x}$$

5. 
$$\frac{5x + 6x}{10} = \frac{11x}{10}$$

6.  $x^2 + 2x - 255 = 0$

$$(x + 17)(x - 15), x = -17 \text{ or } x = 15$$

7. 
$$\frac{8m + 3m}{12} = \frac{11m}{12}$$

8.  $Z = \text{Zahara's age: } Z(Z + 4) = 725$

$$Z^2 + 4Z - 725 = 0$$

$$(Z + 29)(Z - 25) = 0$$

$$Z = -29 \text{ or } Z = 25$$

$$\text{Zahara} = 25, \text{Maddox} = 29$$

Zahara is now 5

= another 20 years.

9. a.  $T = V - dx$

$T = \text{Total volume remaining}$

$V = \text{initial volume}$

$d = \text{number of drippers}$

$x = \text{amount used by each dripper}$

b.  $T = V - dx$

$$60 = 150 - 4x$$

$$4x = 90$$

$$x = 22.5$$

Amount of water used by each

dripper is 22.5 litres.

10.  $4x + 3x^2 + 5x - 2x^2 = 22$

$$9x + x^2 = 22$$

$$x^2 + 9x - 22 = 0$$

$$(x + 11)(x - 2) = 0$$

$$x = -11 \text{ or } x = 2$$

Width of path = 2m



# Algebra Practise

## Advanced Skill Questions - Set A

1.  $x^2 + 6x + c = 0$

1 solution means  $(x + a)^2 = (x + 3)^2$   
equation is  $x^2 + 6x + 9 = 0$

2.  $A = \text{distance Andy}, B = \text{distance Billy}$

$$A + B = 8 \text{ or } B = 8 - A$$

$$B = \frac{15}{10} \text{ Billy travels 1.5km faster}$$

$$1.5A = 8 - A, A = 3.2, B = 4.8$$

3.  $\frac{x^2 - 1}{x^2 + 2x + 1} = \frac{3}{4}$

$$\frac{(x - 1)(x + 1)}{(x + 1)(x + 1)} = \frac{3}{4} \quad (\text{but } x \neq -1)$$

$$4x - 4 = 3x + 3, x = 7$$

4.  $S + A = 38$

$$10s + 12A = 420$$

$$10s + 10A = 380 \text{ (equation } 1 \times 10)$$

$$2A = 40$$

$$\text{Adults} = 20, \text{Students} = 18$$

5.  $4 \times 2^x = 2^{6x+3}$

$$2^2 \times 2^x = 2^{6x+3}$$

$$2 + x = 6x + 3 \text{ (as } A^x + A^y = A^{x+y})$$

$$5x + 1 = 0, x = -0.2 \text{ } (-1/5)$$

6.  $x^2 + 5x - 36 = 144$

$$x^2 + 5x - 150 = 0$$

$$(x + 15)(x - 10) = 0$$

$$x = -15, x = 10$$

$x = 10$  as no negative lengths

7.  $\frac{3}{2x} + \frac{x+4}{4}$  (cross multiply)

