NCEA

Q&A

LEVEL 1
MATHEMATICS

Part 5 - AS90153
Geometric Reasoning

QUESTIONS & ANSWERS

MAHOBE

Contents

Q&A

NCEA Level 1 Mathematics – Questions & Answers

4

About This Book

Q&A eResources are recognised as the leading study guides for NCEA. Each freely available title has been compiled by a team of experienced educators to meet the study and revision needs of NCEA students. They are proving to be valuable resources in the hands of students who want to work ahead of their regular class programme. They also serve as effective revision programmes in the run up to the final examinations.

This book carefully explains the mathematical concepts that will be tested in NCEA then illustrates them with Achievement, Merit and Excellence examplars. It allows students to practise on NCEA-type questions and provides detailed solutions. After working through this programme, all students will be well prepared for their final assessments.

$$A = \pi r^2$$
$$= 3.14 \times 4^2$$
$$= 12.56^2$$
$$= 157.8 \ cm^2$$

4cm

The student who wrote the above answer on a recent assessment paper did not use a Q&A Level 1 Mathematics eResource.

Q&A

MAHOBE

YOU BEAUTY

Don't FRY this summer get RAD!

RAD is not just a suncreen but a vitamin enhanced lotion that will protect from both UVA and UVB radiation and will nourish and maintain a natural youthful and beautiful skin.

Purchase Environ RAD online at:

www.youbeauty.co.nz

A SAFE TAN can also be a natural looking tan!

For those who need a safe tan before summer we recommend Environ Self Tan. This is a moisturizing gel that leaves the skin moist and builds up a light, natural and healthy looking tan. Results will be noticeable after 2 hours.
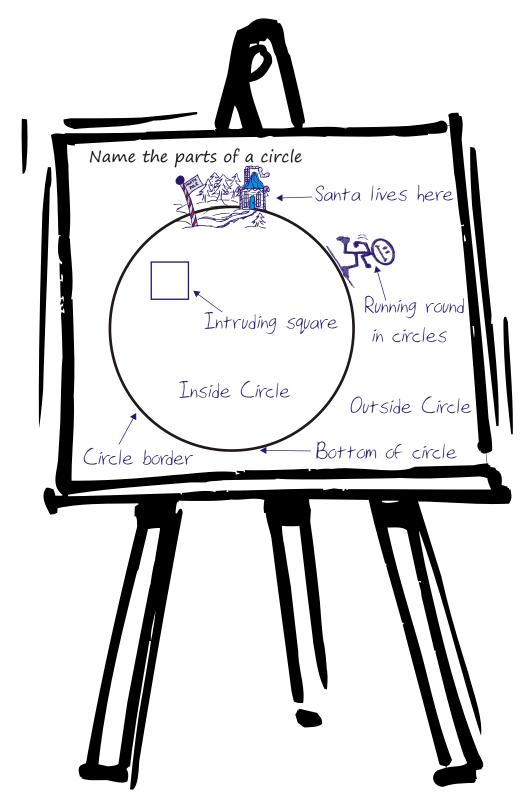
Purchase Environ Self Tan online at:

www.youbeauty.co.nz

MATHEMATICS 1.9 - AS90153
Geometric reasoning

Always understand what the examiner wants! A past examination answer is shown below. The student who wrote this answer on a recent assessment paper did not use a Q&A Level 1 Mathematics eResource.
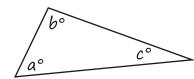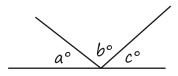
Geometric Reasoning
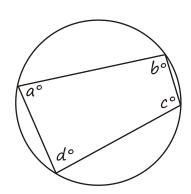
The Facts to Learn

Angles in a triangle add up to 180°.

a + b + c = 180°

Angles on a straight line add up to 180°.

a + b + c = 180°

Angles in a quadrilateral add up to 360°.
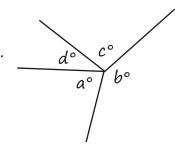
a + b + c + d = 360°

A cyclic quadrilateral has all its vertices (corners) touching a circle.
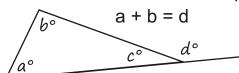
Opposite angles of a cyclic quadrilateral add up to 180°

a + c = 180°, d + b = 180°
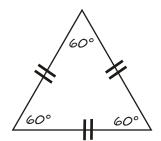
Angles round a point add up to 360°.

a + b + c + d = 360°

Exterior angles of a triangle
= sum of the opposite interior angles

a + b = d

Isosceles triangle
2 sides the same length
2 angles the same size

Equilateral triangle
3 sides the same length
3 angles the same size (60°)
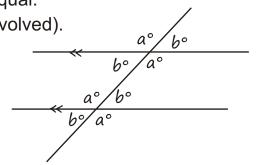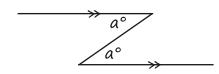
More Geometric Reasoning
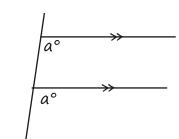
Vertically opposite angles are equal.
(Even when parallel lines are involved).

