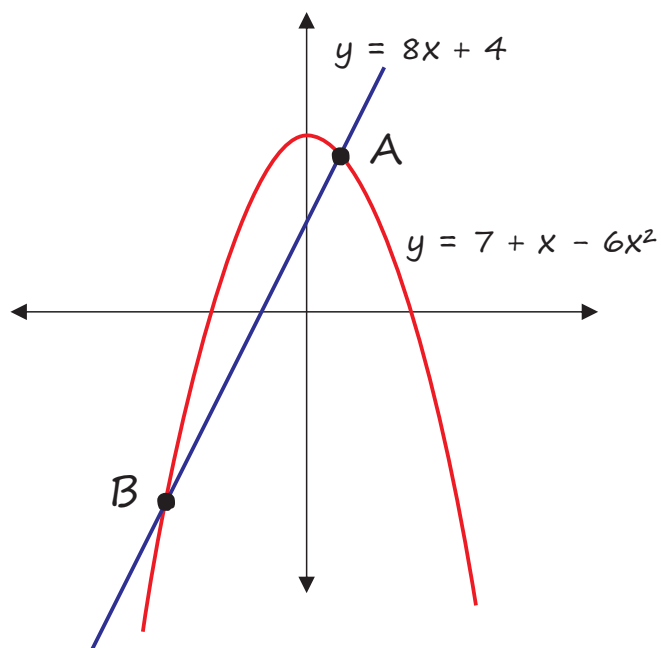
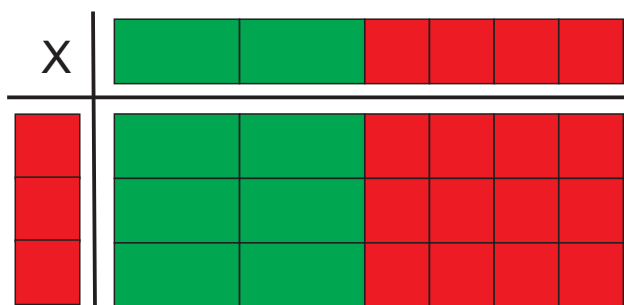
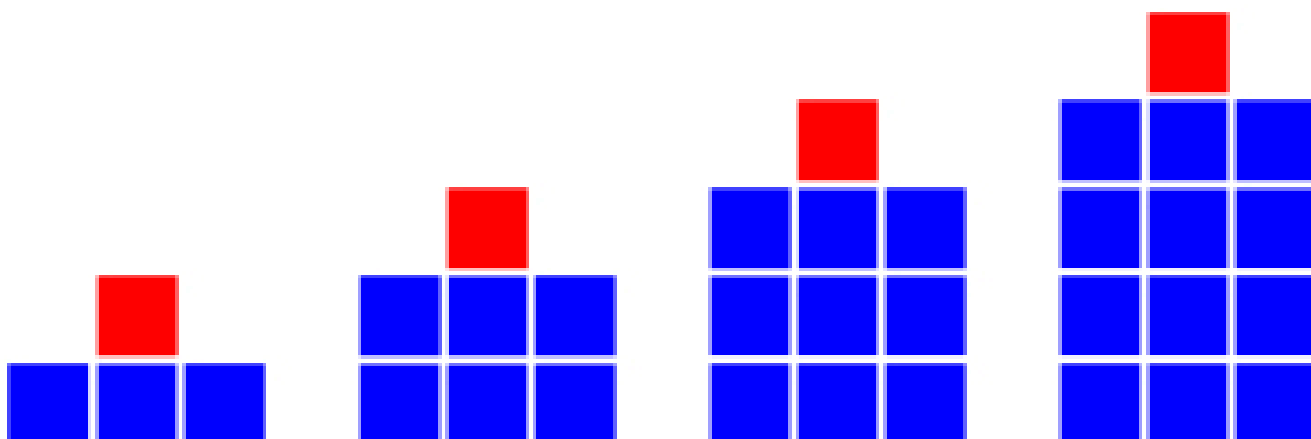
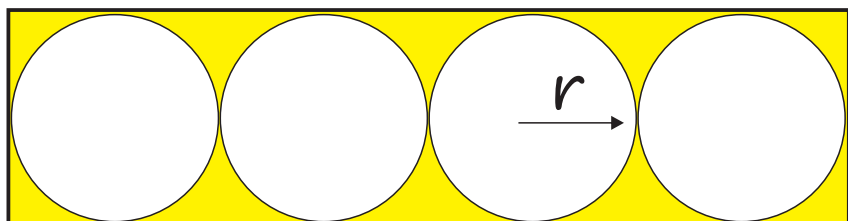


Expand  $3(2x + 4) = 6x + 12$



# THE ALGEBRA PROJECT

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**The Algebra Project**  
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# Algebra Like Terms

Simplify all these expressions

- $a + a + a =$
- $x + x =$
- $p + p + p + p + p =$
- $y + y + y =$
- $i + i + i + i + i + i + i =$
- $k + k + k + k =$
- $4x + x =$
- $10x + 2x =$
- $7y + 3y + y =$
- $8j + j + j =$
- $15d + 3d + 2d =$
- $12a + 5a + 2a =$
- $13c - 4c =$
- $17u - 8u =$
- $8g - 10g =$
- $6r + 6r - r =$
- $5t - 3t + 2t =$
- $10h + 8h - 12h =$
- $6x - 8x + x =$
- $-8y + 6y - 5y =$
- $3r - 8r + 2r =$
- $-5w - 6w =$
- $3y - 6y + y =$
- $f - 6f - 2f =$
- $4\pi - 7\pi + \pi =$
- $3e + 8s - 2s - e =$
- $9i + 4j - 6i - 2j =$
- $-3t - 5s + 5t - 2s =$
- $4d + 2e - 4e - 8d =$
- $-4f + 3h - 7h + 5f =$
- $11 + 3n + 6 + 9n =$
- $8 + 3d - 6 - 5d =$
- $r - 5 - 5r + 10 =$
- $g - 12g + 7h =$
- $7j + 5k - 2j - 5k =$
- $4w - 6x + 8w - 6w =$
- $-4m + 6n - 10n + 4n =$
- $5j + 5k =$
- $2s - 7t - 10s - 2t =$
- $3x^2 + 2x^2 - 3y =$
- $5x + 6y - 7y - 8x =$
- $-2q + 2p - 7p + 8q =$
- $14ab - 7b - 3ab + 8b =$
- $-2xy + 5x - 8xy - 9x =$
- $8y^2 - 2y + 5y - 6y^2 =$
- $16gh + 8g - 7gh =$
- $6bg - 2b - 5bg =$
- $-6mn - 12n + 8n =$
- $-5xy + 12yx =$
- $-9ab + 8ab + 9b =$

# Algebra Expressions

Simplify all these expressions

- $a \times a =$
- $y \times y \times y \times y =$
- $p \times p \times p =$
- $w \times w \times w \times w \times w =$
- $i \times i \times i \times i \times i \times i \times i =$
- $k \times k \times k \times k =$
- $5a \times b =$
- $8f \times 2x =$
- $9y \times 3z =$
- $5j \times 3k =$
- $7d \times 3e =$
- $12a \times 5b =$
- $15c \times 2c =$
- $7u \times 8u =$
- $8g \times 5g =$
- $6r^2 \times r =$
- $5p^3q \times 3p =$
- $10h^2j \times 8j =$
- $6b^2 \times 9b^3 \times b =$
- $-8y \times 6y^2 =$
- $4r^2 \times 9r^2 \times r =$
- $-5w^3 \times -6w^2 =$
- $3y^2 \times 6y^2 \times y =$
- $2g^3f \times 6gf^2 =$
- $4p^2qr \times 5r \times 2p =$

- $(y^2)^3 =$
  - $(5a)^2 =$
  - $(2y)^5 =$
  - $(3y)^2 =$
  - $(5xy)^3 =$
  - $-(2x)^2 =$
  - $(-2x)^2 =$
  - $(x^2)^7 =$
  - $(k^5h^3)^2 =$
  - $w^0 =$
  - $\sqrt{49a^{12}} =$
  - $\sqrt{x^{100}} =$
  - $\sqrt{9a^{10}b^6} =$
  - $\sqrt{100g^2h^{100}} =$
- If  $a = 2, b = 5, c = 10$  calculate:
- $(a + b)^2 =$
  - $4b^2 - (3a)^2 =$
  - $(a^2)^2 =$
  - $c^2 - a^2 + b^2 =$
  - $(2a^2)^3 =$
  - $2c^2 - 2b^2 =$
  - $4(a^4 - 10) =$
  - $b(5c - a^2) =$
  - $3a + 4c^3 =$
  - $\sqrt{c^2} =$

# Algebraic Fractions

Simplify all these fractions!

■  $\frac{3}{x} + \frac{5}{x} =$

■  $\frac{7}{w} - \frac{5}{w} =$

■  $\frac{x}{5} + \frac{x}{15} =$

■  $\frac{b}{3} - \frac{b}{6} =$

■  $\frac{3}{x^2} - \frac{2}{x} =$

■  $\frac{2}{x^2} + \frac{5}{x^3} =$

■  $\frac{7}{x^3} + \frac{4}{x} =$

■  $\frac{s}{4} + \frac{5}{3} =$

■  $\frac{x}{3} - \frac{x}{7} =$

■  $\frac{r}{5} + \frac{r}{8} =$

■  $\frac{x}{a} + \frac{2}{d} =$

■  $\frac{4}{m} - \frac{x}{n} =$

■  $\frac{5}{a} + \frac{c}{c} =$

■  $\frac{7}{xy} + \frac{2}{y} =$

■  $\frac{5}{ab} - \frac{3}{a} =$

■  $\frac{7}{x} + \frac{7}{xy} =$

■  $\frac{3h}{gh} =$

■  $\frac{12m}{4n} =$

■  $\frac{6x}{18y} =$

■  $\frac{mp}{6m} =$

■  $\frac{3m^4}{6m^2} =$

■  $\frac{2w^5}{8w^2} =$

■  $\frac{8x^5}{4x^3} =$

■  $\frac{50r^4}{5r^2} =$

■  $\frac{7e^7}{21e^3} =$

■  $\frac{5m^5}{25m^2} =$

■  $\frac{12r^9}{3r^3} =$

■  $\frac{27m^4}{9m} =$

■  $\frac{8a}{15n} \times \frac{5c}{4a} =$

■  $\frac{3c}{4e} \times \frac{2e^3}{4c} =$

■  $\frac{5ab}{2a} \times \frac{5a}{10ab} =$

■  $\frac{3}{4b} \times \frac{15}{8b^2} =$

■  $\frac{3ab^2}{5cd} \times \frac{5c}{7a^2} =$

If  $a = \frac{1}{2}$ ,  $b = 4$ ,  $c = 10$  calculate:

■  $\frac{2a + b}{c} =$

■  $\frac{4a^2}{b} =$

■  $\frac{ac}{bc} \times \frac{bc}{ac} =$

■  $\frac{c}{b} \times \frac{a}{b} =$

■  $\frac{c}{b} \div \frac{a}{b} =$

■  $\frac{4a + c}{b} =$

■  $a(c^2 + b^2) =$

# Algebra Expansions

Expand all these expressions!