$$\frac{12}{8x} + \frac{2x^2 + 8x}{8x}$$

$$= \frac{2x^2 + 8x + 12}{8x}$$

8.  $6x - y = 21$  or  $y = 6x - 21$

$$-x + 6y = 14 \text{ or } 6y = 14 + x$$

$$36x - 126 = 14 + x$$

$$35x = 140, x = 4, y = 3$$

$$x - y = 1$$

9.  $x^3 = mx,$

$$x^2 = m$$

$$x = \sqrt{m}$$

$$x^2 = n + x \quad \text{and } m = n + x$$

$$m = n + m$$

$$n = m - \sqrt{m}$$

10.  $2(12x + 1) + (4x - 2)(2x + 3) = 92$

$$24x + 2 + 8x^2 + 12x - 4x - 6 = 92$$

$$8x^2 + 32x - 96 = 0$$

$$8(x^2 + 3x - 12) = 0$$

$$8(x + 6)(x - 2) = 0$$

$$x = -6, x = 2$$

In this situation  $x = 2$

11.  $(x + 6)(x - 4) = x^2 + 2$

$$x^2 + 2x - 24 = x^2 + 2$$

$$2x = 2 + 24$$

$$x = 13$$

Tiles at the start =  $13^2 = 169$

# Algebra Practise

## Advanced Skill Questions - Set B

1.  $\frac{1}{2}x(4x + 6) = 35$

$$2x^2 + 3x - 35 = 0$$

$$(2x - 7)(x + 5) = 0$$

$$x = 3.5, \text{ or } x = -5$$

Disregard  $x = -5$

2.  $8 \times 2^{x-4} < 20$

$$2^3 \times 2^{x-4} < 20$$

$$2^{x-1} < 20$$

$$2^4 = 16 \text{ and } 2^5 = 32$$

$$x \leq 5$$

3.  $A + S = 27$  and  $S = 27 - A$

$$30A + 20S = 650$$

$$30A + 20(27 - A) = 650$$

$$30A + 540 - 20A = 650$$

$$10A = 110 \text{ and } A = 11, S = 16$$

$R = \text{Arabella}$  and  $S = \text{Isabella}$

$$\frac{1}{2}R + \frac{2}{3}S = 16 \quad \times 6$$

$$\frac{1}{2}R + \frac{1}{3}S = 11 \quad \times 6$$

$$3R + 4S = 96$$

$$\underline{3R + 2S = 66}$$

$$2S = 30$$

$$S = 15, R = 12$$

4.  $C = 2\pi r$  and  $r_1 + r_2 = 10$

$$C = 2\pi r_1 + 2\pi r_2$$

$$C = 2\pi(r_1 + r_2)$$

$$C = 2\pi 10$$

$$C = 20\pi$$

5.  $5^1 \times 5^{3x} = 5^{-2x^2}$

$$3x + 1 = -2x^2$$

$$2x^2 + 3x + 1 = 0$$

$$(2x + 1)(x + 1) = 0$$

$$x = -\frac{1}{2} \text{ or } x = -1$$

6.  $(2y)(3y - 1) = \frac{1}{2}(2y - 2)(5y + 9)$

$$6y^2 - 2y = \frac{1}{2}(10y^2 + 8y - 18)$$

$$y^2 - 6y + 9 = 0$$

$$(y - 3)^2 = 0, y = 3$$

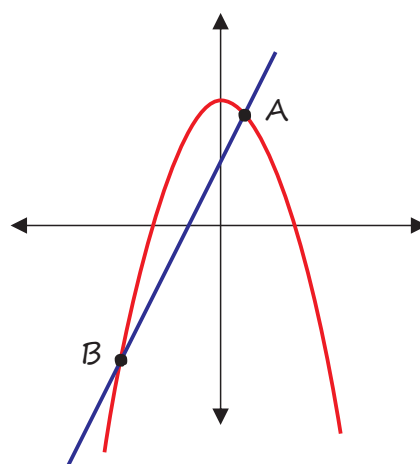
7.  $7 + x - 6x^2 = 8x + 4.$

$$6x^2 + 7x - 3 = 0$$

$$(3x - 1)(2x + 3) = 0$$

$$x = -\frac{1}{3}, x = -1.5$$

$$y = 6\frac{2}{3}, y = -8$$



$$A = (0.33, 6.67)$$

$$B = (-1, 5, -8)$$

# Algebra Practise

## Advanced Skill Questions - Set C

1.  $5^2 \times 5^{2y+13} = 5^{y^2}$

$$2 + 2y + 13 = y^2$$

$$y^2 - 2y - 15 = 0$$

$$(y - 5)(y + 3) = 0$$

$$y = 5, y = -3$$

2.  $U = \text{under } 65, V = 65 \text{ or over}$

$$U + V = 21$$

$$14U + 10V = 258$$

$$\underline{10U + 10V = 210}$$

$$4U = 48$$

$$U = 12, V = 9$$

3.  $x^2 + cx + 16 = 0$

solution must be  $(x + a)^2$

$$(x + 4)^2 = 0, c = 8$$

4.  $A + B = 15$  and  $B = 15 - 8$

$$B = 18/12A$$

$$B = 1.5A$$

$$1.5A = 15 - A$$

$$2.5A = 15, A = 6\text{km}, B = 9\text{km}$$

5.  $D - 20 = G + 20$

$$2(G - 22) = D + 22$$

$$2G - 44 = D + 22$$

$$D = 2G - 66$$

$$2G - 66 - 20 = G + 20$$

$$G = \$106, D = \$146$$

6.  $A = (-5, -10), B = (9, 11)$

Using point A  $-10Q = -5P + 5$  ( $\times 9$ )

Using point B  $11Q = 9P + 5$  ( $\times 5$ )