$a°$ $b°$
$b°$ $a°$

$a°$ $b°$
$b°$ $a°$
$a°$ $b°$
$b°$ $a°$

$a°$
$a°$
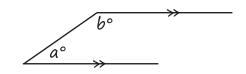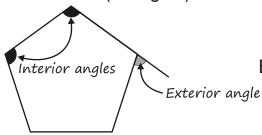
Alternate angles
(Z angles) are equal.

$a°$
$a°$

Corresponding angles
(F angles) are equal.

$b°$
$a°$
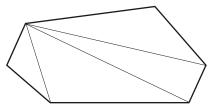
Co-interior angles
(C angles) add to 180°.

Interior angles

Exterior angle

Exterior angles of a polygon add up to 360°.

The sum of the interior angles of a polygon = (n - 2) × 180º

(Where n = number of sides)
This formula comes from splitting a polygon
into triangles by using diagonals. There will
always be 2 less triangles then there are sides.
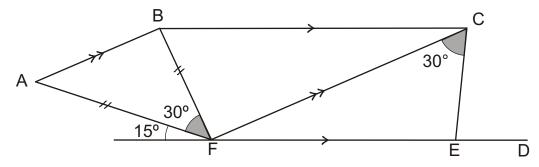
When given a geometry problem don't just concentrate on the angle you have been
asked to find. Try and find all the obvious angles according to the rules that you
can remember. Sooner or later the answer you need will be found.

Geometric Reasoning Example

Find all the other angles in the diagram below.

Triangle FAB is isosceles therefore ∠FAB = ∠FBA = 75°

AB and FC are parallel. ABFC is a Z-shape.

Therefore if ∠FBA = 75° then ∠BFC = 75° (alternate or "Z" angles)

∠ EFC = 60° (straight line angles sum to 180°)
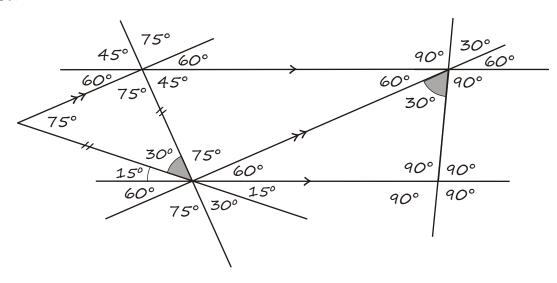
BC and FE are parallel, therefore ∠FCB = 60° (alternate or "Z" angles)

∠FBC = 45° (triangle angles sum to 180°)

∠FEC = 90° (triangle angles sum to 180°)

Angles on a straight line mean that ∠CED = 90° (straight line angles sum to 180°)
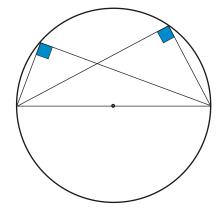
If needed extend all the lines on the diagram. This can sometimes help you see all the related angles – especially when parallel lines are concerned.
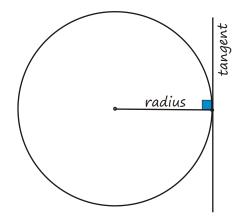
Circle Geometry
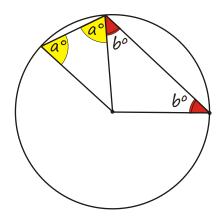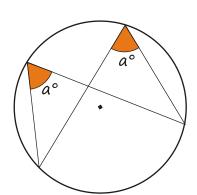
Angle in a semicircle = 90º.
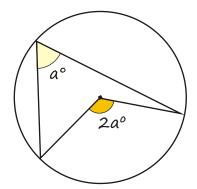
Tangent and radius meet at 90º.

radius   *tangent*

A triangle formed using two radii is an isosceles triangle.

$a°$ $a°$ $b°$ $b°$
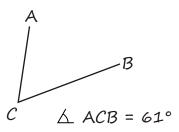
Angles in the same segment are equal.
i.e angles formed at the circumference by chords are equal.

$a°$ $a°$

$a°$ $2a°$

The angle at the centre is twice the angle at the circumference.

A

B

C   ∠ ACB = 61°

When writing angles use three letters.
The middle letter is where the angle is.

Q&A   YEAR 11 MATHEMATICS

MAHOBE

Circle Geometry Examples

1. In the diagram below, O is the centre of the circle.
   Find the angles: U, V, W.
   The diagram is not drawn to scale.