▣  $2(6 + 3x) =$

▣  $5(3a - 5) =$

▣  $10(2b + 4) =$

▣  $4(5c - 7) =$

▣  $7(4d + 5e) =$

▣  $4f(f + 2) =$

▣  $5g(2g - 6) =$

▣  $6h(h - 5) =$

▣  $5x(x^2 - x + 2)$

=

▣  $3(a^2 - 18a + 4)$

=

▣  $2(a + 2b) + (a - b)$

=

▣  $4(c - 3d) - 3(c + 2d)$

=

▣  $3(2e - f) + 2(e - 2f)$

=

▣  $3x(x + 8) - x(2x - 5)$

=

▣  $(x + 5)(x + 1)$

▣  $(x - 7)(x - 2)$

▣  $(a + 6)(a - 4)$

▣  $(x - 3)(x + 8)$

▣  $(x - 9)(x + 9)$

▣  $(x + 6)^2$

▣  $(x + 9)(x + 4)$

▣  $(x - 12)(x + 1)$

▣  $(x + 11)(x - 5)$

▣  $(x - 15)(x - 3)$

▣  $(7 - x)(7 + x)$

▣  $(9 - x)^2$

▣  $(3x + 2)(2x - 1)$

▣  $(4x - 7)(x + 5)$

# Quadratic Expansions

Want extra practise? Expand all these expressions!

$$\square (x + 3)(x - 8)$$

$$\square (x + 12)(x + 10)$$

$$\square (x - 9)(x - 8)$$

$$\square (x - 7)(x + 11)$$

$$\square (x + 8)(x + 1)$$

$$\square (x - 2)(x + 6)$$

$$\square (x - 5)(x - 13)$$

$$\square (x + 15)(x - 4)$$

$$\square (x + 3)(6 - x)$$

$$\square (2 - x)(3 - x)$$

$$\square (10 - x)(4 + x)$$

$$\square (x + 8)(5 + x)$$

$$\square (2x + 3)(3x + 4)$$

$$\square (5x + 1)(x - 2)$$

$$\square (4x - 5)(2x - 3)$$

$$\square (1 - 2x)(3x + 7)$$

$$\square (x - 5)(x + 5)$$

$$\square (1 + 2x)(1 - 2x)$$

$$\square (3 - 2x)(2x + 3)$$

$$\square (x + 7)^2$$

$$\square (x - 5)^2$$

$$\square (3x - 1)^2$$

$$\square (2 + 4x)^2$$

$$\square (2x + 3y)^2$$

$$\square (6x - 2y)^2$$

$$\square 2(3x - 5)^2$$

# Factorising

Factorise all these expressions!

▶  $10a - 15b =$

▶  $3xy - 12xz =$

▶  $4\pi x - 4\pi y =$

▶  $4a^2 + 3a =$

▶  $x^3 + x^2y =$

▶  $a^2 - 6y =$

▶  $2x^3 + 2xy^2 =$

▶  $y^2 + y^3 =$

▶  $x^2 + 2xy - x =$

▶  $36 + 6x^3 =$

▶  $2\pi r^2 + \pi rh =$

▶  $x^2y - xy^2 =$

▶  $6a^2 - 12a =$

▶  $5x^2 - 10xy =$

▶  $4a^2b + 8ab^2 =$

▶  $21x^3 - 14x^2 =$

▶  $t^2 + 4t =$

▶  $20x + 10x - 5y + 25y$

▶  $14a^2b + 21ab^2$

▶  $3c + 6d + 12e$

▶  $14a - 21b =$

▶  $3xy + 9yz =$

▶  $2\pi d - 2\pi r =$

▶  $15x^2 - 20x =$

▶  $r^3 + r^2 =$

▶  $9 + 3n^2 =$

▶  $2x^2 + 2xy =$

▶  $5x^2 + 5y^2 =$

▶  $3x^3 + xy =$

▶  $4ab - 8bc =$

▶  $ax + bx + ay + by$

▶  $1 + x + y + xy$

▶  $2xy + 6y - x - 3$

▶  $a^2 + ab - ac - bc$

▶  $2ax - 4ay - 3bx + 6by$



# Quadratic Factoring 1

Factorise all these quadratic expressions!

- $x^2 + 11x + 24$

- $x^2 + 11x + 18$

- $x^2 - 100$

- $x^2 + 9x - 36$

- $x^2 + 5x - 84$

- $x^2 + 6x - 16$

- $x^2 - 3x - 40$

- $x^2 - 11x + 18$

- $2x^2 - 20x + 32$

- $3x^2 - 24x + 48$

- $2x^2 - 2x - 84$

- $36 - x^2$

- $4x^2 + 21x + 5$

- $3x^2 + 16x + 13$

- $2x^2 - 2x - 12$

- $8x^2 - 18x + 9$

- $5x^2 - 13x - 6$

- $6x^2 + 16x + 10$

- $6x^2 + 5x - 6$

- $10x^2 + 6x - 4$

- $3x^2 - 6x - 24$

# Quadratic Factoring 2

Factorise all these quadratic expressions!

- $3x^2 + 33x + 84$

- $2x^2 + 20x + 32$

- $4x^2 + 12x - 72$

- $2x^2 + 14x - 36$

- $2x^2 - 6x - 80$

- $3x^2 - 12x - 15$

- $2x^2 - 36x + 160$

- $5x^2 - 20x + 15$

- $4x^2 - 88x + 484$

- $2x^2 + 20x + 50$

- $4x^2 - 400$

- $6x^2 - 6$

- $2x^2 + 5x + 3$

- $3x^2 + 8x + 4$

- $8x^2 + 14x + 3$

- $3x^2 + 5x - 2$

- $2x^2 + 3x - 5$

- $5x^2 - 8x + 3$

- $3x^2 - 17x + 10$

- $2x^2 - 3x - 9$

- $2x^2 + 9x - 18$

- $15x^2 + x - 6$

- $4x^2 + 2x - 20$

- $8x^2 + 14x + 6$

# Quadratic Factoring 3

Factorise all these quadratic expressions!

- $6x^2 + 14x + 4$

- $4x^2 + 6x + 2$

- $8x^2 - 10x + 3$

- $3x^2 + x - 2$

- $3x^2 + 16x + 5$

- $3x^2 + 11x + 10$

- $14x^2 + x - 3$

- $4x^2 - 8x - 5$

- $6x^2 + 13x - 28$

- $2x^2 + 5x + 3$

- $5x^2 + 12x + 4$

- $4x^2 - 12x - 7$

- $2x^2 - x - 10$

- $6x^2 + x - 2$

- $8x^2 + 14x + 3$

- $2x^2 + 9x - 5$

- $2x^2 - 24x + 40$

- $16x^2 - 49$

- $2x^2 - 50$

- $3x^2 - 147$

- $4 - 36x^2$

Factorise and solve to show that  
the solutions to the equation:

$$2x^2 + 4x - 48 = 0$$

are 2 times the solutions of the  
equation:  $2x^2 + 2x - 12 = 0$

# Algebra (1)

Simplify all these expressions

- $3a + 8a =$

- $b + b + 5b =$

- $4c + 6c - 8c =$

- $5d - d - 3e - 2e =$

- $5e + 2f + 3e - 4f =$

- $3g - 3h + 4g - 5h =$

- $a \times a \times 5 =$

- $9b \times b \times b \times b =$

- $14y \times 2$

- $c \div c =$

- $12d \div 4 =$

- $5e \times 6 =$

- $4f \times f =$

- $2g \times 8h =$

- $20m \div 10m =$

- $15xy \div 3 =$

- $6r \times 7r =$

- $5g^2 \times 4g =$

- $2(a + 4b) =$

- $5(3c - 8) =$

- $-10(2d - 3) =$

- $-2(4f + 3e) =$

- $2(a + 2) + (a - 1)$

- $4(b - 1) + 2(b + 2)$

- $(c - d) + 6(c + 2d)$

- $3(e + 9) - 2e$

- $f(2f - 4) + 4f$

---


$$x = 3 \text{ and } y = 4$$

Find the values:

---

- $2x + 3y =$
- $2(x + y) =$

- $5x - 2y =$
- $x^2 - y^2 =$

- $3x^2 =$
- $\frac{6x + y}{2} =$

- $xy^2 =$
- $\sqrt{4x + y} =$

# Algebra (2)

Simplify all these expressions

$$3a + 2a - 4a =$$

$$6b - 4b - 3b =$$

$$2c - 3d + c - d =$$

$$ab + ba =$$

$$a^2 \times a \times 5 =$$

$$9b \times 6b =$$

$$14y \times 2$$

$$c^2 \div c =$$

$$d^3 \div d^2 =$$

$$5e \times 8e =$$

$$4f \times 9g =$$

$$2g \times 8g^2 =$$

$$8m^3 \div 2m^2 =$$

$$15x^3 \div 3x^3 =$$

$$4d^2 + 4d^2 =$$

$$ef + 5ef - 4ef =$$

$$4ab \times 4ac \div 4bc =$$

$$(3x^2y^3)^2 =$$

$$\sqrt{25r^2s^6} =$$

$$a^3 \times a^5 =$$

$$(b^5)^2 =$$

$$c^5 \div c^3 =$$

$$d^7 \times d^7 =$$

$$(3e^4)^3 =$$

$$f \times f^3 \times f^3 =$$

$$3g^5 \times 2g^3 =$$

$$3h^6 \div h^2 =$$

$$2(3a - 5) =$$

$$4(6b + 8c) =$$

$$5d(d^2 - d + 2) =$$

$$5a + (-2a) =$$

$$b - (-2b) =$$

$$(-5c) - (-8c) =$$

$$(-2d) + (-2d) =$$

$$-8y \times 4y =$$

$$6 \times -4r^2 =$$

$$-3g^2 \times -4g^3 =$$

$$-6x \div -6x =$$

# Algebra (3)

Simplify all these expressions

- $13a + (2a - 8a) =$

- $2b - 3c + b - 2c =$

- $6a^2 \times 7 \times a =$

- $9b \times 6b^3 =$

- $14y \times 2y =$

- $c^3 \div c^2 =$

- $(8d)^2 =$

- $e^3 + e^3 =$

- $4f^2 + 4f^2 =$

- $5ab \times 5ac \div 5bc =$

- $(8e^2f^3)^2 =$

- $\sqrt{(16g^2h^8)} =$

- $(-2x)^2 =$

- $-(3x)^3 =$

- $2(a + 2b) + (a - b)$

- $x(2x + 3) - 4(3x - 1)$

- $x(x^2 + 1) - x^2(x + 1)$

- $\frac{12a^2b}{3ab} =$

- $\frac{6cd^2}{2c^2d} =$

- $\frac{10e}{3} - \frac{2e}{9}$

- $\frac{d}{21} + \frac{4d}{7}$

- $\frac{3x}{8} - \frac{x}{6} =$

- $\frac{5a}{2} + \frac{a}{10} =$

- $\frac{4c}{15d} = \frac{5e}{8}$

- $\frac{3}{4e} \div \frac{15}{8e^2}$

- $\frac{3ab}{10cd} - \frac{2c^3}{9b^2}$

- $\frac{8}{9x} \div \frac{2}{3x}$

- $\frac{a+1}{2} - \frac{a-1}{3}$

- $\frac{b+c}{2} - \frac{b-2}{5} =$

- $\frac{3x-1}{3} + \frac{2-x}{4} =$

- $\frac{4a+1}{2} - \frac{a+3}{5} =$

---

$x = 5 \quad y = 3 \quad z = 1$

---

- $6x + 2y = \dots\dots\dots$       •  $x^2 + y^2 = \dots\dots\dots$

- $2x^2 + 3z = \dots\dots\dots$       •  $(4x - 2z)^2 = \dots\dots\dots$

---

$a = \frac{1}{2} \quad b = \frac{1}{8} \quad c = \frac{3}{4}$

---

- $a + b = \dots\dots\dots$       •  $24a^2 + 4c = \dots\dots\dots$

- $\frac{a}{b} = \dots\dots\dots$       •  $4c(12a^2b) = \dots\dots\dots$

# Equations (1)

Solve all these equations!

