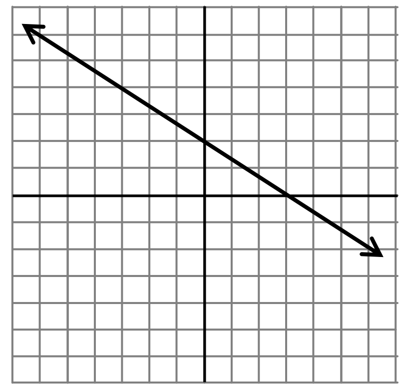
Building Polynomial Functions

NAME ____________________________



1. What is the equation of the linear function shown to the right?

2. How did you find it?

3. The slope – y-intercept form of a linear function is $y = mx + b$.
If you've written the equation in another form, rewrite your equation in slope – y-intercept form.

4. Now, factor out the slope, and rewrite the function as $y = m\left(x + \frac{b}{m}\right)$.

5. Choose a second linear function and write it in slope – y-intercept form.

6. Graph the function on the axis above, and be sure to label it.

7. Rewrite your second function with the slope factored out (just like you did in Question 4).

8. For each function, what does $\frac{b}{m}$ represent on the graph?

If you let $c = -\frac{b}{m}$, then the form $y = m(x - c)$ could be called the slope – x-intercept form of a linear equation, where c is the x-intercept. The factor theorem states that if c is a root (x-intercept) of a polynomial function, then $(x - c)$ must be a factor of that polynomial function. Note that $(x - c)$ is a factor of the expression. The only other factor is the slope m.

9. From their slope – y-intercept form, multiply the two functions together.

10. Graph the resulting function on the same axis as the two lines on the previous page.

11. What kind of function did you get?

12. What relationship do you see between the graph from Question 10 and the lines?

- …and the x-intercepts?
- …and the y-intercepts?

13. Identify the left-most x-intercept on the graph. With a straight-edge, cover everything to the right of that point. What connections do you see relating the signs of the y-values?

14. Identify the right-most intercept on the graph. With a straight-edge, cover up everything to the left of that point. What connections do you see relating to the signs of the y-values?

Complete the following sentences.

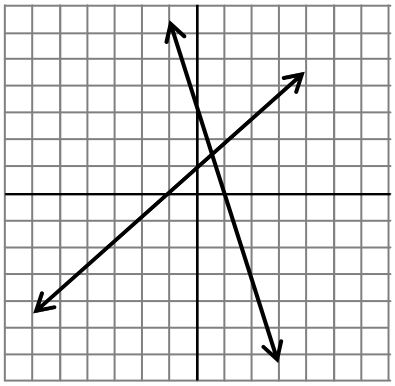
15. When both lines are **above** the x-axis, the y-values are ______________________ and the parabola __________________________.

16. When both lines are **below** the x-axis, the y-values are ______________________ and the parabola __________________________.