   Find all the obvious angles.

   Angles in the same segment
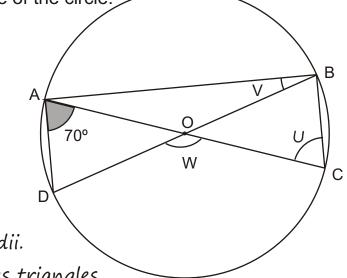
   $\triangle CAD = \triangle CBD = 70°$

   Angles in a semicircle = 90°

   $\triangle ABC = 90°, \triangle DAB = 90°$

   Isosceles Triangles. Sides are radii.

   AOD, BOC, BOA are all isosceles triangles.

   $\triangle BDA = 70°, \triangle CAB = 20°$ *and* $\triangle V = 20°$

   (Angle in a semicircle, angles in an isosceles triangle.)

   Angle U = 70° (identical isosceles triangle to AOD)

   $\triangle BOC$ *and* $\triangle AOD = 40°$ *(angles in a triangle = 180°)*

   W = 140° (straight line = 180° or isosceles triangle AOB)

2. Find, with geometric reasons, the size of angle d°.

   AB and CD are parallel lines.
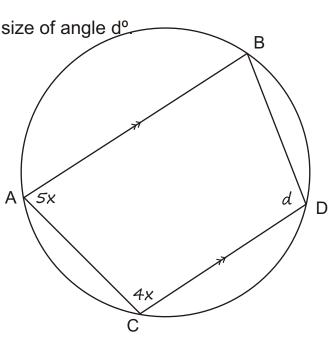
   4x + 5x add to 180°.

   (co-interior angles.)

   9x = 180° therefore x = 20°.

   If x = 20°, 5x = 100°.
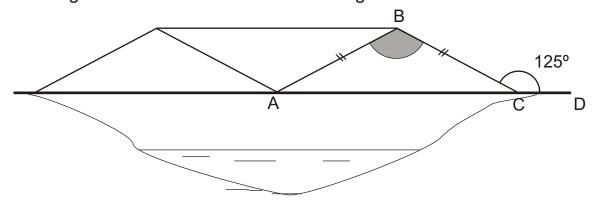
   This means that d = 80°.

   Opposite angles in a cyclic triangle are supplementary.

Geometric Reasoning - Achievement Examples

1. The diagram shows the side view of a bridge over a stream.
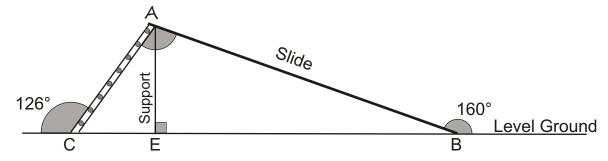
AB = BC, △BCD = 125º.
Calculate the size of angle ABC

△BCA = 180° – 125° (supplementary angles sum to 180°)

    = 55°

△BAC = 55° (base angles of an isosceles triangle are equal)

△ABC = 70° (triangle angles sum to 180°)

2. The diagram below shows the side view of a water slide.
Find the angle △CAB the angle between the ladder and the slide.

△ACE = 54° as angles on a straight line sum to 180°

△ABE = 20° as angles on a straight line sum to 180°

△CAB = 180° – 20° – 54°

    = 106° as angles in a triangle sum to 180°
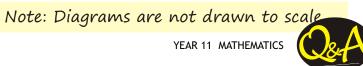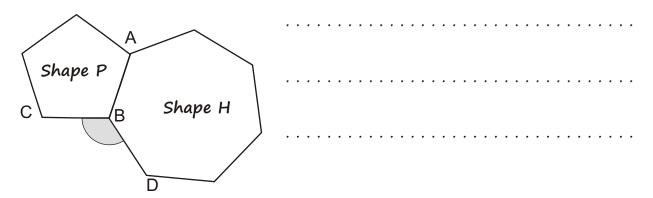
Exercises

1.

a. Calculate the size of angle △CDE

. . . .

. . . .

. . . .

b. Calculate the size of angle △ HIJ.

. . . .

. . . .

. . . .

c. Calculate the size of angle c.

. . . .

. . . .

. . . .

d. Calculate the size of △VUW

. . . .

. . . .

. . . .

Note: Diagrams are not drawn to scale
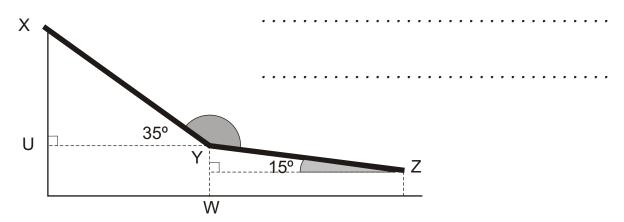
2. Shape A is a regular pentagon. Shape H is a regular heptagon. Both shapes share the side AB.

    a. Calculate the size of the angle △CBD.