$$-90Q = -45Q + 45$$

$$\underline{55Q = 45Q + 25}$$

$$-35Q = 70, Q = -2 \text{ and } P = -3$$

7.  $(x - 3)(x + 3) < (x - 4)(x + 2)$

$$x^2 - 9 < x^2 - 2x - 8$$

$$-1 < -2x, x < \frac{1}{2}$$

8.  $10 \times 2^{p-1} < 165$

$$2^{p-1} < 16.5$$

$$2^4 = 16 \text{ and } 2^5 = 32$$

$p < 5$  assuming  $p$  is an integer

9.  $M = 5(a^2 - 3a + 4) + a^2$

$$N = (3a - 5)(2a - 4) + 7(a + 5)$$

expand and simplify

$$M = 6a^2 - 15a + 20$$

$$N = 6a^2 - 15a + 55$$

$$\text{then } M = N - 35$$

10.  $3^3 \times 3^{x-4} > 200$

$$3^{x-1} > 200 \quad (3^5 = 243), x \geq 6$$

11.  $A - 2 = B + 2$  or  $A = B + 4$

$$B - 2 = (A + 2) \div 2$$

$$B - 2 = (B + 4 + 2) \div 2$$

$$2B - 4 = B + 6,$$

$$B = 10, A = 14$$

# Algebra Practise

## Advanced Skill Questions - Set D

1.  $x^4 - 18x^2 + 81 = 0$

$$(x^2 - 9)(x^2 - 9) = 0$$

$$(x^2 - 9) = (x + 3)(x - 3)$$

$$x = 3 \text{ or } -3$$

2.  $Ry - 2Rx = 5y - 4x$

$$4x - 2Rx = 5y - Ry$$

$$x(4 - 2R) = y(5 - R)$$

$$x = \frac{y(5 - R)}{4 - 2R}$$

3. Perimeter A  $6W + 8L = 20$

Perimeter B  $8W + 10L = 26$

multiply A  $\times 8$  and B  $\times 6$

$$48W + 64L = 160$$

$$\underline{48W + 60L = 156}$$

$$4L = 4, L = 1, W = 2$$

Rec A Length = 4, width = 6

Rec B, Length = 5, width = 8

4.  $(3 + 2x)(2 + 2x) = 2(2 \times 3)$

$$6 + 6x + 4x + 4x^2 = 12$$

$$4x^2 + 10x - 6 = 0$$

$$2x^2 + 5x + 3 = 0 \quad (2 \times 3 = 6)$$

$$2x^2 + 6x - 1x - 3 = 0$$

$$2x(x + 3) - 1(x + 3) = 0$$

$$(2x - 1)(x + 3) = 0$$

$$x = \frac{1}{2}, x = -3 \text{ (ignore)}$$

5.  $(3x + A)^2 = 9x^2 + Bx + 16, A = 4$

$$(3x + 4)(3x + 4)$$

$$9x^2 + 24x + 16, A = 4, B = 24$$

6.  $(2^3)^x \times (2^2)^{x^2 - 6} = 2^2$

$$2x^2 + 3x + 14 = 0 \quad (2 \times 14 = 28)$$

$$2x^2 + 7x - 4x - 14 = 0$$

$$(x - 2)(2x + 7) = 0, x = 2 \text{ or } -7/2$$

7.  $3x + 2y = 20$

$$2y = 20 - 3x$$

$$xy = 14, x \times \frac{20 - 3x}{2} = 14$$

$$20x - 3x^2 = 28$$

$$3x^2 - 20x + 28 = 0 \quad (3 \times 28 = 84)$$

$$3x^2 - 6x - 14x + 28 = 0$$

$$(3x - 14)(x - 2) = 0, x = 14/3 \text{ or } 2$$

8.  $(3x - 5)^2 \leq 3x^2 + 1$

$$6x^2 - 30x + 24 \leq 0$$

$$x^2 - 5x + 4 \leq 0, 1 \leq x \leq 4$$

9.  $(-3, 6) \quad -6A = -3B + 6$

$$(3, 10) \quad \underline{10A = 3B + 6}$$

$$4A = 12, A = 3, B = 8$$

10.  $x^2 + 4x - 140 = 0$

$$(x + 14)(x - 10) = 0, x = 10$$

11.  $6^2 \times 6^{2x+6} = 6^{x^2}$

$$2x + 8 = x^2, x = 4 \text{ or } x = -2$$

12. Any odd number =  $2n + 1$

Next consecutive odd =  $2n + 3$

Using  $n^2 - 2$ .

$$[(2n+3)^2 - 2] - [(2n+1)^2 - 2]$$

$$= 4(2n + 2)$$

4 is a factor and it is therefore divisible by 4.

# Algebra Practise

## Advanced Skill Questions - Set E

1. 2 points where the ball is 3m above the ground

$$3 = 4x - x^2$$

$$x^2 - 4x + 3 = 0, x = 3 \text{ or } x = 1$$

This means the ball is 3m or above for 2m of horizontal distance.

Finding the total horizontal distance zero metres or above.

$$x(x - 4) = 0, x = 0 \text{ or } x = 4$$

Total horizontal distance = 4m

$$\% \text{ distance} = 2\text{m} \div 4\text{m} (50\%)$$

2.  $2n^2 - n + 5$  (consecutive =  $n+1$ )

$$2(n+1)^2 - (n+1) + 5$$

$$2n^2 + 3n + 6$$

$$- \underline{2n^2 - n + 5}$$

$$4n + 1$$

or if  $n = 1, 2, 3, 4, 5$

$$2n^2 - n + 5 = 6, 11, 20, 33, 50$$

Difference = 5, 9, 13, 17

$$\text{Rule} = 4n + 1$$

3.  $x^2 + 6x + c = 0$

solution must have form  $(x + a)^2$

$$(x + 3)^2 \text{ and } c = 9$$

Final equation  $x^2 + 6x + 9 = 0$ .