17. When one line is above the x-axis and the other is below the x-axis, the parabola __________________________.

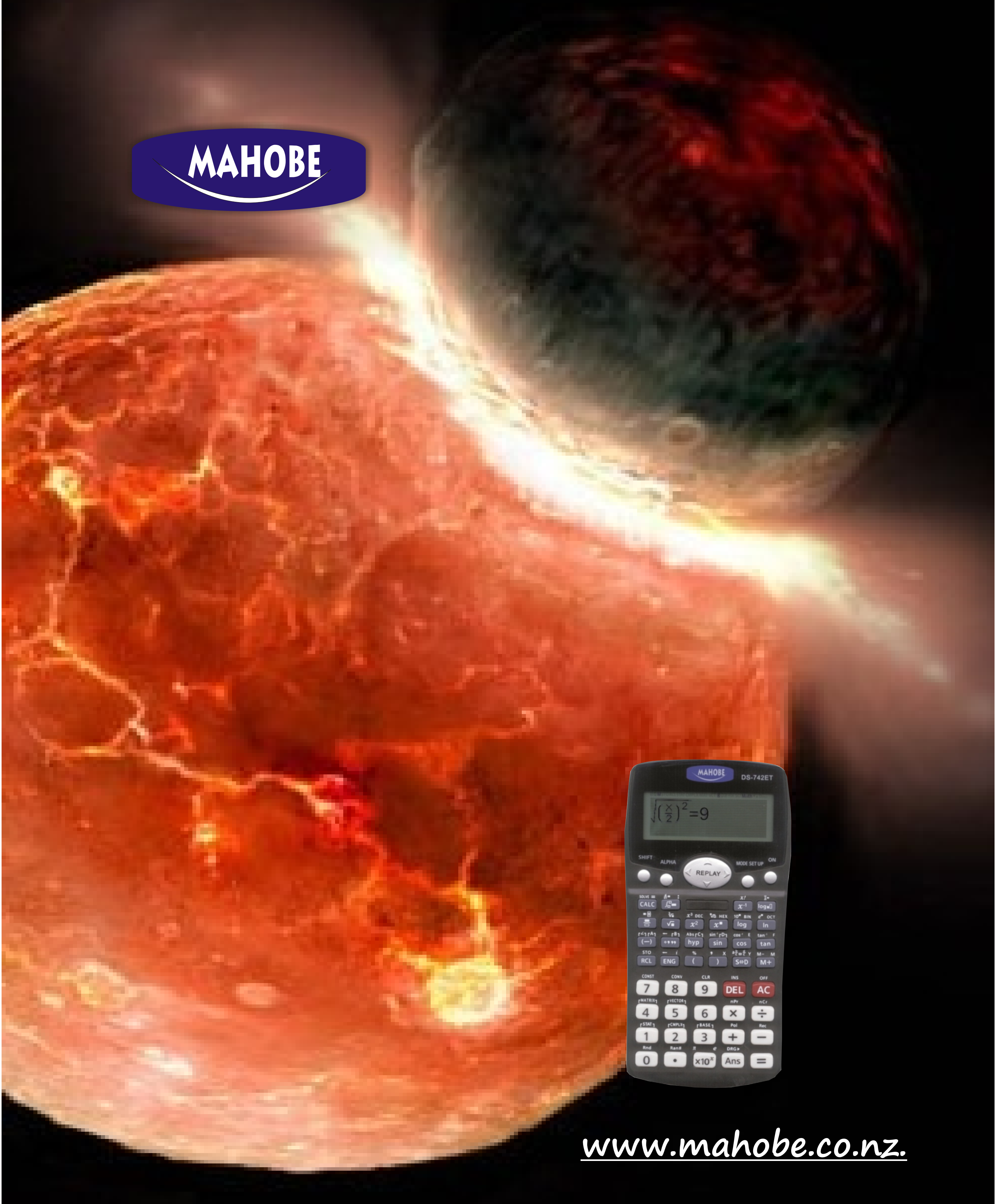
y-VALUE OF L_1	y-VALUE OF L_2	PARABOLA IS ABOVE/BELOW THE x-AXIS
+	+	
+	-	
-	+	
-	-	

18. Based on the patterns you saw on the previous page, draw a sketch of the quadratic function that would be obtained from the linear expressions of these lines.



19. Write the equation for each line.

20. To check your sketch in Question 18, multiply the expressions together, and graph the resulting function on the grid above. How accurate was your sketch?



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Working Backwards

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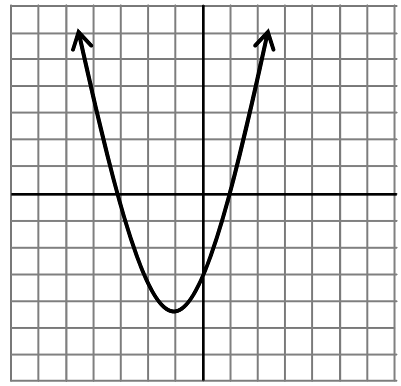
To the right, a parabola is given. Working backwards, we want to find two lines that could represent the linear factors.



1. What is the left-most x-intercept of the parabola?

2. To the left of that point, what do we know about the two lines that represent the linear factors?

3. What is the right-most x-intercept of the parabola?

4. To the right of that point, what do we know about thetwo lines that represent the linear factors?

5. What do we know about the two lines between the two linear factors?

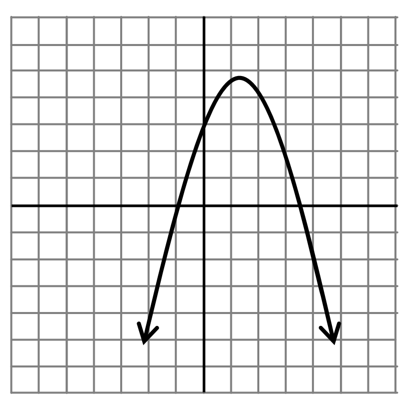
6. On graph sketch two lines that could represent the linear factors.

7. Write the equations for the lines you sketched.

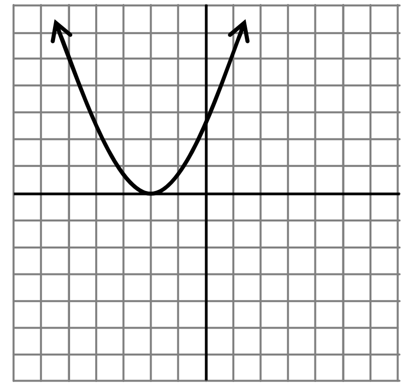
8. Multipty the two expressions together.