▶  $4x = 44$        $x = \dots\dots\dots$

▶  $x - 8 = 9$        $x = \dots\dots\dots$

▶  $\frac{1}{4}x = 8$        $x = \dots\dots\dots$

▶  $x + 7 = 19$        $x = \dots\dots\dots$

▶  $6x = 48$        $x = \dots\dots\dots$

▶  $\frac{1}{2}x = 20$        $x = \dots\dots\dots$

▶  $x - 5 = 25$        $x = \dots\dots\dots$

▶  $5x = 0$        $x = \dots\dots\dots$

▶  $x - 14 = 5$        $x = \dots\dots\dots$

▶  $x + 8 = 24$        $x = \dots\dots\dots$

▶  $3x + 2 = 14$

▶  $14 + 4x = 22$

▶  $12x - 5 = 31$

▶  $8x - 21 = 11$

▶  $3x = x + 10$

▶  $7x + 3 = 33 + x$

▶  $15x - 4 = 3x + 8$

▶  $4 + 6x = 12 + 2x$

▶  $3(x + 2) - x = 30$

▶  $4(1 + 2x) = 52$

▶  $6(x + 1) - 6 = 30$

▶  $2(x - 9) = x - 4$

▶  $8x = 2(3x + 6)$

▶  $5(x + 8) = 50$

▶  $(x + 1) + 2(x + 2) = 29$

▶  $4(3x - 1) + 10 = 18$

▶  $x + (x + 2) + (x + 4) = 21$

▶  $6x - 3 = 2x + 25$

▶  $4(x + 5) = 3(16 - x)$

▶  $8x - 5 = 2x + 13$

▶  $6x - 4 = 2(x + 14)$

# Equations (2)

Solve all these equations!

$$\blacksquare a - 5 = 19$$

$$\blacksquare b + 19 = 38$$

$$\blacksquare 7c = 91$$

$$\blacksquare \frac{d}{5} = 20$$

$$\blacksquare 11e - 5 = 72$$

$$\blacksquare 2f + 4 = 80$$

$$\blacksquare g + 5g = 72$$

$$\blacksquare 3h = 90 - 2h$$

$$\blacksquare 3j + 2 = 27 - 2j$$

$$\blacksquare 36 - 4k = 0$$

$$\blacksquare 12m - 5 = 15 + 8m$$

$$\blacksquare 2n + 7 = 31 - 4n$$

$$\blacksquare 3(p + 2) - p = 26$$

$$\blacksquare 4(2r - 6) + 3(r + 5) = 35$$

$$\blacksquare \frac{1}{2}(t - 4) = 12$$

$$\blacksquare \frac{w + 7}{5} = 5$$

$$\blacksquare 2(2a - 5) + 4(a + 7) = 82$$

$$\blacksquare 5(c + 3) + (c - 6) = 27$$

$$\blacksquare 2(e + 5) - 3(6 - e) = 42$$

$$\blacksquare 5(f - 1) + 3(f - 4) = 23$$

$$\blacksquare 22 - 5k - (k + 10) = 0$$

$$\blacksquare 2(3m - 4) - 2(m + 7) = 0$$

$$\blacksquare 5n - 2 = n - 3(n + 3)$$



# Equations (3)

Solve all these equations!

■  $3a + 5 = 17 - a$

■  $25 - 7c = -10$

■  $12d - 5 = 15 + 10d$

■  $2e + 7 = 31 - 4e$

■  $3(f + 2) - f = 22$

■  $\frac{1}{4}(g - 12) = 9$

■  $\frac{h+7}{4} = 10$

■  $\frac{3}{4}j - 2 = 10$

■  $4k - 5 = 2k + 43$

■  $5(m + 3) + (m - 5) = 100$

■  $3(2n - 5) - 4(n + 7) = 13$

■  $\frac{2a - 5}{3} = \frac{a - 2}{2}$

■  $\frac{c - 1}{3} = c - \frac{3(c + 2)}{5}$

■  $\frac{3}{4}(5e - 12) - \frac{1}{4}(4 + e) = 32$

■  $x^2 = 81$

■  $x^3 = 64$

■  $x^2 = 0.36$

■  $\sqrt[3]{x} = 2$

■  $3^x = 81$

■  $4^x = \frac{1}{16}$

■ Let  $x = 2y$ .

Find the value of  $\frac{x^2 - 2y^2}{x^2 + 2y^2}$

# Equations (4)

Solve all these equations

- $3^{x+1} = 81$

- $2^{x-1} = 64$

- $5 \times 5^{3x} = 5^{-2x^2}$

- $7 \times 7^{1-x} = 7^{3x^2}$

- $2^{3x+4} > 2^x$

- $5^{x+6} = 5^{x^2}$

- $9 \times 3^{x-4} < 27$

- $10 \times 2^{p-1} < 1280$

- $25 \times 5^{(2y+13)} = 5^{y^2}$

- $9 \times 3^{(x-4)} > 81$

- $216 \times 6^{(2x+5)} = 6^{x^2}$

- $8 \times 2^{(2y+13)} = 2^y$

- $64 \times 4^{x-5} > 512$

- $16 \times 2^x = 2^{6x+3}$

- $6 \times 2^{2n+1} > 96$

- $9 \times 3^x = 3^{5x+4}$

# Simultaneous Equations

Solve all these equations on your own paper!

1.  $y = 2x + 4$   
 $y = 5x - 5$

2.  $x = -y + 12$   
 $x = 2y$

3.  $y = 7$   
 $y = 3x - 20$

4.  $x = 1 - 2y$   
 $x = y - 5$

5.  $x = 2$   
 $x = 3y - 1$

6.  $y = 6x - 1$   
 $y = 10x - 1$

7.  $y = 3x + 1$   
 $y = 2x - 2$

8.  $x = 3y$   
 $x = 5 - 2y$

9.  $y = 10$   
 $y = 2 - 4x$

10.  $x = y - 4$   
 $x = 3 - 6y$

11.  $y = 5x - 6$   
 $2x + y = 8$

12.  $x = 3y + 6$   
 $2y + x = 1$

13.  $x = 5 - 3y$   
 $x + 4y = 21$

14.  $x = 13 - 2y$   
 $3y - 27 = x$

15.  $y = 6x - 5$   
 $y + 2x = -5$

16.  $x + y = 3$   
 $2x - 3 = y$

17.  $2x + y = 7$   
 $y = 4x - 5$

18.  $5y - x = 2$   
 $x = 4y - 1$

19.  $5 = x + 8y$   
 $x = 7 - 10y$

20.  $x = 3 - 3y$   
 $x = 2y + 18$

21.  $x + y = 14$   
 $x - y = 10$

22.  $-x + y = 2$   
 $x + y = 28$

23.  $x - 2y = 4$   
 $x + 2y = 14$

24.  $-3x + y = -5$   
 $3x + y = 19$

25.  $y + 2x = 1$   
 $4x - y = -25$

26.  $4y - x = -5$   
 $x + 3y = 5$

27.  $x + 2y = 1$   
 $x - 2y = -11$

28.  $5x + y = 17$   
 $-5x + y = -3$

29.  $-y + 3x = 8$   
 $3x + y = 16$

30.  $7x + y = 11$   
 $y + 3x = 3$

*Solve all these equations on your own paper!*

1. *Samantha bought apples and oranges. She bought a total of 8 fruit. The apples cost \$3 each, and the oranges cost \$2 dollars each. If she spent a total of \$20, how many apples and how many oranges did she buy?*
2. *A bakery sells muffins and cupcakes. The total number of baked goods sold is 27. Muffins cost \$6 each, cupcakes cost \$4.50 each. Total revenue was \$117. How many muffins and how many cupcakes were sold?*
3. *A bookshop sells paperback and hardcover books. The total number of books sold is 12. Paperbacks cost \$30 each, and hardcovers cost \$45 each. The total revenue was \$960 dollars. How many paperbacks and how many hardcovers were sold?*
4. *A store sells pens and pencils. The total number of writing instruments sold is 18. Pens cost \$12 dollars each, and pencils cost \$8 each. Total revenue was \$224. How many pens and how many pencils were sold?*
5. *A school choir consists of sopranos and altos. The total number of choir members is 28. Sopranos each receive a score of 4 points, and altos receive a score of 3 points. The total score of the choir was 101 points. How many sopranos and how many altos are in the choir?*
6. *A box contains red and blue marbles. The total number of marbles is 40. Red marbles are priced at \$8 each, and blue marbles are priced at \$3 each. The total value of the marbles is \$225. How many red and how many blue marbles are in the box?*
7. *A classroom has a total of 25 students divided into boys and girls. For the mufti fund-raiser, boys each brought in \$20, and girls each brought in \$25. The total amount collected was \$550 dollars. How many boys and girls are in the class?*

*Solve all these equations on your own paper!*

8. *1000 tickets to the Selwyn Benefit Concert were sold. Adult tickets cost \$20, children's cost \$12, and a total of \$17,600 was collected. How many tickets of each kind were sold?*
9. *Mrs. B. invested \$30,000; part at 5%, and part at 8%. Total interest on the investment was \$2,100. How much did she invest at each rate?*
10. *A mother is now 30 years older than her son. 15 years ago, she was twice as old. What are the present ages of the mother and her son?*
11. *A total of 725 concert tickets were sold for \$11,500. If adult tickets cost \$18.50, and children's tickets cost \$10, how many tickets of each kind were sold?*
12. *Mr. Barlow. has \$20,000 to invest. He invests part at 6%, the rest at 7%. He earns \$1,280 interest. How much did he invest at each rate?*
13. *How many litres of 20% alcohol solution and how many litres of 50% alcohol solution must be mixed to produce 9 litres of 30% alcohol solution?*
14. *It takes a boat 2 hours to travel 24 km and 3 hours to travel 18 km upstream. What is the speed of the boat in still water, and how fast is the current?*
15. *Andre has more money than Bob. If Andre gave Bob \$20, they would have the same amount. While if Bob gave Andre \$22, Andre would then have twice as much as Bob. How much does each one actually have?*

# Changing the Subject

Make  $x$  the subject of the equations

▶  $2x + 5y = 15$

▶  $4y = 2x + 8$

▶  $3x - y = 12$

▶  $xy = 19$

▶  $ax = \frac{b}{10}$

▶  $\frac{3}{x} = \frac{4}{2}$

▶  $\sqrt{\frac{x}{5}} = y$

▶  $2x^2 = 3y$

▶  $12y = \frac{x}{6}$

▶  $x^2 = 5y$

▶  $y^2 = x^2 + 2a$

▶  $T = \frac{1}{4}\pi x^2$

▶  $\frac{3}{8}kx = y$

▶  $\frac{x}{11} = \frac{y}{5}$

▶  $x^2 + y^2 = r^2$

▶  $s = 4.9x^2$

# Simultaneous Equations

Solve all these simultaneous equations!