. . . .

. . . .

. . . .
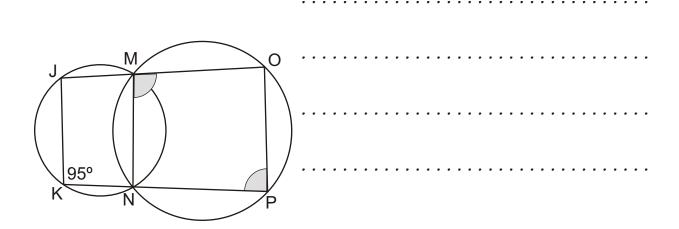
The diagram below gives the angles of a playground slide.

    b. Calculate the size of the shaded angle △ XYZ.

. . . .

. . . .

Two cyclic quadrilaterals are shown below. The size of angle △ JKN is 95º.
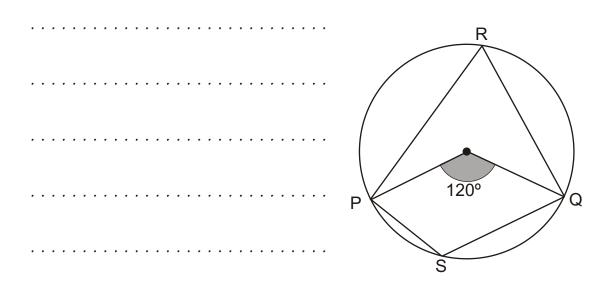
    c. Find the size of the angles △ OMN, and △OPN.

    d. How do we know that JK is parallel to OP?

. . . .

. . . .

. . . .

. . . .
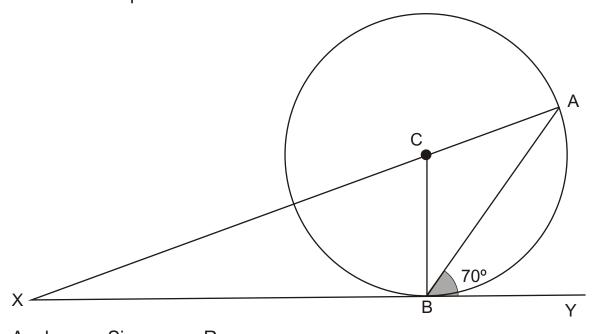
3. **a.** PQRS is a cyclic quadrilateral. O is the centre of the circle.
Find the size of angles △PRQ and △PSQ.

. . . .

. . . .

. . . .

. . . .

. . . .

b. In the diagram below (which is not drawn to scale), C is the centre of
the circle, and XY is a tangent to the circle. Angle △ABY = 70º.
Complete the sentences below to find, in 4 logical steps the angle
which equals 50º.

Angle	Size	Reason
XBC	90º	. . . .
CBA	. . . . . .	Adjacent angles on a line add up to 180º.
CAB	20º	. . . .
XCB	40º	. . . .
. . . . . .	50º	. . . .

Geometric Reasoning - Merit Examples
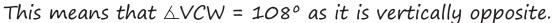
1. The diagram below shows a pentagram - a five sided star.
The shaded shape formed by ABCDE is a regular pentagon.
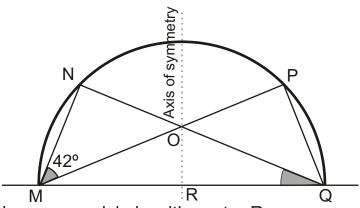
Calculate the angle △VCW.
Give geometric reasons with each of your calculations.

The total of all the interior angles
of a regular pentagon can be
calculated by the formula:
(n–2) × 180 where n is the
number of sides (5).
Therefore 3 × 180° = 540°
Each angle = 108° (as 540° ÷ 5 = 108°).
This means that △VCW = 108° as it is vertically opposite.

2. The figure above shows a semicircle with centre R.
RO is an axis of symmetry. The angle △NMP is 42°.
Find the angle △NQM. Support your answer with geometric reasons.

△QNM = △MPQ = 90° (angle in a semicircle)
This means that △NOM = 48° (angles in a triangle sum to 180°)
△NOP and △MOQ = 132° (angles around a point sum to 360°)
*   (Vertically opposite angles are equal)*
△OQM = △OMQ = 24° (base angles of an isosceles triangle)
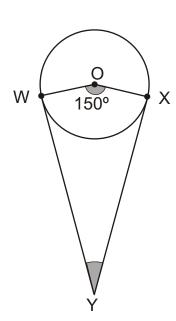
4. In the diagram below the points A, B, C and D lie on a circle with centre O. Calculate the size of angle $\triangle$BCD. Give geometric reasons for your answer.