9. Graph the resulting parabola. How does it compare to the graph given?

10. Follow the same steps for the parabola to the right to determine a possible equation. Below, describe how you got the equation you've found.



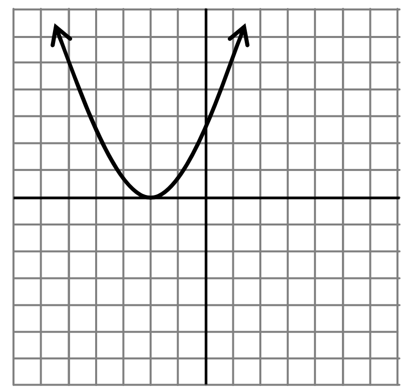
11. What is different about the x-intercept(s) of the graph to the right?



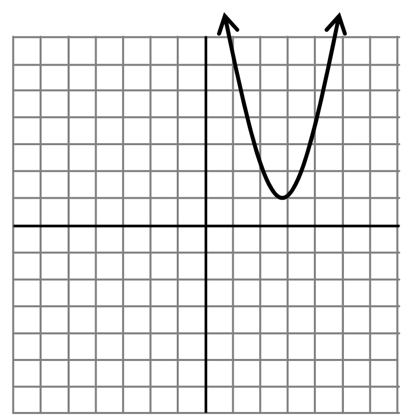
12. Sketch the lines that could represent the factors.

13. Describe the process of how you determined your answer for Question 12.

14. To the right is another graph just like the one above. Find an alternative pair of lines that could also satisfy the requirements for representing the factors. Graph them over the parabola to the right.



15. For the graph to the right, sketch the lines that could represent the factors.



16. Describe the process of how you determined your answer for Question 15.

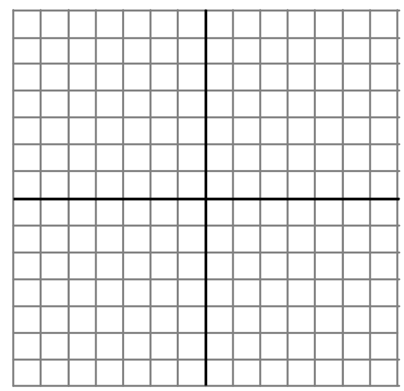
Building Polynomial Functions
Polynomials of Degree 3

NAME ____________________________

1. Graph each of the following three linear functions on the axis provided.

$y_1 = \frac{1}{2}x - 2$ $y_2 = x + 3$ $y_3 = -x + 1$

2. Multiply the three functions together.

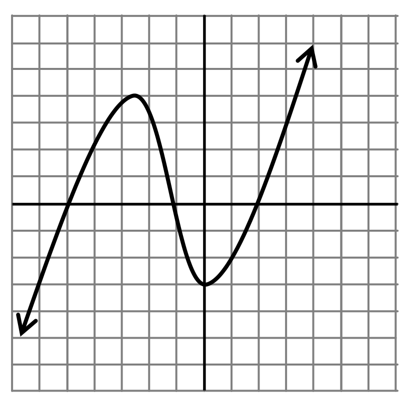


3. What type of function is the result?

4. Using what you know about multiplying signed numbers and the graphs of the lines, sketch your prediction for the graph of the product of the three linear functions.

5. Describe in your own words how you chose your graph.

6. Work-backwards: Find three lines that could be the components for the cubic represented by the graph to the right.



7. Describe in your own words how you chose your lines.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Answers to *Building Polynomials* Activity Sheets

Sheet 1

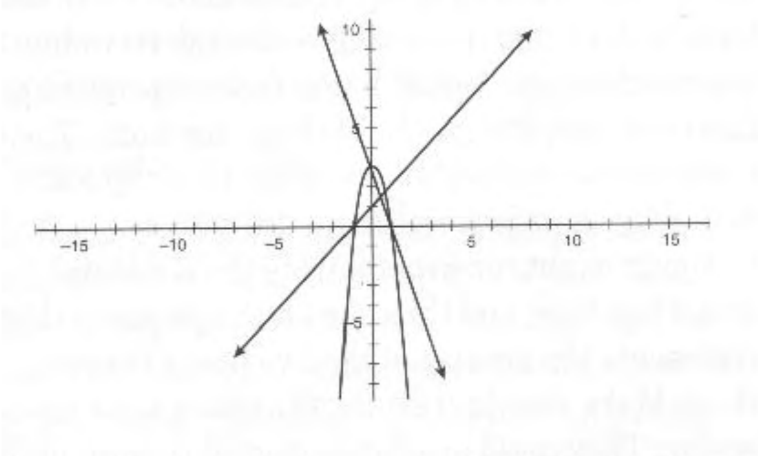
1. $y = (-2/3)x + 2$ or an equivalent form.
2. Answers will vary. For example, students can use the slope-intercept form of the point-slope form to find the equation.
3. $y = (-2/3)x + 2$
4. $y = (-2/3)(x - 3)$
5. Lines will vary.
6. $-b/m$ is the x-intercept.
7. Answers will vary.