4.  $x^2 - ax - 24 = 0$

factors of 24: -6, 4; -12, 2; -8, 3

$$(x - 12)(x + 2) = 0, x = 12, x = -2$$

5.  $R = 5n^2 + n - 4 - 4n^2 - 2n + 1$

$$R = n^2 - n - 3,$$

$$= (n^2 - n + 5) - 8$$

$$R = T - 8$$

6. Substitute point G into equation

$$13 = A(-1)^2 + B(-1) + 2$$

$$13 = A - B + 2$$

$$A - B = 11$$

Substitute point H into equation

$$18 = A(4)^2 + B(4) + 2$$

$$18 = 16A + 4B + 2$$

$$16A + 4B = 16$$

Simultaneous Equation

$$4A - 4B = 44$$

$$\underline{16A + 4B = 16}$$

$$20A = 60, A = 3, B = -8$$

7. Area  $(4x + 2)(6x + 3)$

$$= 24x^2 + 24x + 6$$

$$= 6(4x^2 + 4x + 1)$$

8.  $(y + 3)^2 = \frac{A(x^2 - 7)}{B}$

$$B(y + 3)^2 = A(x^2 - 7)$$

$$\frac{B(y + 3)^2}{A} + 7 = x^2$$

$$x = \sqrt{\frac{B(y + 3)^2}{A} + 7}$$

# Algebra Practise

## Advanced Skill Questions - Set F

1.  $3x - y = 8$

$$\frac{(2^3)^x}{2^y} = \frac{2^{3x}}{2^y}$$

$$= 2^{3x-y} \quad (2^8) = 256$$

2a.  $(\sqrt{3} + \sqrt{15})^2 = 3 + 2\sqrt{3}\sqrt{15} + 15$

$$= 18 + 2\sqrt{45}$$

$$= 18 + 2 \times \sqrt{9} \times \sqrt{5}$$

$$= 18 + 6\sqrt{5}$$

2b.  $(3 + \sqrt{5})^2 = 9 + 3\sqrt{5} + 3\sqrt{5} + 5$

$$= 14 + 6\sqrt{5}$$

$$k^2 + 14 + 6\sqrt{5} = 18 + 6\sqrt{5}$$

$$k^2 = 4$$

$$k = 2 \text{ (ignore } -2)$$

3.  $x+y+z = \frac{m+9}{2} + \frac{2m+15}{2} + \frac{3m+18}{2}$

$$= \frac{6m+42}{2}$$

$$= 3m + 21$$

$$= \frac{3m+21}{3}$$

$$= m + 7$$

4.  $3x + 1 = x + 4 - x^2$

$$x^2 + 2x - 3 = 0$$

$$(x+3)(x-1), x = -3 \text{ or } 1,$$

coordinates  $(-3, -8), (1, 4)$

5.  $2x(2x-1) = \frac{1}{2}(2x-4)(7x+9)$

$$4x^2 - 2x = (x-2)(7x+9)$$

$$4x^2 - 2x = 7x^2 - 5x - 18$$

$$3x^2 - 3x - 18 = 0$$

$$(3x+6)(x-3) = 0$$

$$x = -2 \text{ (ignore) or } x = 3$$

6. Let  $u = 3^x$  and  $u^2 = 3^{2x}$

$$u^2 - 4u - 45 = 0$$

$$(u-9)(u+5) = 0, u = 9 \text{ or } u = -5$$

$$9 = 3^2 \text{ and } 81 = 3^4 \text{ then } x = 2$$

$$-5 = 3^x \text{ and } 25 = 3^{2x} \text{ } x \text{ not +integer}$$

7.  $3^2 \times 3^{x+y} = (3^3)^{2x}$

$$2 + x + y = 6x$$

$$y = 5x - 2$$

8.  $= (2n)^2 + (2n+2)^2 + (2n+4)^2 - 2$

$$= 12n^2 + 24n + 18$$

$$= 2(6n^2 + 12n + 9), 2 \text{ is a factor}$$

$$= 3(4n^2 + 8n + 6), 3 \text{ is a factor}$$

$$= 6(2n^2 + 4n + 3), 6 \text{ is a factor}$$

9.  $y = 4a^2 - 4a - 4b^2 + 4b$

$$y = 4[(a^2 - b^2) - a + b]$$

$$y = 4[(a-b)(a+b) - a + b]$$

$$y = 4[(a-b)(a+b) - 1(a-b)]$$

$$y = 4(a-b)(a+b-1)$$

10.  $n(2n+6) > n(n^2+3)$

$$2n+3 > n^2+3$$

$$n^2 - 2n - 3 < 0$$

$$(n-3)(n+1) < 0$$

$$n-3 < 0, n < 3 \text{ \& } n+1 < 0, n < -1$$

11.  $33\pi = 2\pi r(r+9.5)$

$$33 = 2r^2 + 19r$$

$$2r^2 + 19r - 33 = 0$$

$$(2r-3)(r+11) = 0$$

$$r = 1.5 \text{ or } r = -11 \text{ (ignore)}$$

## 1 - Substitution

1.  $a = 3, b = -2, c = 13$  and  $z = \frac{1}{2}$ 
  - a.  $abc - 3b \rightarrow 3 \times (-2) \times 13 - 3 \times -2 = -72$
  - b.  $b^2 + 4c - 3z \rightarrow (-2)^2 + 4 \times 13 - 3 \times \frac{1}{2} = 54.5$
  - c.  $(a^4 + b + \frac{1}{2}c) \div z \rightarrow (3^4 + (-2) + \frac{1}{2} \times 13) \div \frac{1}{2} = 171$
  
2.  $h = 2g^2 + 0.15d - 5l; g = 6, d = 8, l = 10$ 

$$h = 2(6)^2 + 0.15 \times 8 - 5 \times 10$$

$$h = 23.2$$
  
3.  $C = 80 + 75h; h = 5\frac{1}{2}$ 

$$C = 80 + 75 \times 5.5$$

$$C = \$492.50$$
  
4.  $A = LW; L = 5 \text{ cm}, W = 14.25 \text{ cm}$ 

$$A = 5 \times 14.25$$

$$A = 71.25 \text{ cm}^2$$
  
5.  $P = 2(W + L); L = 5 \text{ cm}, W = 14.25 \text{ cm}$ 

$$P = 2(14.25 + 5)$$

$$P = 38.5 \text{ cm}$$
  
6.  $V = \pi r^2 h; r = 6 \text{ cm}, h = 14 \text{ cm}$ 

$$V = \pi \times 6^2 \times 14$$

$$V = 1583.4 \text{ cm}^3, (\text{or } 504\pi \text{ cm}^3)$$
  
7.  $SA = 2\pi r^2 + 2\pi rh; r = 6 \text{ cm}, h = 14 \text{ cm}$ 

$$SA = 2 \times \pi \times 6^2 + 2 \times \pi \times 6 \times 14$$

$$SA = 754 \text{ cm}^2 (\text{or } 240\pi \text{ cm}^2)$$
  
8.  $E = \frac{1}{2}mv^2; v = 45 \text{ metres per second}, m = 210\,000 \text{ kg}$ 

$$E = \frac{1}{2} \times 210000 \times 45^2$$

$$E = 212\,625\,000$$
  
9.  $M = 0.0625s + s; s = \$2000$ 

$$M = 0.0625 \times 2000 + 2000$$

$$M = \$2125$$
  
10.  $z^2 = 11 + x^2; x = -5.$ 

$$z^2 = 11 + (-5)^2$$

$$z = \sqrt{36}$$

$$z = 6$$

## 2 - Expanding Single Brackets and Simplifying Expressions

Expand each expression

1.  $3(5x - 8) = 15x - 24$
2.  $xy(10 + z) = 10xy + xyz$
3.  $-5(y - 2) = -5y + 10$
4.  $-3a(12 - 2b) = -36a + 6ab$
5.  $-xy(3 - y) = -3xy + xy^2$
6.  $5b(abc + 8) = 5ab^2c + 40b$
7.  $-4(-3 + c) = 12 - 4c$
8.  $-6r(2 - 3r) = -12r + 18r^2$
9.  $3xy(x + 7) = 3x^2y + 21xy$
10.  $xy(y + 18) = xy^2 + 18xy$
11.  $8xy(2xy - xy) = 16x^2y^2 - 8x^2y^2$
12.  $4x^2(y^3 + 2x^3) = 4x^2y^3 + 8x^5$
13.  $3(x + 1) + 4(4x + 2)$   
 $= 3x + 3 + 16x + 8$   
 $= 19x + 11$
14.  $4(3 - x) - 3(2x + 1)$   
 $= 12 - 4x - 6x - 3$   
 $= 9 - 10x$
15.  $3(5x - 0.5) - 4x(-y + 0.25)$   
 $= 15x - 1.5 + 4xy - x$   
 $= 14x + 4xy - 1.5$