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

5. The diagram below shows a circle with two tangents at W and X. O is the centre of the circle. Calculate the angle $\triangle$WYX. Give geometric reasons for your answer.

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .
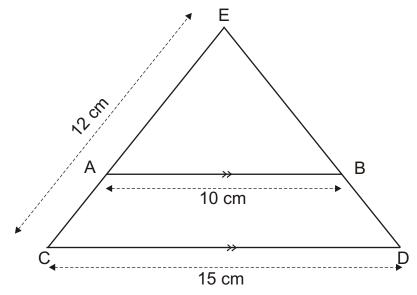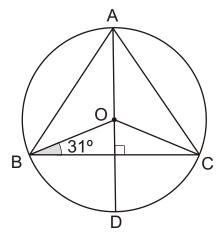
6. In the figure below ABDC is an isosceles trapezium.
Find the length of BD. Give geometric reasons for your answer.

E

12 cm

A   B
10 cm

C   D
15 cm

. . . .

. . . .

. . . .

. . . .

7. In the diagram below ABC is an isosceles triangle.
O is the centre of the circle. The angle ∠OBC = 31º
        a.   Calculate the size of ∠BAC.
        b.   Calculate the size of ∠ODC.
           Give reasons for your answers.

A

O

31º

B   C

D

. . . .

. . . .

. . . .

. . . .

. . . .

YEAR 11 MATHEMATICS

MAHOBE

8. The diagram below is NOT drawn to scale.
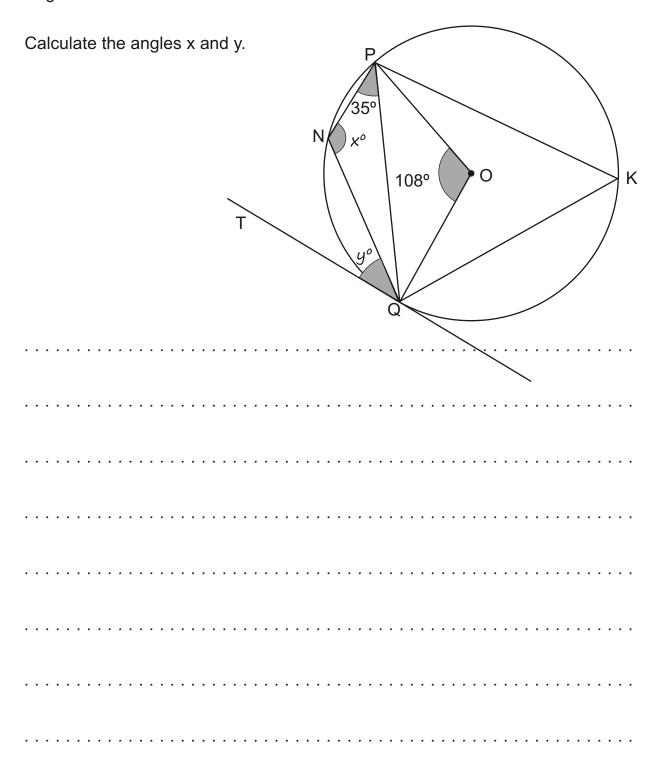
O is the centre of the circle.

P, K, Q and N are points on the circumference.

QT is the tangent to the circle at Q.

Angle POQ = 108°

Angle NPQ = 35°

Calculate the angles x and y.

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

Geometric Reasoning - Excellence Examples

1. The diagram below shows a circle with points A, B and C on the circumference.
Centre of the circle is O.
Calculate the size of △CAB.
Give geometric reasons for your answer.
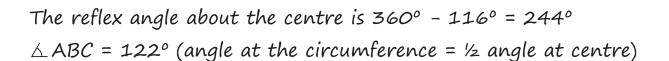
Triangle AOC is isosceles (OA = OC)

△OAC = 32°

(base angles of an isosceles triangle)
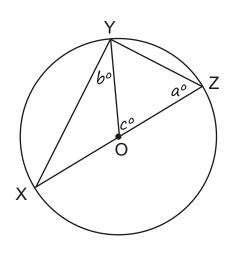
△AOC = 116°

(angle sum of a triangle = 180°)

The reflex angle about the centre is 360° – 116° = 244°

△ABC = 122° (angle at the circumference = ½ angle at centre)

△BCA = 32°. (alternate angles are equal i.e. △BCA and △OAC)

△BAC = 26° (angle sum of a triangle = 180°)

2. In the figure below, O is the centre of the circle. XZ is the diameter.
Prove that c = 2b. Give geometric reasons for your answer.