▶  $x + y = 10$   
 $x - y = 12$

▶  $5x + y = 14$   
 $4x - y = 22$

▶  $2x + 3y = -2$   
 $3x - y = 13$

▶  $-4x + 5y = 20$   
 $4x - 3y = -4$

▶  $5x + 8y = 71$   
 $7x - 3y = 71$

▶  $3x + 5y = 5x - 3y$   
 $3x + 5y = 19 - 2y$

▶  $2x + y = 13$   
 $x = 8 - y$

▶  $x = 4y - 1$   
 $3x + 2y = 11$

▶  $2x = 20 + 2y$   
 $4x + y = 20$

▶  $y = 6x - 5$   
 $y + 2x = -5$

▶  $y = 5x - 2$   
 $y = 4x + 4$

▶  $-2x + 3y = 3$   
 $7x + 2y = 27$

# Sequences

Use  $n = 1, 2, 3$  and  $4$  to find the first 4 terms of each sequence

- $3n - 1$

$n$	1	2	3	4

- $100 - 10n$

$n$	1	2	3	4

- $5n + 4$

$n$	1	2	3	4

- $-2n + 3$

$n$	1	2	3	4

- $10^n$

$n$	1	2	3	4

- $2^n$

$n$	1	2	3	4

- $n(n + 3)$

$n$	1	2	3	4

- $n^2 + n$

$n$	1	2	3	4

- $6n + 4$

$n$	1	2	3	4

- $25 - 3n$

$n$	1	2	3	4

- $2n^2 - n$

$n$	1	2	3	4

- $n + 2^n$

$n$	1	2	3	4

- $n^3 - 1$

$n$	1	2	3	4

- $n^2(n - 2)$

$n$	1	2	3	4

- $\frac{n}{n+2}$

$n$	1	2	3	4

- $\frac{n}{2n+1}$

$n$	1	2	3	4



# Sequences

Find the next 2 terms and the rule for all these sequences!

■ 3, 7, 11, 15, 19, ....., .....

■ 6, 9, 12, 15, 18, ....., .....

■ -3, -1, 1, 3, 5, ....., .....

■ 6, 11, 16, 21, 26, ....., .....

■ 4, 2, 0, -2, -4, ....., .....

■ 95, 90, 85, 80, 75, ....., .....

■ 7, 4, 1, -2, -5, ....., .....

■ 6.5, 7, 7.5, 8, 8.5, ....., .....

■ 1, 4, 9, 16, 25, ....., .....

■ 3, 6, 11, 18, 27, ....., .....

■ 0, 3, 8, 15, 24, ....., .....

■ 3, 10, 21, 36, 55 ....., .....

■ 3, 8, 15, 24, 35, ....., .....

■ 2, 10, 24, 44, 70, ....., .....

■ 6, 12, 20, 30, 42, ....., .....

■ 8, 14, 24, 38, 56, ....., .....

# Sequences

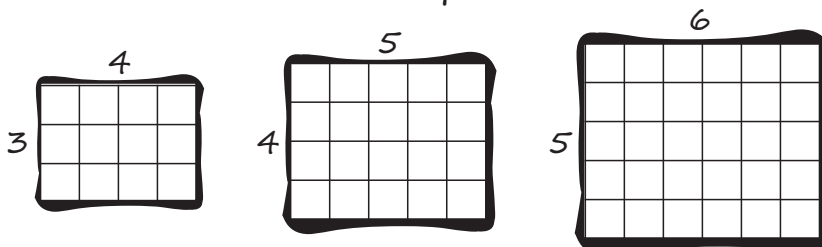
Can you answer these sequence questions?

1. The first five terms in a number sequence are: 7, 10, 13, 16, 19
  - (a) Write an expression, for the  $n$ th term of the sequence.
  - (b) Find the 100th term in this number sequence.
  
2. A number sequence has  $n$ th term  $6n + 5$ 
  - (a) Write down the first four terms of this sequence.
  - (b) Kim says that 1008 is a term in this sequence. Is Kim correct?
  
3. The first 5 terms of a number sequence are: 1, 5, 9, 13, 17.
  - (a) Write an expression for the  $n$ th term of the sequence.
  - (b) Is 97 a term in this sequence?
  
4. The  $n$ th term of a number sequence is  $n^2 + 5$ .
  - (a) Write the first three terms of this sequence.
  - (b) Calculate the difference between the 5th and 10th terms.
  
5. The first 5 terms in a number sequence are: 13, 10, 7, 4, 1.
  - (a) Write an expression for the  $n$ th term of the sequence.
  - (b) Find the 50th term of the sequence.
  
6. Kim calculates first 50 terms of the sequence  $150 - 4n$ .  
Which will be first negative term?
  
7. The  $n$ th term of a sequence is  $(n + 2)(n + 5)$   
Write first 5 terms of the sequence.
  
8. The  $n$ th term of a sequence is  $5n - 9$ 
  - (a) Write down the first 5 terms of the sequence.
  - (b) What is the difference between the 99th and 100th terms?  
The last term of this sequence is 741.
  - (c) How many terms are there in this sequence?

# Sequences

Can you answer these harder sequence questions?

1. A linear sequence starts 300, 296, 292, 288 ...  
Which term will be the first to have a negative value?
  
2. The  $n$ th term of a sequence is  $4n - 10$   
Write an expression for the term  $n + 1$
  
3. A linear sequence starts off 4, 11, 18, 25, ...  
Write an expression for the  $n$ th term.  
How many terms in the sequence will be less than 150?
  
4. Find the  $n$ th term of Sequence A: 4, 7, 10, 13, ....  
Find the  $n$ th term of Sequence B: 16, 49, 100, 169, ....  
Show that the 30th term of sequence B equals  $49 \times 169$ .
  
5. A linear sequence has the terms  $a+b$ ,  $a+3b$ ,  $a+5b$ ,  $a+7b$ ,  $a+9b$ , ....  
Term 5 of the sequence = 35.  
Term 8 of the sequence = 59.  
Find the  $n$ th term of the sequence.
  
6. A pattern of rectangles continues as below. The areas of each rectangle form a sequence: 12, 20, 30, 42, ...  
Find the  $n$ th term of the sequence of areas.



# Algebra Practise

## Basic Skill Questions - Set A

1. Expand  $(x + 2)(4x - 5)$ .
2. Factorise  $2x^2 - 15x + 18$ .
3. Give the coordinates where the graph  $y = x(x + 3)$  cuts the  $x$  axis.
4. If  $x = 2$  and  $h = x(x - 1) + 2$   
Find the value of  $h$ .
5. Expand  $(3x + 7)(x - 2)$
6. Factorise  $3x^2 - 11x + 6$
7. If  $x = 2$  and  $h = 2x(x - 1.5) + 1$   
Find the value of  $h$ .
8. Give the coordinates where the graph  $y = x(x + 9)$  cuts the  $x$  axis.
9. Kim thinks of a number, adds 5 and multiplies the result by 4. He gets an answer of 24. What is the number Kim is thinking of?
10. You think of a number, add 7 and then divide the answer by 4 to get an answer of 5. What is the number you were thinking of?
11. A rectangular garden is  $W$  metres wide. Its length is 2 metres longer than the width. Write a formula for the area of the garden.
12. A certain garden is a rectangle  $L$  metres long. The width is 3 metres shorter than its length. Write a formula for the area of the garden.
13. A rectangle has area:  $x^2 - x - 2$ .  
One side has the length  $(x + 1)$ .  
Find the length of the second side.
14. A rectangle has area  $x^2 + 5x - 36$ .  
Find the length of each side.
15. A parabola has the equation  
 $y = 3x^2 - 2x + 5$   
What is the value of  $y$  when  $x = 4$ ?
16. A parabola has the equation  
 $y = 3x^2 - 5x + 7$   
What is the value of  $y$  when  $x = 2$ ?
17. A rectangle area is  $n^2 - 4n - 5$ .  
If one side has the length  $n + 1$ ,  
find the length of the second side.
18. A rectangle area is  $x^2 + 4x - 12$ .  
Find the length and width of the rectangle.
19. Kim kicks a ball. The flight path of the ball can be modelled by:  
 $y = -(x^2 - 4x)$   
What does  $x$  measure?
20. The distance ( $d$  metres) travelled by an object is given by:  
 $d = ut + 3t^2$   
If  $u = 3$  and  $t = 5$ , calculate the distance that the object has travelled.

# Algebra Practise

## Basic Skill Questions - Set B

1. The area (A) of a pathway and barbecue area is given by:

$$A = xy + 5y^2$$

If  $x = 2$  and  $y = 4$  calculate A.

2. A rectangle has an area:

$$3x^2 - 4x - 32$$

Find expressions that give the length and width of the rectangle.

3. Rewrite the equation  $h = 9 - 4x^2$  to make  $x$  the subject.

4.  $n = 9m^2 - 16$ . Rewrite the equation to make  $m$  the subject.

5. A rectangle has an area:

$$3x^2 + 2x - 40$$

Find expressions that give the length and width of the rectangle.

6. The sides of a rectangle are:

$$2x + 3 \text{ and } x - 2$$

Write an expression for the area of the rectangle in the form:

$$ax^2 + bx + c$$

7. Find the value of  $x$  if  $3^{x+1} = 81$ .

8. Find the value of  $x$  if  $2^{x-1} = 64$

9.  $A = \frac{25}{q^2}$ . Rewrite the equation to make  $r$  the subject.

10.  $A = \frac{9}{4}r^2$ . Rewrite the equation to make  $r$  the subject.

11. The area of a deck on a house is given by:  $A = 3(2x^2y - 2x)$

If  $x = 3$  and  $y = 5$  give the area of the deck.

12. The area of a garden is given by:

$$A = 2(x + 3xy^2)$$

If  $x = 5$  and  $y = 2$ , give the area of the garden.

13. The sides of a rectangle are  $3x - 2$  and  $x + 2$ . Write an expression for the area of the rectangle written in the form:  $ax^2 + bx + c$

14. What is the value of  $2x^4 - 3x + 5$  when  $x = -2$ .

15. When 60 is divided by  $x$  and then 12 is added to the answer the result is 14. What is the value of  $x$ ?

16.  $w = pq^2 + r$

Rewrite to make  $p$  the subject.