Simplify each expression

1.  $5x + x = 6x$
2.  $3a + 4b + 4a = 7a + 4b$
3.  $7a + 5b - 16a + 12b$   
 $= -9a + 17b$
4.  $4x^2 + x^2 + 3x - 11x$   
 $= 5x^2 - 8x$  or  $x(5x - 8)$
5.  $6ab - 3a - 3b + 4b$   
 $= 6ab - 3a + b$   
 or  $3a(2b - 1) + b$
6.  $7pq + 10pq^2 - pq + pq^2$   
 $= 6pq + 11pq^2$  or  $pq(6 + 11q)$
7.  $3x - 4x^2 + 9x - 10x^2 =$   
 $= 12x - 14x^2$  or  $2x(6 - 7x)$
8.  $b^3 - 10ab^3 + 8b^3$   
 $= 9b^3 - 10ab^3$  or  $b^3(9 - 10a)$
9.  $-5z^3 - 2z - 10z + 3z - 4z^3$   
 $= -9z^3 - 9z$  or  $9z(z^2 - 1)$
10.  $x^2 + 9x - 52$   
 $= (x + 13)(x - 4)$
11.  $-9t^2 + t - 21t^2$   
 $= -30t^2 + t$  or  $t(-30t + 1)$
12.  $2x^2 - 11x^2 - 21$   
 $= -9x^2 - 21$
13.  $x - x - x + 0.5x - 3x$   
 $= -3.5x$
14.  $3x(2 + y) + 7x(2 + y)$   
 $= 6x + 3xy + 14x + 7xy$   
 $= 20x + 10xy$
15.  $-5x^2 - (-2x) - (-5x^2) + 8x$   
 $= 10x$



## 3 - Simplifying Expressions and Expanding Single Brackets

Simplify each expression:

$$1. \quad 40y \div 10y = 4$$

$$2. \quad 30x^2 \div 5x^2 = 6$$

$$3. \quad 3d \times 7d^2 = 21d^3$$

$$4. \quad 6e^3f^2 \times 4ef = 24e^4f^3$$

$$5. \quad 3x^3 \div 12x^2 = \frac{x}{4}$$

$$6. \quad 3pq^2 \times 5p^4q^{-1} = 15p^5q$$

$$7. \quad 30d^4 \times 4cd^2 = 120cd^6$$

$$8. \quad 200x^2yz^3 \div 100x^3y^3z^2 = 2x^{-1}y^{-2}z$$

$$= \frac{2}{x} \cdot \frac{1}{y^2} \cdot z$$

$$9. \quad -12a^4c \times -3a^4c^2 = 36a^8c^3$$

$$10. \quad 2(0.5x + 2.7x) - 3x = 3.4x$$

$$11. \quad 70x \div 140x^2 = \frac{1}{2x}$$

$$12. \quad 50x^3y^5z^2 \times 3x^2yz^4 = 150x^5y^6z^6$$

$$13. \quad 21a^3b^{-4}c^{-2} \div 28a^{-4}b^5c^{-1} = \frac{3a^7}{4b^9c}$$

$$14. \quad 30y^2 \times 5y^{-3} = 150y^{-1}$$

$$= \frac{150}{y}$$

$$15. \quad 15z^3 \times 15z^{-4.5} = 225y^{-1.5}$$

$$= \frac{225}{y^{1.5}}$$

Expand each expression:

$$1. \quad x(x + y) = x^2 + xy$$

$$2. \quad 4a(a - a^2) = 4a^2 - 4a^3$$

$$3. \quad 2bc(3bc + 2b) = 6b^2c^2 + 4b^2c$$

$$4. \quad 5x(x^2 - xy) = 5x^3 - 5x^2y$$

$$5. \quad bc(a + b^2) = abc + b^3c$$

$$6. \quad x^2y(y + xy) = x^2y^2 + x^3y^2$$

$$7. \quad 3yz(-3y - z) = -9y^2z - 3yz^2$$

$$8. \quad 8x(x^3 + x^2y) = 8x^4 + 8x^3y$$

$$9. \quad a^3(7a - a^2b) = 7a^4 - a^5b$$

$$10. \quad 4x(2x^4 - 3x^2) = 8x^5 - 12x^3$$

$$11. \quad xz(x^2z - xyz) = x^3z^2 - x^2yz^2$$

$$12. \quad x(a + bc) = ax + bcx$$

$$13. \quad x^2(2x + y) - x(x^2 - xy)$$

$$= x^3 + 2x^2y$$

$$14. \quad x^3(2x - 4) + x^2(x^2 - 2)$$

$$= 3x^4 - 4x^3 - 2x^2$$

## 4 - Simple Factorising and Expanding Brackets

Factorise each expression:

1.  $5x + 5 = 5(x + 1)$
2.  $20x + 12 = 4(5x + 3)$
3.  $6xy - 12x = 6x(y - 2)$
4.  $5x + 10 + 20xy = 5(x + 2 + 4xy)$
5.  $9x^2 + 5x = x(9x + 5)$
6.  $3x^2y + y^2x = xy(3x + y)$
7.  $32xy + 40x = 8x(4y + 5)$
8.  $15y + 45xyz = 15y(1 + 3xz)$
9.  $12x - 16 = 4(3x - 4)$
10.  $60y + 12y^2 = 12y(5 + y)$
11.  $8x^2 + 16x^5 = 8x^2(1 + 2x^3)$
12.  $abc^4d - a^2bc^3 = ab(c^4d - ac^3)$
13.  $3xy^2 - 4xy + xy = xy(3y - 3)$
14.  $9a^3b^2 + 3ab = 3ab(3a^2b + 1)$
15.  $x^2 + (xy)^2 = x^2 + x^2y^2$   
 $= x^2(1 + y^2)$

Expand each expression:

1.  $(x + 8)(x + 4)$   
 $= x^2 + 12x + 32$
2.  $(x + 20)(x + 6)$   
 $= x^2 + 26x + 120$
3.  $(p - 1)(4 - p)$   
 $= -p^2 + 5p - 4$
4.  $(a + 9)(a - 7)$   
 $= a^2 + 2a - 63$
5.  $(3y + 3)(y + 2)$   
 $= 3y^2 + 9y + 6$
6.  $(2x - 5)^2$   
 $= 4x^2 - 20x + 25$
7.  $(9x + 2)(9x - 2)$   
 $= 81x^2 - 4$
8.  $(2x - 5)(x - 4)$   
 $= 2x^2 - 13x + 20$
9.  $(3x - 7)(5x + 6)$   
 $= 15x^2 - 17x - 42$
10.  $(2x + 3)^2$   
 $= 4x^2 + 12x + 9$
11.  $(4x + 3)(4x - 3)$   
 $= 16x^2 - 9$
12.  $(2 - x)(2 + x)$   
 $= 4 - x^2$
13.  $(ax + b)(ax - 2b)$   
 $= a^2x^2 - axb - 2b^2$
14.  $(\frac{1}{2}x + \frac{3}{4})^2$   
 $= \frac{4x^2 + 12x + 9}{16}$
15.  $(x + 2)(4x - 9)$   
 $= 4x^2 - x - 18$