8.-10. Answers will vary. Students may not immediately see that the product of the two linear expressions gives a parabolic expression, nor do they necessarily recognize that the parabola and the lines share the same x-intercepts. Questions 11 and 12 are meant to highlight the intercepts for those students who have not made the connection.

11. The lines have the same x-intercepts as the parabola.
12. The y-intercept of the parabola is the product of the y-intercepts of the lines.

13.-18. Answers will vary. The teacher should assist students who are placing their paper strip incorrectly.

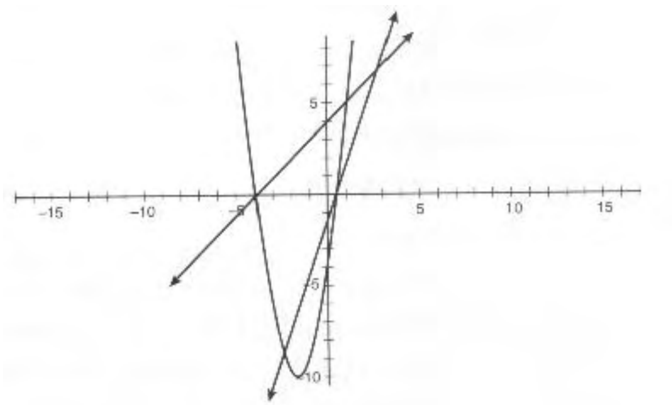
19. For any section of the graph, the product of the signs of the y-coordinates of the linear functions is the same as the sign of the y-coordinates of the parabola in that section.
20. A sample graph is given below. The parabola should have the same x-intercepts as the lines have, and the y-intercept should be 3. It is inverted.



21. Yes. Explanations will vary, but students should state again at least some of the relationships among the graphs found in questions 10 to 20.
22. One line will have a positive slope, and the other will have a negative slope.

Sheet 2

1. The lines drawn should go through the x-intercepts of the parabola. The product of the y-intercepts of the lines should equal the y-intercept of the parabola. The parabola should have negative y-coordinates when just one of the lines has negative y-coordinates. A possible pair of lines is given below.



2. Students may not have considered both the x-intercepts and the y-intercepts when sketching their graphs. They certainly could have used a combination of y-coordinates for a value of x 0, but using the y-intercepts is easier.
3. Students should also consider the signs of the y-coordinates for various sections of the graph.
4. Equations will vary. One possibility is $y = x + 4$ and $y = 2x - 1$. Another possibility is $y = (1/2)x$ and $y = 4x - 2$. For all equations, the x-intercepts should be -4 and 1/2 and the product of the y-intercepts should be -4.
5. The quadratic expression should be the product of the linear expressions given in question 4. If the linear expressions were $x + 4$ and $2x - 1$, then the quadratic function would be $y = 2x^2 + 7x - 4$.
6. Yes. See remarks in the text about generating other sets of lines by distributing unit factors.
7. The answer will be the same as the one for question 5.
8. Students should notice that the product is the same even though the lines have changed.
9.-10. The graph should show two lines, both having an x-intercept of 3. The product of the y-intercepts of the lines is 9. Two lines are needed, and since only one x-intercept exists, both lines must go through that intercept. The y-intercepts of the lines must have a product of 9. Note that the two lines could both be $y = -x + 3$; could both be $y = x - 3$; or could differ, for example, $y = (1/3)x - 1$, and $y = 3x - 9$.
11.-12. No lines can be drawn that would be components of the quadratic function given.
13. The absence of x-intercepts implies that no real roots exist, that is, lines cannot be drawn, because the quadratic equation cannot be factored into linear expressions over the real numbers.

Sheet 3

1. Equations and graphs will vary.
2. A cubic expression
3.-4. The graph drawn should go through the x-intercepts of all three lines. The product of the y-intercepts of the lines gives the y-intercept of the cubic. Help in graphing the cubic can also be obtained by observing the signs of the y-coordinates.
5.-6. Each line should pass through one of the x-intercepts. The product of the y-intercepts of the lines should be -6. Exactly one or exactly three of the lines will have negative y-coordinates when the y-coordinates on the cubic are negative. One such set of lines would be $y = x - 1$, $y = -x - 3$, and $y = -2x - 2$.