17. Simplify  $5m^2n \times m^3n^2$ .

18. Solve:

a.  $4a^3 = 32$

b.  $(b^3)^2 = 64$

c.  $5^c = 125$

d.  $(2d^2)^2 = 324$

19. Simplify  $12a^2b + 6ab^2 - 7a^2b$

20. Solve  $3(2x + 7) = 9$

# Algebra Practise

## Proficiency Skill Questions - Set A

1. James hires a bike. It costs \$8 for two hours and then \$3 for each additional hour. His afternoon ride costs him \$23. How long did James hire the bike?
2. Jane has an operation and needs to build her fitness. Her goal is to walk for 160 or more minutes. The first week she walks for 10 minutes each day. Each Monday she doubles the time she walks. The time spent walking each day is modelled by the equation:
 
$$T = 10 \times 2^{n-1}$$
 How many weeks will it take for her to reach her goal?
3. Five people on a camp have a stomach bug which spreads at a constant rate ( $r$ ). At the end of 3 days, 320 people have the stomach bug. This can be modelled by:  $320 = 5r^3$ . Find the rate ( $r$ ) at which the bug is spreading.
4. A rectangle has an area  $x^2 - x - 2$ . The rectangle has a length of  $x + 1$  metres.  
What is the width of the rectangle?
5. Show that  $\frac{2}{x} + \frac{3+x}{5} = \frac{x^2 + 3x + 10}{5x}$
6. Simplify  $\frac{3ab^2 - 4a^3b + ab^2}{4ab^2}$
7.  $y = x^2 + 3x - 10$ . For what values of  $x$  will  $y$  be negative?
8.  $y = x^2 + 4x - 12$ . For what values of  $x$  will  $y$  be positive?
9. Jethro is training for a marathon. The first week he runs 7km. Each week he doubles the distance that he runs. The distance run each week can be modelled by the equation:  $D = 7 \times 2^{n-1}$   
 $D$  = distance run each week  
 $n$  = number of weeks  
 How long will it take for Jethro to run 112km per week?
10. Jess is paid \$38 to babysit her cousin for 2 hours. She is then paid \$13 per hour after. Altogether she was paid \$77. How many hours was she paid for?
11. Simplify  $\frac{5xy^2 - 2x^3y + xy^2}{4xy^2}$
12. Weed is growing on a lake at the rate of  $r$  m<sup>2</sup>. When first measured 6m<sup>2</sup> of the lake was covered in weed. Four weeks later the area covered was 486 m<sup>2</sup>. This can be modelled by  $486 = 6r^4$ .  
Use algebra to find the rate ( $r$ ) at which the weed is spreading.



## Proficiency Skill Questions - Set B

1. Solve  $x^2 + 2x - 8 = 0$

2. A rectangle has an area:

$$x^2 + 4x - 12$$

If the area =  $128 \text{ cm}^2$ , find the side lengths of the rectangle.

3. Solve  $y^2 - 3y - 10 = 0$

4. Solve  $3x^2 + 8x - 16 = 0$

5. Solve  $2a^2 - 3a - 9 = 0$

6. Solve  $10x^2 - 27x - 9 = 0$

7. Solve  $15y^2 - 4y - 4 = 0$

8. The area of a rectangle can be represented by  $3x^2 + 2x - 40$ .

What are the possible values of  $x$ ?

9. Simplify  $\frac{6x^2 - 18x}{2x^2 - 7x + 3}$

10. Simplify  $\frac{x^2 - 5x + 4}{5x^2 - 20x}$

11. Write  $\frac{2}{x+1} + \frac{3}{x-2}$  as a single fraction.

12. Write  $\frac{8x-1}{4} + \frac{3x-5}{3}$  as a single fraction.

13. Solve the equation  $\frac{x+12}{x+4} = \frac{x+4}{x+2}$

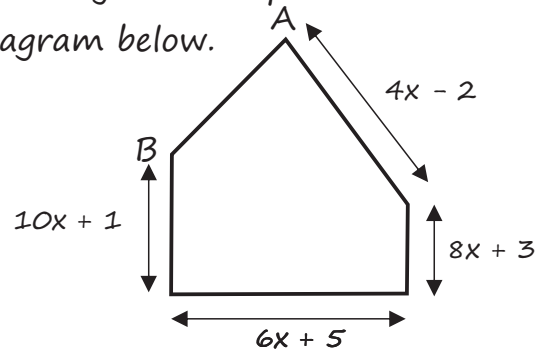
14. Solve the inequality:

$$6(5 - 2x) - 4(5 - 3x) > 5(x + 4)$$

15. Solve the inequality:

$$6(2 - 4y) + 4(6y - 2) < 4(y + 4)$$

16. A courtyard is shaped as in the diagram below.

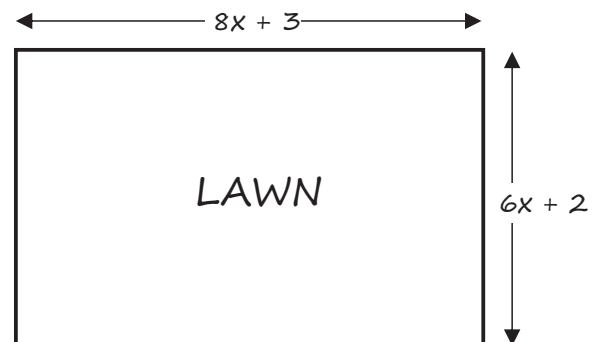


The courtyard perimeter is:

$$32x - 7.$$

What is the length of  $AB$ ?

17. The plan of a rectangular garden is shown in the diagram below.



The lawn perimeter = 290 metres.

Find the value of  $x$ .

18.  $a \bullet b$  is defined by:  $10ab \div (a + b)^2$

Use this to find the value of  $-4 \bullet 3$

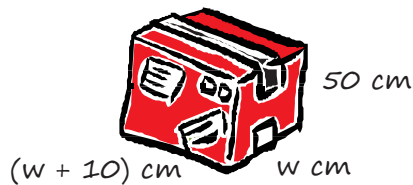
19. If  $(2a)^3 \times 2a^k = 16a^8$ , what is the value of  $k$ ?

20.  $A = \frac{1}{2}(a+b)h$ . Use this to find  $h$  when  $A = 36$ ,  $a = 15$  and  $b = 9$ .

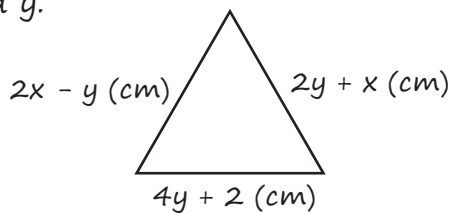
## Proficiency Skill Questions - Set C

1. One solution of  $4x^2 + 8x + 3 = 0$  is  $x = -1.5$ . Use this to find the second solution of the equation

2. The volume of the box shown below is 60 litres. Find the box's dimensions.



3. The triangle below is equilateral with a perimeter of 30 cm. Find the values of  $x$  and  $y$ .



4. Simplify  $\frac{x^2 - 4y^2}{x^2 - 2xy}$

5. Express  $\frac{x}{2} + \frac{3x}{5}$  as a single fraction.

6. Solve the equation  $x^2 + 2x = 255$   
Hint: 2 factors of 255 are 15 & 17.

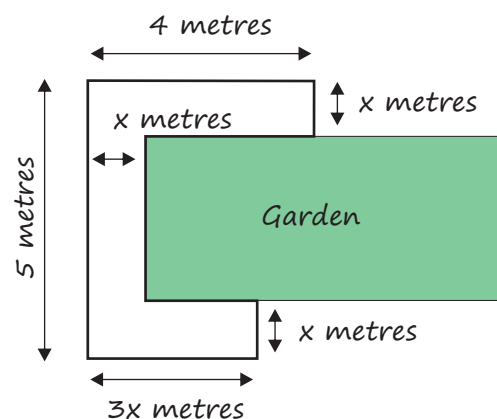
7. Simplify  $\frac{2m}{3} + \frac{m}{4}$

8. Zahara is five years old and Maddox is four years older. Form an equation to find how many years it will take until Zahara's and Maddox's ages multiplied together make 725 years.

9. There are  $V$  litres in Claudia's water tank. There are  $d$  "drippers" on the irrigation hose from the tank to the garden. Each dripper uses  $x$  litres of water per day. (a) Write an expression to show the total amount of water,  $T$ , left in the tank after one day.

At the end of the day on the 1st of April there were 150 litres of water in the tank. The next day, 4 drippers were used to irrigate the garden and at the end of the day there were 60 litres of water left. (b) Use your expression show how much water each dripper used on that day.

10. Graeme is designing a path around the front of his garden. The design is shown below. The width of the path is  $x$  m. Graeme has sufficient pavers to make a path with a total area of  $22\text{m}^2$ . The area of the path can be written as  $4x + 3x^2 + (5 - 2x)x = 22$ . Solve to find the width of the path around the front of the garden.





# Algebra Practise

## Advanced Skill Questions - Set A

1. Winston is trying to find a value for  $c$  so that  $x^2 + 6x + c = 0$  has only one solution for  $x$ .

Find the value for  $c$  and the solution to the equation.

2. Andy and Billy live 8km from each other. Andy skateboards 10km in the same time that Billy rides his bike 15km. If they both leave home at the same time and travel towards each other, how far from Billy's home will they meet?

3. Solve the equation below and explain why there is only 1 valid solution.

$$\frac{x^2 - 1}{x^2 + 2x + 1} = \frac{3}{4}$$

4. A group of families go to a fun park. There are 38 in the group. Students cost \$10.

Adults cost \$12

Total cost of the group is \$420.

How many students were in the group?

5. For what value(s) of  $x$  will

$$4 \times 2^x = 2^{6x+3}?$$

6. A rectangle has an area formula:

$$x^2 + 5x - 36$$

If the area of the rectangle is  $114\text{cm}^2$ , what is the value of  $x$ ?

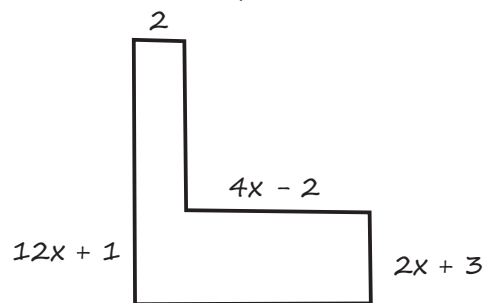
7. Show that  $\frac{2x^2 + 8x + 12}{8x}$  is the same as  $\frac{3}{2x} + \frac{x+4}{4}$

8. If  $6x - y = 21$  and  $-x + 6y = 14$ , what is the value of  $x - y$ ?

9. Woodsy thinks of a number  $x$ .

When the number is cubed, the answer is  $m$  times  $x$ . When the number is squared the answer is  $n$  more than  $x$ . Give an expression for  $n$  in terms of  $m$  only.

10. Look at the shape below:



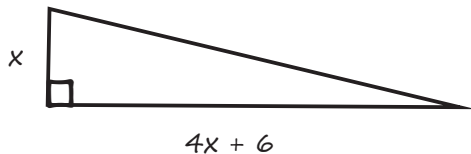
The area of the shape is  $92\text{ cm}^2$ . What is the value of  $x$ ?

11. Jazz is laying square concrete tiles for a deck. He lays them in a square pattern. His friend, Aussie, thinks it would be better to have a rectangular deck so he changes the layout to make the length 6 tiles longer and the width 4 tiles shorter. The rectangular deck needs 2 extra tiles. How many tiles were needed for each of the configurations?