## 5 - Factorise Quadratics and Solving Equations

Factorise these expressions

$$1. \quad x^2 + 5x + 6 = (x + 3)(x + 2)$$

$$2. \quad x^2 - 10x + 25 = (x - 5)(x - 5)$$

$$3. \quad 3x^2 - 30x + 27 = 3(x^2 - 10x + 9) \\ = 3(x - 9)(x - 4)$$

$$4. \quad x^2 + 2x - 8 = (x + 4)(x - 2)$$

$$5. \quad 2x^2 - 4x - 30 = 2(x^2 - 2x - 15) \\ = 2(x - 5)(x + 3)$$

$$6. \quad x^2 + 12x + 27 = (x + 9)(x + 3)$$

$$7. \quad 2x^2 + 32x + 128 = 2(x^2 + 16x + 64) \\ = 2(x + 8)^2$$

$$8. \quad x^2 + 8x + 16 = (x + 4)^2$$

$$9. \quad 3x^2 - 24x + 48 = 3(x^2 - 8x + 16) \\ = 3(x - 4)^2$$

$$10. \quad x^2 + 22x - 48 = (x + 24)(x - 2)$$

$$11. \quad 5x^2 - 15x + 10 = 5(x^2 - 3x + 2) \\ = 5(x - 2)(x - 1)$$

$$12. \quad x^2 - 0.25 = (x + 0.5)(x - 0.5)$$

$$13. \quad 50 - 2x^2 = 2(5 + x)(5 - x)$$

$$14. \quad 3x^2 + x - 4 = (3x + 4)(x - 1)$$

$$15. \quad 10x^2 - 6 + 11x = (5x - 2)(2x + 3)$$

Solve each equation:

$$1. \quad 2x + 6 = x + 3 \\ x = -3$$

$$2. \quad 3x + 6 = 48 \\ x = 14$$

$$3. \quad 4x - 8 = 5x - 2 \\ x = -6$$

$$4. \quad 6x + 7 = 2x + 20 \\ x = 3.25$$

$$5. \quad x + 6 = 2x - 8 \\ x = 14$$

$$6. \quad 3(x + 2) = 5(x - 2) \\ x = 8$$

$$7. \quad 4 = 8 - \frac{x}{3} \\ x = 12$$

$$8. \quad 3 - \frac{x}{3} = -5 \\ x = 24$$

$$9. \quad 5x - 6 = -39 \\ x = -6.6 \text{ or } \frac{-33}{5}$$

$$10. \quad 2 - \frac{x}{2} = -1 - \frac{x}{4} \\ x = 12$$

## 6 - Practise the Basics of "Expand" and "Factorise"

1.  $u(u + 1) = u^2 + u$
2.  $v(v - 6) = v^2 - 6v$
3.  $-w(3w - 2) = -3w^2 + 2w$
4.  $x(4x + 5) = 4x^2 + 5x$
5.  $3y(2y - 3) = 6y^2 - 9y$
6.  $-z(-5z + 3) = 5z^2 - 3z$
7.  $3 + 2(x - 8) = 3 + 2x - 16$   
 $= 2x - 13$
8.  $5(x + 7) - 12 = 5x + 35 - 12$   
 $= 5x + 23$
9.  $3(x - 6) + 2(4x - 5)$   
 $= 3x - 18 + 8x - 10$   
 $= 11x - 28$
10.  $4(a + 6) - 2(a - 2)$   
 $= 4a + 24 - 2a + 4$   
 $= 2a + 28$
11.  $2x(x + 1) - x(7 - x)$   
 $= 2x^2 + 2x - 7x + x^2$   
 $= 3x^2 - 5x$
12.  $x^2(x + 1) = x^3 + x^2$
13.  $\frac{1}{2}(4x + 12) = 2x + 6$
14.  $\frac{3}{4}(12x - 6) = 9x - 4.5$
15.  $3x(2x^2 - 4) = 6x^3 - 12x$
16.  $x(x^2 + 4) + x(3x + 2)$   
 $= x^3 + 4x + 3x^2 + 2x$   
 $= x^3 + 3x^2 + 6x$
17.  $6x + 24 = 6(x + 4)$
18.  $5x - 25 = 5(x - 5)$
19.  $11x^2 - 66x = 11x(x - 6)$
20.  $10x + 25xy = 5x(2 + 5y)$
21.  $100x + 20y = 20(5x + y)$
22.  $27 - 33x = 3(9 - 11x)$
23.  $5x^2 + x = x(5x + 1)$
24.  $6a^2 + 3a = 3a(2a + 1)$
25.  $15b^2 - 30b = 15b(b - 2)$
26.  $14y^2 + 21y = 7y(2y + 3)$
27.  $5 + 5n^2 = 5(1 + n^2)$
28.  $6x^2 + 18xy = 6x(x + 3y)$
29.  $2xy - 4ab = 2(xy - 2ab)$
30.  $3p^2 - 9pq = 3p(p - 3q)$
31.  $(x + 1)(x + 6) = x^2 + 7x + 6$
32.  $(x + 2)(x + 8) = x^2 + 10x + 16$
33.  $(x - 5)(x + 7) = x^2 + 2x - 35$
34.  $(x - 2)(x + 9) = x^2 + 7x - 18$
35.  $(x + 4)(x - 5) = x^2 - x - 20$
36.  $(x + 7)(x - 3) = x^2 + 4x - 21$
37.  $(x - 10)(x - 15) = x^2 - 25x + 150$
38.  $(x - 8)(x - 11) = x^2 - 19x + 88$
39.  $(x + 6)^2 = x^2 + 12x + 36$
40.  $(x - 9)^2 = x^2 - 18x + 81$
41.  $(x + 1)^2 + 10 = x^2 + 2x + 11$
42.  $(x - 5)^2 - 20 = x^2 - 10x + 5$
43.  $x^2 + 10x + 21 = (x + 7)(x + 3)$
44.  $x^2 + x - 12 = (x + 4)(x - 3)$
45.  $x^2 - 2x - 15 = (x - 5)(x + 3)$
46.  $x^2 - 14x + 40 = (x - 10)(x - 4)$
47.  $x^2 + 11x + 30 = (x + 6)(x + 5)$
48.  $x^2 + x - 2 = (x + 2)(x - 1)$
49.  $x^2 - 3x - 10 = (x - 5)(x + 2)$
50.  $x^2 - 4x - 96 = (x - 12)(x + 8)$
51.  $x^2 - 5x - 14 = (x - 7)(x + 2)$
52.  $x^2 - 16 = (x - 4)(x + 4)$
53.  $x^2 - 81 = (x - 9)(x + 9)$
54.  $(x - 3)^2 - 16 = x - 6x + 9 - 16$   
 $= x - 6x - 7$   
 $= (x - 7)(x + 1)$
55.  $x^2 + 2x - 15 = (x + 5)(x - 3)$
56.  $x^2 - 6x + 8 = (x - 4)(x - 2)$
57.  $2x^2 - 2x - 220 = 2(x^2 - x - 110)$   
 $= 2(x - 11)(x + 10)$
58.  $4x^2 - 100 = 4(x^2 - 25)$   
 $= 4(x - 5)(x + 5)$