△XYZ = 90° (angle in a semicircle)
OX, OZ and OY are radii therefore there
are two isosceles triangles formed, OYZ
and OYX. This means that △OYZ = a°
and △OXY = b°.
b = 90 – a (angle in a semicircle)
c = 180 – 2a (angles in a triangle sum)
Therefore c = 2b
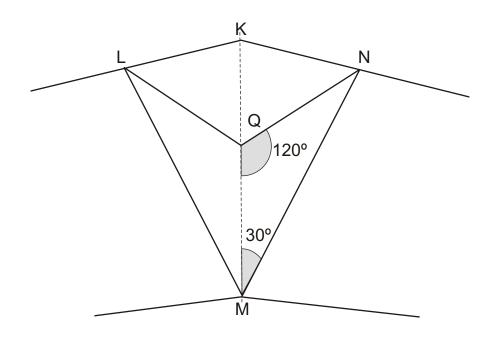
YEAR 11 MATHEMATICS

MAHOBE

9. In the figure below, line KM forms an axis of symmetry.
Length QN = Length QK = Length QL.
Angle NQM = 120°.
Angle NMQ = 30°.

Prove that the quadrilateral KLMN is cyclic.

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .
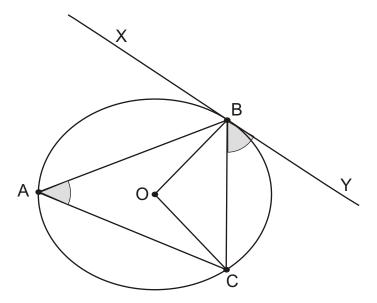
10. The circle below has a centre O and a tangent XY at point B.
The points A and D lie on the circle.

Prove that $\triangle$YBC = $\triangle$BAC

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

Level 1 Mathematics - Sample Exam

AS90153 Geometric Reasoning

You are advised to spend 25 minutes answering the questions in this section.

QUESTION ONE

Diagram is not to scale.

Pauline's garden table is illustrated in the diagram.
ABEF is a straight and horizontal table top.
XY is an axis of symmetry.
Angle BED = 110°

Calculate the size of angle ABC giving reasons for your answers.

. . . .

. . . .

. . . .

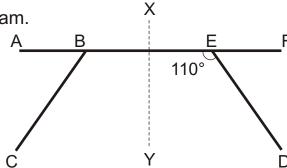. . . . .

. . . .

QUESTION TWO

Stan's step-ladder is shown in the diagram.

Angle LMN = 122°
KL = LN

Calculate the size of the inside angle of the top of the
step-ladder KLM giving reasons.

. . . .

. . . .

. . . .

. . . . K
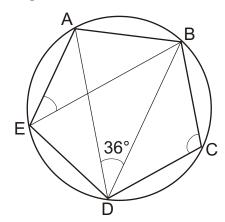
QUESTION THREE

The diagram shows a regular pentagon inside a circle.
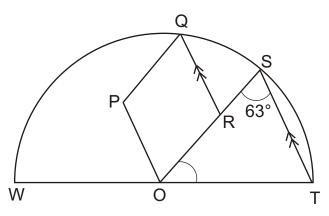
Calculate the size of BEA and DCB giving reasons for your answers.

. . . .

. . . .

. . . .

. . . .

QUESTION FOUR

In the diagram, PQRO is a rhombus QR is parallel to ST.

Angle TSR = 63°. Calculate the angle of ROT.

Give a geometric reason for each step leading to your answer.

. . . .

. . . .
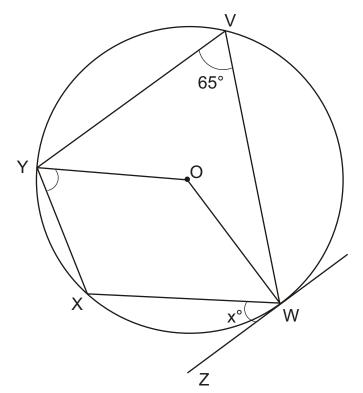
QUESTION FIVE

In the diagram V, W, X and Y lie on a circle, centre O.

ZW is a tangent to the circle at W.

Angle YVW = 65°

Angle XWZ = x°

Find the size of angle OYX.

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

QUESTION SIX

KLM and MNO are two triangles.

Angle KLM = 32°

Angle MNO = 24°

Find the size of angle OMN.

Give geometric reasons for each step leading to your answer.

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

MAHOBE

The Answers
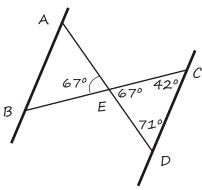
The Answers
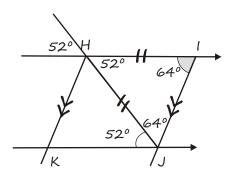
Page 14

1.   a.