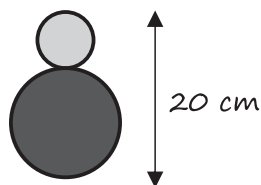
# Algebra Practise

## Advanced Skill Questions - Set B

1. The area of the triangle below is  $35 \text{ cm}^2$ . Use algebra to find the value of  $x$ .



2. Solve  $8 \times 2^{x-4} < 20$
3. Arabella and Isabella are organising 27 people in two groups to go to a hockey game. The tickets cost \$20 for students and \$30 for adults. Half of Arabella's group are students. Two-thirds of Isabella's group are students. The total cost of the tickets (27) people is \$650. How many students are in Isabella's group?
4. Coco is designing a logo made up of two circles. She draws one circle on top of another.

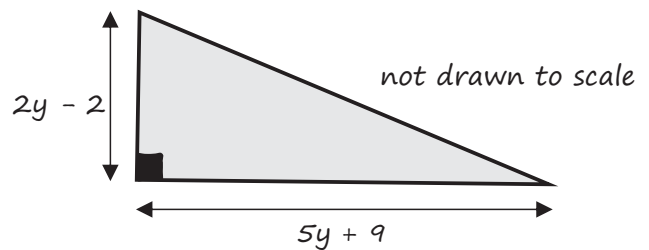
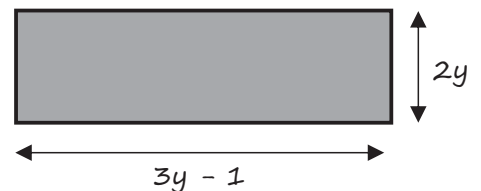


- (a) The total height of the two circles must always be 20 cm. Show that the total circumference of the two circles will always be  $20\pi$  (cm).
- (b) Give an equation for the difference in the area of the two circles.  
Remember  $C = 2\pi r$  and  $A = \pi r^2$

5. For what values of  $x$  will

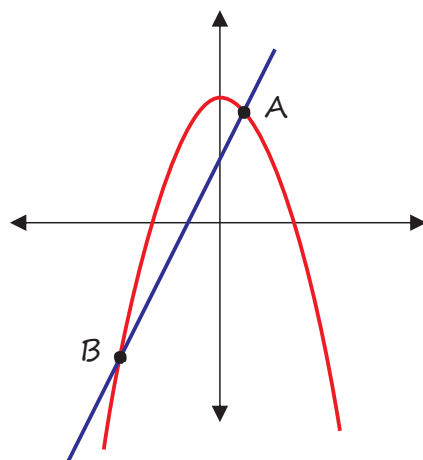
$$5 \times 5^{3x} = 5^{-2x^2}?$$

6. Find the value of  $y$  so that the area of the right angle triangle (below) has the same value as the area of the rectangle.



ALL measurements in cm

7. The diagram below shows the graph  $y = 7 + x - 6x^2$  and  $y = 8x + 4$ .



Find the two points A and B. where the two graphs intersect.

# Algebra Practise

## Advanced Skill Questions - Set C

1. Find the values of  $y$  if:

$$25 \times 5^{2y+13} = 5^{y^2}$$

2. 21 adults go to a movie. The cost is \$14 for people under age 65, and \$10 for people aged 65 or over. The total cost for the group is \$258. How many in the group are over age 65?

3. Winston is trying to find a value for  $c$  so that  $x^2 + cx + 16 = 0$  has only one solution.

Find the value for  $c$  and the solution to the equation.

4. Andy and Billy live 15km from each other. Andy skateboards 12km in the same time that Billy rides his bike 18km. If they both leave home at the same time and travel towards each other, how far from Andy's home will they meet?

5. Ditta has more money than Greta. If Ditta gives Greta \$20 they will both have the same amount. However if Greta gives Ditta \$22 then Ditta will have twice as much as Greta. How much money does each girl actually have?

6. The equation of a straight line passing through points  $(-5, -10)$  and  $(9, 11)$  is given by  $Qy = Px + 5$ . Use algebraic methods to find the values of  $P$  and  $Q$ .

7. For what values of  $x$  is:  
 $(x - 3)(x + 3) < (x - 4)(x + 2)$ ?

8. If  $p$  is a whole number, for what values of  $p$  is  $10 \times 2^{p-1} < 165$ ?

9.  $M = 5(a^2 - 3a + 4) + a^2$

$$N = (3a - 5)(2a - 4) + 7(a + 5)$$

Write an expression for  $M$  in terms of  $N$

10. Solve  $27 \times 3^{x-4} > 200$  when  $x$  is a whole number.

11. Selwyn school uses 2 vans to take a group of students on a field trip.

- If 2 students move from van A to van B, then the vans will have the same number of students in each.
- If, instead, 2 students move from van B to van A, then van B would have half the number of students that were then in van A.

Use this information to find the total number of students on the field trip.

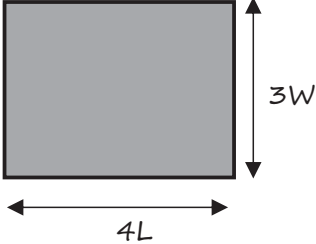
# Algebra Practise

## Advanced Skill Questions - Set D

- Solve  $x^4 - 18x^2 + 81 = 0$
- If  $R = \frac{5y - 4x}{y - 2x}$  give the equation for  $x$  in terms of  $y$  and  $R$ .

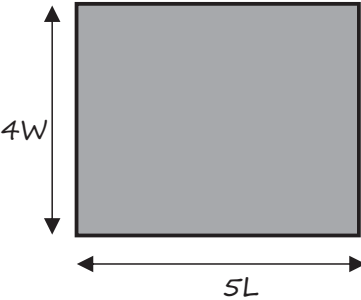
3. Rectangle A

Area =  $4L \times 3W$   
Perimeter = 20



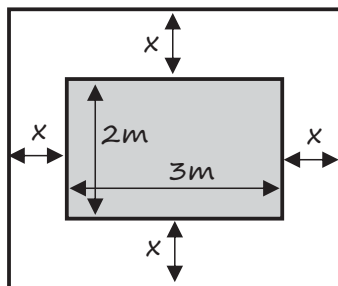
Rectangle B

Area =  $5L \times 4W$   
Perimeter = 26



Find the length and width of Rectangles A and B

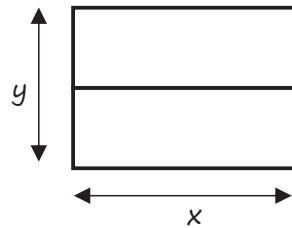
- A rectangular room has a rug measuring  $3\text{m} \times 2\text{m}$  in the middle. The border around the rug is  $x$  metres. The area of the room is double the area of the rug.



What is the value of  $x$ ?

- Find the values of  $A$  and  $B$  that make  $(3x + A)^2 = 9x^2 + Bx + 16$  true.

- Solve the equation:  $8^x \times 4^{x^2 - 6} = 4$
- The total length of all the lines in the diagram below is 20 metres. The total area is  $14\text{m}^2$ .



Find all possible values of  $x$ .

- Solve the inequality  $(3x - 5)^2 \leq 3x^2 + 1$
- The equation of a straight line that passes through the points  $(-3, -6)$  and  $(3, 10)$  has the equation:

$$Ay = Bx + 6$$

Find the values of  $A$  and  $B$

- A rectangle has an area modelled by the expression:  $x^2 + 4x - 12$ . If the rectangle's area is  $128\text{m}^2$  what is the value of  $x$ ?

- Find the values for  $x$  if:

$$36 \times 6^{2x+6} = 6^{x^2}$$

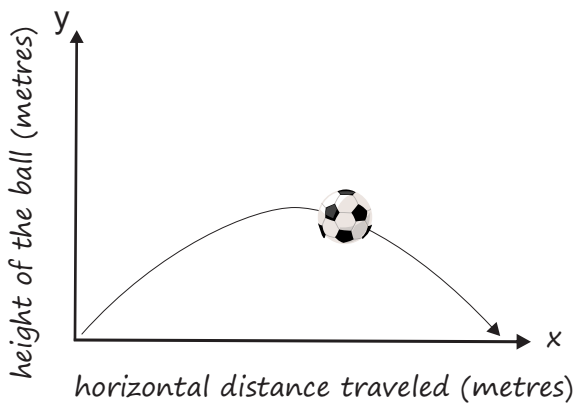
- The number of small squares in the  $n$ th shape of a pattern is given by  $n^2 - 2$ . Show that the difference in the number of small squares used between two consecutive odd-numbered shapes is divisible by 4.

## Advanced Skill Questions - Set E

1. Kim kicks a ball. The flight can be modelled by the equation:

$$y = 4x - x^2$$

$x$  and  $y$  are measured in metres.



For what percentage of the horizontal distance (that the ball travels) will it be 3m or more above the ground?

2. Kim is exploring the sequence of numbers given by the rule:

$$2n^2 - n + 5$$

Give the rule for finding the difference between any two consecutive terms (e.g. the 4th and 5th terms) from the sequence:

$$2n^2 - n + 5$$

3. Find the value of  $c$  so that there is only 1 solution for  $x$  in the equation:  $x^2 + 6x + c = 0$ .

Write the final equation.

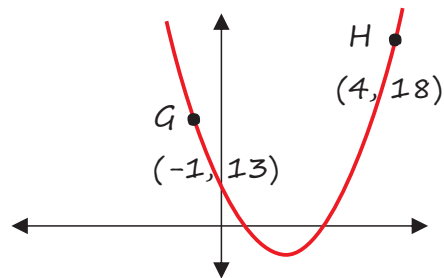
4. Find the value of  $a$  ( $a$  positive number) for  $x^2 - ax + 6 = 30$  if the difference between the solutions of the equation is 10.

5.  $T = n^2 - n + 5$

$$R = (5n-4)(n+1) - 2n(2n+3) + 4(n+1) - 3$$

Write an equation for  $R$  in terms of  $T$

6.

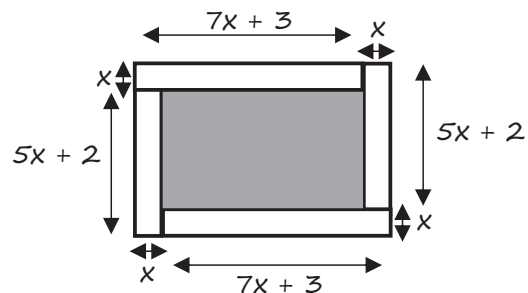


The diagram above shows a parabola formed by the equation:

$$y = Ax^2 + Bx + 2$$

Find the values of  $A$  and  $B$ .

7. A picture is framed with 4 pieces of wood. Find the area of the picture.



8.  $y + 3 = \sqrt{\frac{A(x^2 - 7)}{B}}$

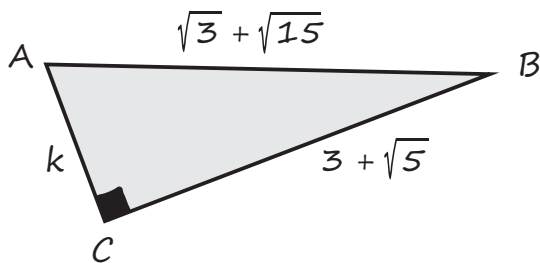
Rearrange the equation for  $x$  in terms of  $A$  and  $B$ .