## 7 - Solving Equations

1. Equations  $2S = A$ ,  $P - 2 = S$ ,  $P = 11$

*Peter = 11, Sue = 9, Ali = 18*

2.  $2x + 10 = 42$

$2x = 32$ ,  $x = 16$

3. equations  $J = 2 + M$ ,  $P = 2M$ ,  $J + M + P = 6$

*Philip = 2 hours, Mary = 1 hour, John = 3 hours*

4.  $(x) + (x+1) + (x+2) + (x+3) = 114$

$4x + 6 = 114$

$x = 27$

5. Area =  $(x + 18)(x - 5)$  and Perimeter  $2(x - 5) + 2(x + 18) = 54$

$4x + 26 = 54$ ,  $x = 7$

*Dimensions are 2cm and 25cm*

6.  $(2x + 1)(2y + 1) = 4xy + 2x + 2y + 1$

$= 2(2xy + x + y) + 1$

*It is a multiple of 2 and adding 1 at the end shows it must be odd*

7. Consecutive numbers  $= a-2, a-1, a, a+1, a+2$

Summing the numbers together  $(a-2) + (a-1) + a + (a+1) + (a+2)$

$= 5a$

8. Consecutive numbers  $= a, a+1, a+2$

Square and sum each.  $= a^2 + (a+1)^2 + (a+2)^2$

$= a^2 + a^2 + 2a + 1 + a^2 + 4a + 4$

$= 3a^2 + 6a + 5$

Subtract 2, divide the result by 3  $= 3a^2 + 6a + 3$

$= a^2 + 2a + 1$

$= (a + 1)^2$

9. Sung's birth month equation is:  $4x + (12 - x) - 2(5 + x) = 10$

Simplifying gives  $x + 2 = 10$  or  $x = 8$

*Kim's birth month is 8 (August)*

10. Because the graph is quadratic the highest point will be at the mid way point between where the water starts and finishes i.e. when  $x = 25$  metres

Therefore  $0.5(25) - 0.01(25)^2$

$= 12.5 - 0.01(625)$

$= 12.5 - 6.25$

$= 6.25$  metres high

## 8 - Solving Quadratic Equations

Solve these equations:

1.  $(x + 3)(x - 2) = 0$

$x = 3, x = 2$

2.  $(x + 0.5)(4x + 6) = 0$

$x = -0.5, x = -1.5$

3.  $(2x + 8)(x - 4) = 0$

$x = -4, x = 4$

4.  $(4x - 10)(3x + 12) = 0$

$x = 2.5, x = -4$

5.  $(4x - 20)(2x + 2.5) = 0$

$x = 5, x = -1.25$

6.  $2x(x - 4) = 10$

$x = -1, x = 5$

7.  $(4x + 2)(2x - 6) = 0$

$x = -\frac{1}{2}, x = 3$

8.  $3x(2x + 4) = 0$

$x = 0, x = 2$

9.  $x^2 + 5x = -6$

$x = -2, x = -3$

10.  $(x - 4)(x + 5) = 0$

$x = 4, x = -5$

11.  $5x^2 - 3x = 0$

$x = 0.6, x = 0$

12.  $6x^2 - 2 = 0$

$x = 0.58, x = -0.58$

13.  $100 - 4x^2 = 0$

$x = 5, x = -5$

14.  $2x^2 - 6x + 4 = 0$

$x = 2, x = 1$

15.  $(x + 3)(x - 2) = 14$

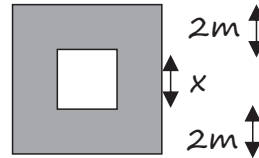
$x = 4, x = -5$

16.  $2x^2 + 5x = -3$

$2x^2 + 5x + 3 = 0 \quad (2x + 3)(x + 1) = 0$

$x = -1.5, x = -1$

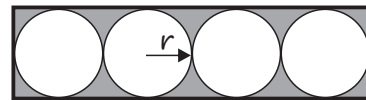
17.