CED = 67° – vertically opposite
Angles in a triangle sum to 180°
180° – 67° – 42° = 71°

b.
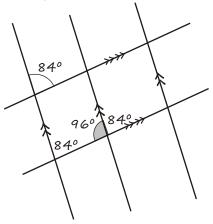

Angle IHJ = 52° alternate angles,
parallel lines.
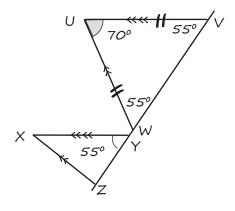HIJ = 64° – Angles in an isosceles
triangle.

c.

See the diagram above for all the
corresponding angles.
c = 96° angles on a straight line

d.

See diagram above for the
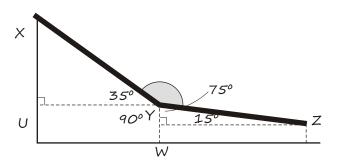corresponding angles. The 70° is
then calculated from the isosceles
triangle WUV.

Page 15

2.   a.   Sum interior angles = (n–2) × 180°
Sum interior angles of pentagon
(5 – 2) × 180° = 540°
Each angle = 108°
Sum interior angles heptagon
(7–2) × 180° = 900°
Each angle = 128.57°
Angle CBD = 360 – (108 + 128.57)
              = 123.43°

b.

Angle UYW = 90° right angled
rectangle.
The angle ZYW = 75° angle sum
of a triangle.
XYZ = 360° – (75° + 90° + 35°)
       = 160°
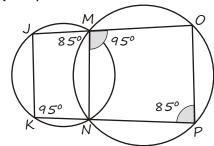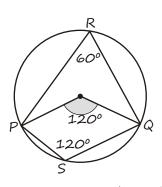
Page 15 (cont)

2.   C.

NMJ = 85° opposite angles of a cyclic triangle sum to 180°
This means angle NMO = 95° angles on a straight line and NPO = 85° as it is the opposite angle in the cyclic triangle.
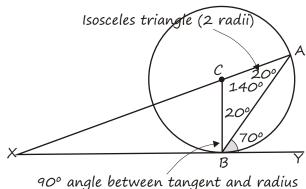
d.   JK is parallel to OP as all the angles indicate that they are corresponding.

Page 16

3.   a.   PRQ = 60° angle at the centre
PSQ = 120° opposite angles in a cyclic quadrilateral.

B.

Isosceles triangle (2 radii)

90° angle between tangent and radius

b.   XBC = Angle between a tangent and a radius
CBA = 20°
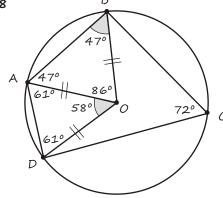CAB = Base angles of an isosceles triangles are equal
XCB = 40° angles on straight line
CXB = Angles in a triangle sum to 180°

Page 18

4.

OB, OA and OD are all radii.
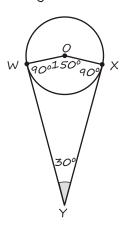This means that there are 2 isosceles triangles AOB (47°, 47°, 86°) and AOD (61°, 61°, 58°)
BAO + DAO: 61° + 47° = 108°
BCD   = 72° opposite angles cyclic quad.
or   BOA + AOD: 86° + 58° = 144°
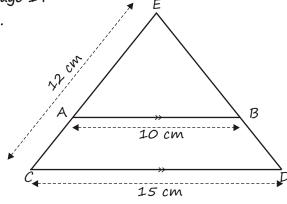BCD = 72° angle at circumference

5.

OWY AND OXY are 90° as the angle between the tangent and radius are perpendicular.
WYX = 30° interior angles of a quadrilateral sum to 360°

Q&A

MAHOBE

34

6.

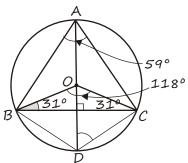EAB and ECD are similar triangles.

This means $\dfrac{EA}{12} = \dfrac{10}{15}$

This means that EA = 8 cm

BD = CE − EA   (CE = DE, AE = BE)

    = 12 cm − 8 cm

    = 4 cm

7.

BCO = 31° base angles isosceles triangle.

COB = 118° angle sum of a triangle
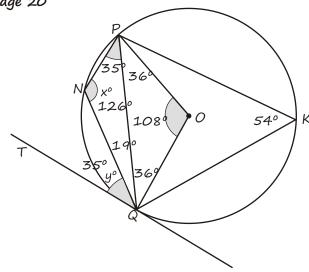
BAC = 59° angle at circumference is half

that at the centre.