## Advanced Skill Questions - Set F

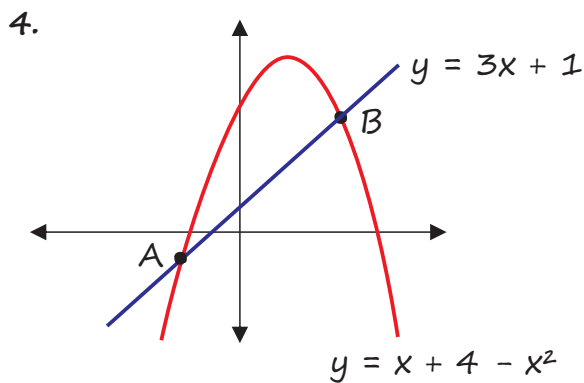
1. What is the value of  $\frac{8^x}{2^y}$  when  $3x - y = 8$

2a. Expand  $(\sqrt{3} + \sqrt{15})^2$

2b. If triangle ABC (below) is right angled, find the value of k.

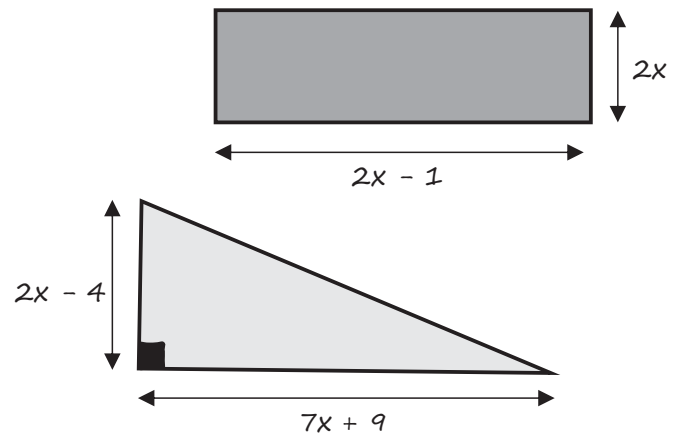


3. Calculate the value of  $\frac{x+y+z}{3}$  when  $x = \frac{m+9}{2}$ ,  $y = \frac{2m+15}{2}$  and  $z = \frac{3m+18}{2}$



The graph above shows the straight line  $y = 3x + 1$  and the parabola  $y = x + 4 - x^2$ . Calculate the coordinates where the lines intersect at the points A and B.

5. Find the value of x so that the area of the both the triangle and rectangle are the same.



6. Find x if  $3^{2x} - 4(3^x) - 45 = 0$  and x is a positive integer

7. Find an equation for y in terms of x.  $9 \times 3^{x+y} = 27^{2x}$

8. Show that if you add the squares of three consecutive even numbers and then subtract 2, the result is a multiple of 2, 3 and 6.

9. Show that if:  
 $y = (2a - 1)^2 - (2b - 1)^2$  then  
 $y = 4(a - b)(a + b - 1)$

10. Solve  $2n^2 + 6n > n^3 + 3n$

11. The formula for the surface area (cm<sup>2</sup>) of a closed circular cylinder, is  $A = 2\pi r(r + h)$ . Find the radius of the cylinder with surface area  $33\pi$  (cm<sup>2</sup>) and a height of 9.5 cm.



## 1 - Substitution

- If the value of  $a = 3$ ,  $b = -2$ ,  $c = 13$  and  $z = \frac{1}{2}$ , what is the value of:
  - $abc - 3b$
  - $b^2 + 4c - 3z$
  - $(a^4 + b + \frac{1}{2}c) \div z$
- To find the height ( $h$ ) of a tree use the formula  $h = 2g^2 + 0.15d - 5l$ .  
 $g$  = distance around base of trunk in metres,  $d$  = depth of root system in metres,  
 $l$  = height above ground of first set of leaves in metres.  
 What is the height of a tree where  $g = 6$ ,  $d = 8$  and  $l = 10$ ?
- A plumber charges a basic fee of \$80 travel plus \$75 per hour. The formula for finding the cost is  $C = 80 + 75h$  (where  $h$  = number of hours).  
 What is the cost of a plumbing job that takes  $5\frac{1}{2}$  hours?
- The formula for the area of a rectangle is  $A = LW$  ( $L$  = length,  $W$  = width).  
 What is the area of a rectangle that has length 5 cm and width 14.25 cm?
- The formula for the perimeter of a rectangle is  $P = 2(W + L)$ . Using the same length and width as in question 4, what is the perimeter of the rectangle?
- The formula for the volume of a cylinder is  $V = \pi r^2 h$  ( $r$  = radius and  $h$  = height).  
 What is the volume of a cylinder with ends radius 6 cm and a height of 14 cm?
- The formula for the surface area of a cylinder is  $2\pi r^2 + 2\pi rh$ . What is the surface area of a cylinder with the same height and radius as in question 6?
- The energy of a moving object is given by the formula  $E = \frac{1}{2}mv^2$  ( $m$  = mass in kg, and  $v$  = velocity in metres per second). What is the energy of a jumbo jet when it lands at a speed of 45 metres per second and has a mass of 210 000 kg?
- The amount of money in a bank account at the end of the year increases by:  
 $M = 0.0625s + s$  where  $s$  = amount of money at the start of the year.  
 If you have \$2000 at the start of the year, how much will you have at the end?
- Find the value of  $z$  if  $z^2 = 11 + x^2$  and  $x = -5$ .

## 2 - Expanding Single Brackets and Simplifying Expressions

Expand each expression

1.  $3(5x - 8) =$

2.  $xy(10 + z) =$

3.  $-5(y - 2) =$

4.  $-3a(12 - 2b) =$

5.  $-xy(3 - y) =$

6.  $5b(abc + 8) =$

7.  $-4(-3 + c) =$

8.  $-6r(2 - 3r) =$

9.  $3xy(x + 7) =$

10.  $xy(y + 18) =$

11.  $8xy(2xy - xy) =$

12.  $4x^2(y^3 + 2x^3) =$

13.  $3(x + 1) + 4(4x + 2)$   
=

14.  $4(3 - x) - 3(2x + 1)$   
=

15.  $3(5x - 0.5) - 4x(-y + 0.25)$   
=

Simplify each expression

1.  $5x + x =$

2.  $3a + 4b + 4a =$

3.  $7a + 5b - 16a + 12b =$

4.  $4x^2 + x^2 + 3x - 11x =$

5.  $6ab - 3a - 3b + 4b =$

6.  $7pq + 10pq^2 - pq + pq^2 =$

7.  $3x - 4x^2 + 9x - 10x^2 =$

8.  $b^3 - 10ab^3 + 8b^3$

9.  $-5z^3 - 2z - 10z + 3z - 4z^3$

10.  $x^2 + 9x - 52$

11.  $-9t^2 + t - 21t^2$

12.  $2x^2 - 11x^2 - 21$

13.  $x - x - x + 0.5x - 3x$

14.  $3x(2 + y) + 7x(2 + y)$

15.  $-5x^2 - (-2x) - (-5x^2) + 8x$



## 3 - Simplifying Expressions and Expanding Single Brackets

Simplify each expression:

1.  $40y \div 10y$

2.  $30x^2 \div 5x^2$

3.  $3d \times 7d^2$

4.  $6e^3f^2 \times 4ef$

5.  $3x^3 \div 12x^2$

6.  $3pq^2 \times 5p^4q^{-1}$

7.  $30d^4 \times 4cd^2$

8.  $200x^2yz^3 \div 100x^3y^3z^2$

9.  $-12a^4c \times -3a^4c^2$

10.  $2(0.5x + 2.7x) - 3x$

11.  $70x \div 140x^2$

12.  $50x^3y^5z^2 \times 3x^2yz^4$

13.  $21a^3b^{-4}c^{-2} \div 28a^{-4}b^5c^{-1}$

14.  $30y^2 \times 5y^{-3}$

15.  $15z^3 \times 15z^{-4.5}$

Expand each expression:

1.  $x(x + y)$

2.  $4a(a - a^2)$

3.  $2bc(3bc + 2b)$

4.  $5x(x^2 - xy)$

5.  $bc(a + b^2)$

6.  $x^2y(y + xy)$

7.  $3yz(-3y - z)$

8.  $8x(x^3 + x^2y)$

9.  $a^3(7a - a^2b)$

10.  $4x(2x^4 - 3x^2)$

11.  $xz(x^2z - xyz)$

12.  $x(a + bc)$

13.  $x^2(2x + y) - x(x^2 - xy)$

14.  $x^3(2x - 4) + x^2(x^2 - 2)$

## 4 - Simple Factorising and Expanding Brackets

Factorise each expression:

1.  $5x + 5$

2.  $20x + 12$

3.  $6xy - 12x$

4.  $5x + 10 + 20xy$

5.  $9x^2 + 5x$

6.  $3x^2y + y^2x$

7.  $32xy + 40x$

8.  $15y + 45xyz$

9.  $12x - 16$

10.  $60y + 12y^2$

11.  $8x^2 + 16x^5$

12.  $abc^4d - a^2bc^3$

13.  $3xy^2 - 4xy + xy$

14.  $9a^3b^2 + 3ab$

15.  $x^2 + (xy)^2$

Expand each expression:

1.  $(x + 8)(x + 4)$

2.  $(x + 20)(x + 6)$

3.  $(p^2 - 1)(4 - p)$

4.  $(a + 9)(a - 7)$

5.  $(3y + 3)(y + 2)$

6.  $(2x - 5)^2$

7.  $(9x + 2)(9x - 2)$

8.  $(2x - 5)(x - 4)$

9.  $(3x - 7)(5x + 6)$

10.  $(2x + 3)^2$

11.  $(4x + 3)(4x - 3)$

12.  $(2 - x)(2 + x)$

13.  $(ax + b)(ax - 2b)$

14.  $(\frac{1}{2}x + \frac{3}{4})^2$

15.  $(x + 2)(4x - 9)$

## 5 - Factorise Quadratics and Solving Equations

Factorise these expressions

1.  $x^2 + 5x + 6$

2.  $x^2 - 10x + 25$

3.  $3x^2 - 30x + 27$

4.  $x^2 + 2x - 8$

5.  $2x^2 - 4x - 30$

6.  $x^2 + 12x + 27$

7.  $2x^2 + 32x + 128$

8.  $x^2 + 8x + 16$

9.  $3x^2 - 24x + 48$

10.  $x^2 + 22x - 48$

11.  $5x^2 - 15x + 10$

12.  $x^2 - 0.25$

13.  $50 - 2x^2$

14.  $3x^2 + x - 4$

15.  $10x^2 - 6 + 11x$

Solve each equation:

1.  $2x + 6 = x + 3$

2.  $3x + 6 = 48$

3.  $4x - 8 = 5x - 2$

4.  $6x + 7 = 2x + 20$

5.  $x + 6 = 2x - 8$

6.  $3(x + 2) = 5(x - 2)$

7.  $4 = 8 - \frac{x}{3}$

8.  $3 - \frac{x}{3} = -5$

9.  $5x - 6 = -39$

10.  $2 - \frac{x}{2} = -1 - \frac{x}{4}$

## 6 - Practise the Basics of "Expand" and "Factorise"

Expand the following:

1.  $u(u + 1)$
2.  $v(v - 6)$
3.  $-w(3w - 2)$
4.  $x(4x + 5)$
5.  $3y(2y - 3)$
6.  $-z(-5z + 3)$
7.  $3 + 2(x - 8)$
8.  $5(x + 7) - 12$
9.  $3(x - 6) + 2(4x - 5)$
10.  $4(a + 6) - 2(a - 2)$
11.  $2x(x + 1) - x(7 - x)$
12.  $x^2(x + 1)$
13.  $\frac{1}{2}(4x + 12)$
14.  $\frac{3}{4}(12x - 6)$
15.  $3x(2x^2 - 4)$
16.  $x(x^2 + 4) + x(3x + 2)$

Factorise the following:

17.  $6x + 24$
18.  $5x - 25$
19.  $11x^2 - 66x$
20.  $10x + 25xy$
21.  $100x + 20y$
22.  $27 - 33x$
23.  $5x^2 + x$
24.  $6a^2 + 3a$
25.  $15b^2 - 30b$
26.  $14y^2 + 21y$
27.  $5 + 5n^2$
28.  $6x^2 + 18xy$
29.  $2xy - 4ab$
30.  $3p^2 - 9pq$

Expand and simplify:

31.  $(x + 1)(x + 6)$
32.  $(x + 2)(x + 8)$
33.  $(x - 5)(x + 7)$
34.  $(x - 2)(x + 9)$
35.  $(x + 4)(x - 5)$
36.  $(x + 7)(x - 3)$
37.  $(x - 10)(x - 15)$
38.  $(x - 8)(x - 11)$
39.  $(x + 6)^2$
40.  $(x - 9)^2$
41.  $(x + 1)^2 + 10$
42.  $(x - 5)^2 - 20$

Factorise each expression:

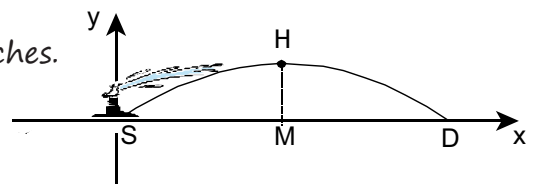
43.  $x^2 + 10x + 21$
44.  $x^2 + x - 12$
45.  $x^2 - 2x - 15$
46.  $x^2 - 14x + 40$
47.  $x^2 + 11x + 30$
48.  $x^2 + x - 2$
49.  $x^2 - 3x - 10$
50.  $x^2 - 4x - 96$
51.  $x^2 - 5x - 14$
52.  $x^2 - 16$
53.  $x^2 - 81$
54.  $(x - 3)^2 - 16$
55.  $x^2 + 2x = 15$
56.  $x^2 = 6x - 8$
57.  $2x^2 - 2x = 220$
58.  $4x^2 - 100$

# The Algebra Project

## 7 - Solving Equations

1. Ali is twice as old as Sue, Sue is 2 years younger than Philip, Philip is 11 years old. How old is Sue? How old is Ali?
2. You ask a friend to think of a number, double it and add 10. His answer is 42. If  $x$  is the number your friend thought of, what is the value of  $x$ ?
3. Three people drive a car on a long journey. John drives for 2 hours more than Mary. Philip drives for twice as long as Mary. The whole journey takes 6 hours. How long does each person drive for?
4. Four consecutive numbers, when added together, give a total of 114. If  $x$  is the lowest number, write down an equation and solve it.
5. The perimeter of a certain rectangle is 54 cm. Calculate the dimensions of the rectangle if the area is  $x^2 + 13x - 90$
6. Use algebra, to show that the product of two odd numbers is always odd.
7. Write an expression that represents 5 consecutive numbers. Use it to show that if you multiply the middle number by 5 you get the same result as if you summed all 5 numbers.
8. Take three consecutive whole numbers. Square each number and sum the three squares. Subtract two from the sum and divide the result by three. Write down an expression of represent any three consecutive numbers. Use this expression to show that if you follow the steps outlined above with any set of three consecutive numbers you will always get as a result the square of the second of the numbers that you first started with.
9. To find Sung's birth month multiply it by 4. Add to this product the difference between 12 and his birth month. Subtract from this result twice the sum of 5 and his birth month. If you successfully follow this equation you should end up with 10. What must Sung's birth month be?
10. The diagram below shows the path of a jet of water from a park's water sprinkler. The furthest distance that the water travels is 50 metres and can be described by the equation:  $y = 0.5x - 0.01x^2$ , where  $x$  is the horizontal distance traveled and  $y$  is the vertical maximum height that the water reaches. The point mid-way between S and D is the highest point of the water (H).

Find the greatest height (MH) that the water reaches.



## 8 - Solving Quadratic Equations

Solve these equations:

1.  $(x + 3)(x - 2) = 0$

2.  $(x + 0.5)(4x + 6) = 0$

3.  $(2x + 8)(x - 4) = 0$

4.  $(4x - 10)(3x + 12) = 0$

5.  $(4x - 20)(2x + 2.5) = 0$

6.  $2x(x - 4) = 10$

7.  $(4x + 2)(2x - 6) = 0$

8.  $3x(2x + 4) = 0$

9.  $x^2 + 5x = -6$

10.  $(x - 4)(x + 5) = 0$

11.  $5x^2 - 3x = 0$

12.  $6x^2 - 2 = 0$

13.  $100 - 4x^2 = 0$

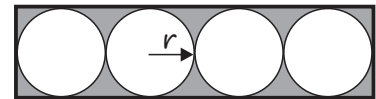
14.  $2x^2 - 6x + 4 = 0$

15.  $(x + 3)(x - 2) = 14$

16.  $2x^2 + 5x = -3$

17. A square garden has a 2m wide path around it. The area of the path is  $40\text{m}^2$ . What are the dimensions of the path and the garden?

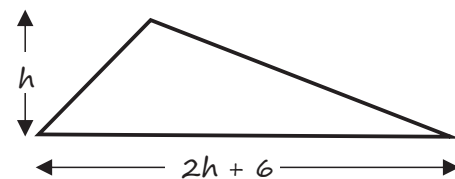
18. Four identical circles are within a rectangle.



The shaded area is  $14\text{cm}^2$ .

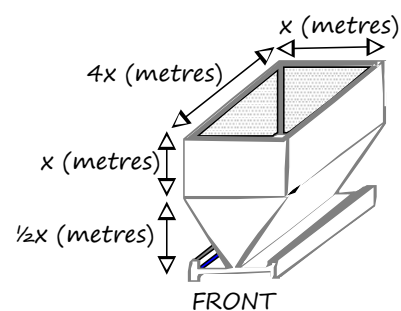
Calculate the radius of the circles.

19. The area of the triangle is  $40\text{cm}^2$ .



Find the dimensions of the triangle (base and height).

20. The diagram shows a rice hopper.



The volume of the hopper can be calculated by finding the cross sectional area of the front and multiplying it by the length.

If the hopper can hold  $40\text{ m}^3$  of rice, calculate the size of  $x$ .

## 9 - Algebraic Fractions

Simplify

$$1. \quad x - \frac{x-2}{2}$$

$$2. \quad \frac{x}{2} - \frac{2x-1}{2}$$

$$3. \quad \frac{x^2 - 3x - 28}{x+4}$$

$$4. \quad \frac{2p^2 - 12pq}{6p^2}$$

$$5. \quad \frac{x^2 - 4x - 5}{x^2 + 6x + 5}$$

$$6. \quad \frac{2}{3a} \times \frac{a^2}{12a}$$

$$7. \quad \frac{5a}{2b} \div \frac{a}{6b}$$

$$8. \quad \frac{3ab^2 - 4a^3b + ab^2}{4ab^2}$$

$$9. \quad \frac{3x}{7} + \frac{2x}{5}$$

$$10. \quad \frac{9x^3}{12x^2}$$

$$11. \quad \frac{x^2 - 5x + 6}{x-3}$$

$$12. \quad \frac{8x^2}{20x - 12x^2}$$

Solve

$$1. \quad \frac{x+2}{3} - \frac{2x-1}{5} = 1$$

$$2. \quad \frac{3}{y-4} - \frac{3}{y+4} = \frac{2}{y+1}$$

$$3. \quad \frac{x}{5} - \frac{2x-1}{2} \geq \frac{-3x}{5}$$

$$4. \quad \frac{2-5x}{4} > 3$$

$$5. \quad \frac{4x-6}{3} > 2x+1$$

$$6. \quad 6x-3 \geq 8x+9$$

## 10 - Quadratic Sequences

Give an expression for the  $n$ th term.

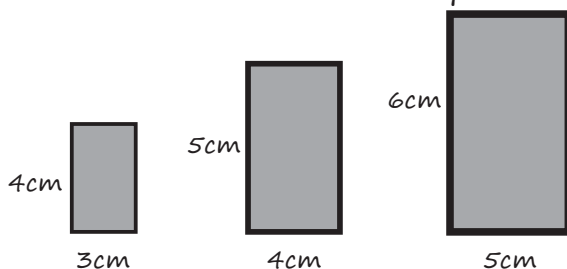
1. 6, 9, 14, 21, 30
2. 11, 14, 19, 26, 35
3. -1, 2, 7, 14, 23
4. -5, -2, 3, 10, 19
5. 4, 10, 18, 28, 40
6. 9, 20, 33, 48, 65
7. 5, 10, 17, 26, 37
8. 3, 3, 5, 9, 15, 23
9. 2, 10, 20, 32, 46, 62
10. -10, -11, -10, -7, -2, 5

11. Show that the  $n$ th term of the sequence 3, 8, 15, 24, 35, 48 is  $n^2 + 2n$

12. The first 4 terms of a sequence are:  
 $\frac{1}{2}$     $\frac{4}{3}$     $\frac{9}{4}$     $\frac{16}{5}$

Find the  $n$ th term for this sequence.

13.

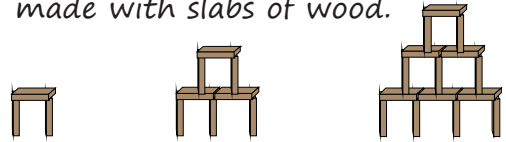


The pattern for these blocks continues. Show that the sequence of areas formed by each block has the  $n$ th term of  $n^2 + 5n + 6$

14. Write the  $n$ th term for:

- a. 5, 12, 21, 32
- b. 3, 9, 17, 27

15. The following structures were made with slabs of wood.



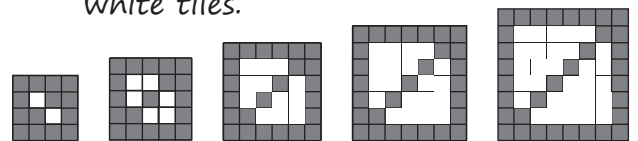
Complete the table below to give the number of slabs needed for each structure.

Storeys (x)	1	2	3	4	5	6	7	8
Slabs needed (y)	3	8	15					

Write the relationship rule for the number of storeys and slabs of wood needed.

If you wanted to build a structure with 25 storeys, how many slabs of wood would be needed?

16. Look at the tile pattern below then complete the table to give the formula for the number of white tiles.



Number of tiles (bottom) =  $n$

Total Number of tiles =  $n^2$

Number of Grey Tiles =  $5n - 6$

Number of White Tiles = .....

17. Sequence  $Q = 3, 8, 15, 24, 35$

The  $n$ th term of the sequence is  $n^2 + kn$  where  $k$  represents a number. Find the value of  $k$ .