$(x + 4)(x + 4) - x^2 = 40$

$x = 3$

18.



total circle area =  $4\pi r^2$

$8r \times 2r - 4\pi r^2 = 14$

$16r^2 - 4\pi r^2 = 14$

$r^2(16 - 4\pi) = 14$

$r = 2.02$

19.  $\frac{1}{2}(2h + 6) \times h = 40$

$h^2 + 3h - 40 = 0$

$(h + 8)(h - 5) = 0$

$h = -8, h = 5$

20. Area Front Square + Area Triangle

= (length)<sup>2</sup> + ( $\frac{1}{2}$  × base × height)

=  $x^2 + \frac{1}{2} \cdot x \cdot \frac{1}{2}x$

=  $x^2 + \frac{1}{4}x^2$

=  $\frac{5}{4}x^2$

Cross Sectional Area (CSA) =  $\frac{5}{4}x^2$

Volume = CSA × Length

$40 = \frac{5}{4}x^2 \times 4x$

$40 = 5x^3$

$8 = x^3,$

therefore  $x = 2 \text{ m}$

# The Algebra Project

## 9 - Algebraic Fractions

Simplify

$$1. \quad x - \frac{x-2}{2} \qquad \frac{x+2}{2}$$

$$2. \quad \frac{x}{2} - \frac{2x-1}{2} \qquad \frac{-x+1}{2}$$

$$3. \quad \frac{x^2 - 3x - 28}{x+4} \qquad x - 7$$

$$4. \quad \frac{2p^2 - 12pq}{6p^2} \qquad \frac{p-6q}{3p}$$

$$5. \quad \frac{x^2 - 4x - 5}{x^2 + 6x + 5} \qquad \frac{x-5}{x+5}$$

$$6. \quad \frac{2}{3a} \times \frac{a^2}{12a} \qquad \frac{1}{18}$$

$$7. \quad \frac{5a}{2b} \div \frac{a}{6b} \qquad 15$$

$$8. \quad \frac{3ab^2 - 4a^3b + ab^2}{4ab^2} \qquad \frac{b-a^2}{b}$$

$$9. \quad \frac{3x}{7} + \frac{2x}{5} \qquad \frac{29x}{35}$$

$$10. \quad \frac{9x^3}{12x^2} \qquad \frac{3x}{4}$$

$$11. \quad \frac{x^2 - 5x + 6}{x-3} \qquad x - 2$$

$$12. \quad \frac{8x^2}{20x - 12x^2} \qquad \frac{2x}{5-3x}$$

Solve

$$1. \quad \frac{x+2}{3} - \frac{2x-1}{5} = 1$$

$$x = -2$$

$$2. \quad \frac{3}{y-4} - \frac{3}{y+4} = \frac{2}{y+1}$$

$$y = 14, y = -2$$

$$3. \quad \frac{x}{5} - \frac{2x-1}{2} \geq \frac{-3x}{5}$$

$$x \leq \frac{5}{2}$$

$$4. \quad \frac{2-5x}{4} > 3$$

$$x < -2$$

$$5. \quad \frac{4x-6}{3} > 2x+1$$

$$x < \frac{-9}{2}$$

$$6. \quad 6x-3 \geq 8x+9$$

$$x \leq -6$$

## 10 - Quadratic Sequences

Give an expression for the  $n$ th term.

1. 6, 9, 14, 21, 30                     $= n^2 + 5$
2. 11, 14, 19, 26, 35                 $= n^2 + 10$
3. -1, 2, 7, 14, 23                     $= n^2 - 2$
4. -5, -2, 3, 10, 19                   $= n^2 - 6$
5. 4, 10, 18, 28, 40                   $= n^2 + 3n$
6. 9, 20, 33, 48, 65                   $= n^2 + 8n$
7. 5, 10, 17, 26, 37                   $= n^2 + 2n + 2$
8. 3, 3, 5, 9, 15, 23                   $= n^2 - 3n + 5$
9. 2, 10, 20, 32, 46, 62               $= n^2 + 5n - 4$
10. -10, -11, -10, -7, -2, 5         $= n^2 - 4n - 7$

11. 3, 8, 15, 24, 35, 48               $= n^2 + 2n$   
     2nd difference = 2 therefore  $n^2$   
         3, 8, 15, 24, 35, 48  
          $n^2$  1, 4, 9, 16, 25, 36  
     difference 2 4 6 8 12 10  
     difference each time = 2 therefore  $+ 2n$

12.  $\frac{1}{2}, \frac{4}{3}, \frac{9}{4}, \frac{16}{5}$

Numerator 1, 4, 9, 16 ...  $n^2$

Denominator 2, 3, 4, 5, ...  $n + 1$

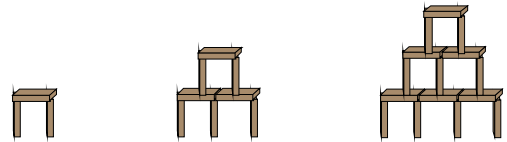
$n$ th term =  $\frac{n^2}{n + 1}$

13. Block base = 3, 4, 5               $n + 2$   
     Block height = 4, 5, 6           $n + 3$   
     Area  $(n + 2)(n + 3) = n^2 + 5n + 6$

14. Write the  $n$ th term for:

- a. 5, 12, 21, 32     $n^2 + 4n$
- b. 3, 9, 17, 27     $n^2 + 3n - 1$

15.



Slabs = 24, 35, 48, 63, 80

Look at the pattern between the storeys and the slabs.

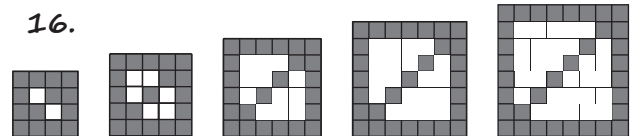
$1 \times 3 = 3, 2 \times 4 = 8, 3 \times 5 = 15, 4 \times 6 = 24, 5 \times 7 = 35$  therefore

$y = x(x + 2)$

$y = x^2 + 2x$

Using this formula, 25 storeys would need 675 slabs of wood.

16.



Total = Grey + white

$n^2 = 5n - 6 + \text{white}$

white =  $n^2 - 5n + 6$

17. Sequence Q = 3, 8, 15, 24, 35

The  $n$ th term of the sequence is

$n^2 + kn$  where  $k$  represents a number. Find the value of  $k$ .

Using term 1,  $n^2 + kn = 3$

$1^2 + k = 3$

$k = 2$