BAC is isosceles triangle

BDC is an isosceles triangle

Angle BDC = 121° opposite angles cyclic

quadrilateral. ODC is half this 60.5°.

8.   Lines OP and OQ are radii.

This means that OPQ is an isosceles

triangle and the base angles OPQ and

OQP = 36°

Angle PKQ = 54° as it is angle at the

circumference (½ POQ)

Angle x (PNQ) = 126° as opposite angles
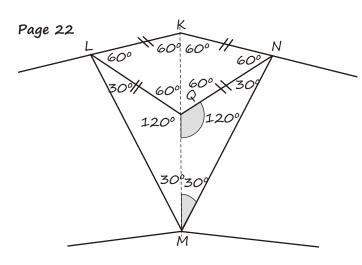
of a cyclic quadrilateral sum to 180°.

OQT is the angle between a radius and a

tangent = 90°.

NQP = 19° angle sum of a triangle

NQT (y°)   = 90° − 36° − 19°

            = 35°

Page 22

K
L 60° 60° 60° N
60°
30° 60° 60° 30°
Q
120° 120°

30° 30°
M

9.   KQN = 60° angles on a straight line
         QKN = 60° isosceles triangle QN = QK
         LKN = 120° line KM is a line of
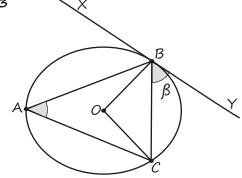   symmetry
         LKN = 2 × QKN
         LMN = 60° due to line of symmetry KM
         LKN + LMN = 180°
         KLMN = a cyclic triangle (opposite angles
         of a cyclic quadrilateral sum to 180°)

Page 23
10.

X
B
β
A   O   Y
C

OB is a radius and the tangent XY is 90°
to the tangent.
Let YBC = β
OBC = 90 – β tangent 90° to radius
OCB = 90 – β base angles of isos triangle
                OB = OC
BOC   = 180 – (90 – β) – (90 – β)
        = 2β
BAC = β angle at circumference is half
that at the centre
BAC = β, YBC = β
Therefore YBC = BAC

Page 27
Question One
         FED = 70° angles on a straight line
         ABC = 70° symmetry through XY
Question Two
         LMK = 58° angles on a straight line
         LKM = 58° as triangle is isosceles
         KLM = 64° angle sum of a triangle

Page 28
Question Three
         BEA = 36° angles in the same segment
         DCB = 108° interior angles of a regular
         pentagon.
Question Four
         OTS is an isosceles triangle
         STO = 63° base angles isosceles triangle
         ROT   = 180° – 2 × 63°
               = 54°

Page 29
Question Five
         OW is at 90° to WZ
         It is a radius and tangent
         This means that OWX = 90° – x
         YXW = 115° opposite angles of a
         cyclic quadrilateral
         WOY = 130° angle at centre is twice the
         angle at the circumference
         Angle sum of a quadrilateral is 360°
         OYX = 360 – 130 – 115 – (90 – x)
             = (25 + x)°

Page 30
Question Six
         Let KML = x°, then OMN = x° as they are
         vertically opposite.
         JOM = x + 45°, JKM = x + 32° as the
         exterior angles of a triangle = the sum of
         the two opposite interior angles
         JOM + JKM = 180° (cyclic quadrilateral)
         x° + 45° + x° + 32° = 180°
         x = 51.5°

Q&A   YEAR 11 MATHEMATICS   MAHOBE
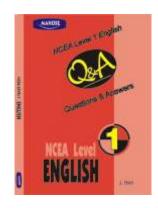
THE NZ CENTRE OF MATHEMATICS

fueling maths education

The Mahobe RedBack Calculator

The NZ Centre of Mathematics recommends the Mahobe RedBack scientific calculator for NCEA. This calculator has advanced features, such as equation solving, to help take the sweat out of such calculations. Purchase it direct from the Mahobe website www.mahobe.co.nz.

NCEA Level 1 English Questions & Answers

Each of the English Achievement Standards are introduced with a wide range of writing topics along with excellent hints on how to structure and write an outstanding essay. There are many sample excellence essays from previous years' successful students. This book shows you how to combine all your teacher and class notes and practise successfully for each assessment. Q&A English is $14.95 and can be purchased from Whitcoulls, Paper Plus or from the Mahobe website www.mahobe.co.nz

NCEA Level 1 Science Questions & Answers

There are explanations and practice NCEA assessments of the type that can be expected in the exam. This book allows you to focus on the exam essentials. It will your boost confidence and maintain motivation in the weeks leading up to the exams. Study with this book and you will enter the exam room with more comfort, confidence and enthusiasm. Q&A Science is $19.95 and can be purchased from Whitcoulls, Paper Plus or from the Mahobe website www.mahobe.co.nz