NCEA

Q&A

LEVEL 1 MATHEMATICS

Geometric Reasoning

QUESTIONS & ANSWERS

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NCEA Level 1 Mathematics, Questions & Answers

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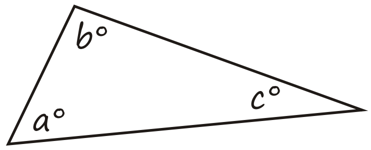
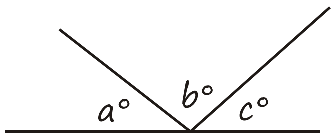
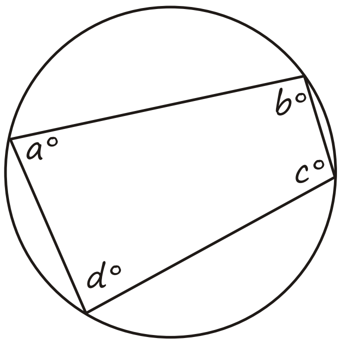
The Facts to Learn



Angles in a triangle add up to 180°.
a + b + c = 180°



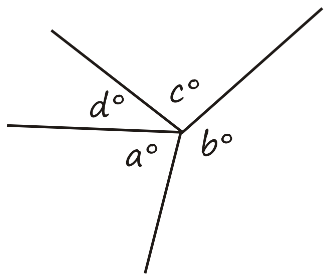
Angles on a straight line add up to 180°.
a + b + c = 180°



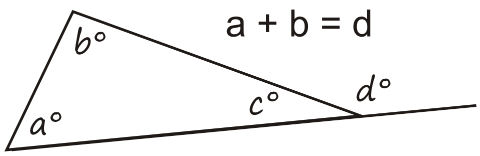
Angles in a quadrilateral add up to 360°.
a + b + c + d = 360°
A cyclic quadrilateral has all its vertices (corners) touching a circle.
Opposite angles of a cyclic quadrilateral add up to 180°
a + c = 180°, d + b = 180°

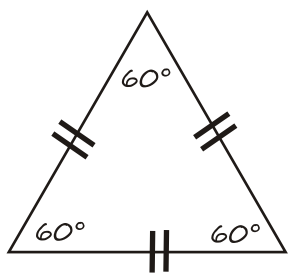
Angles round a point add up to 360°.

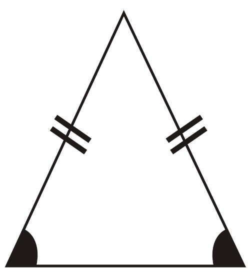
a + b + c + d = 360°



Exterior angles of a triangle
= sum of the opposite interior angles
a + b = d



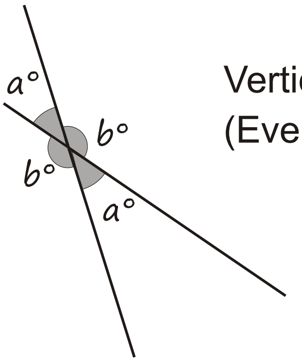




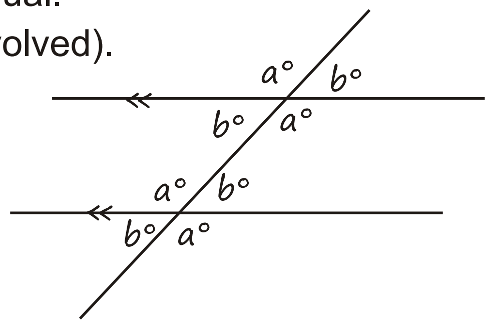
Isosceles triangle
2 sides the same length
2 angles the same size

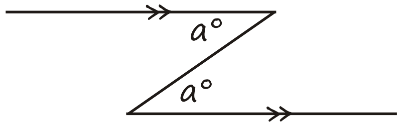
Equilateral triangle
3 sides the same length
3 angles the same size (60°)

More Geometric Reasoning

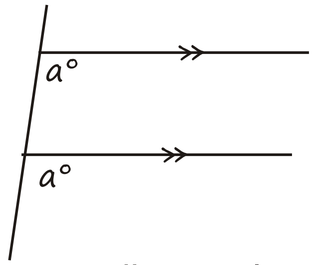


Vertically opposite angles are equal.
(Even when parallel lines are involved).

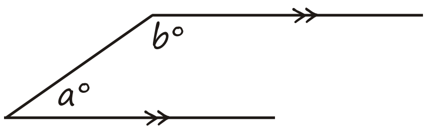




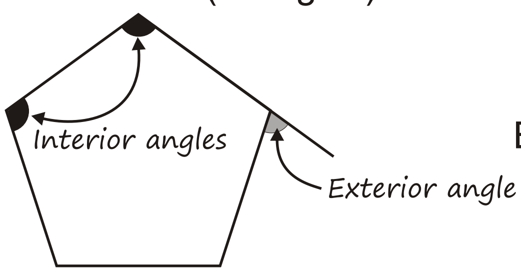
Alternate angles
(Z angles) are equal.



Corresponding angles
(F angles) are equal.

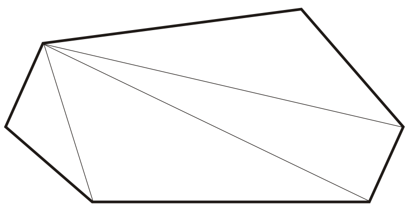


Co-interior angles
(C angles) add to 180°.



Exterior angles of a polygon add up to 360°.

The sum of the interior angles of a polygon = (n - 2) × 180º

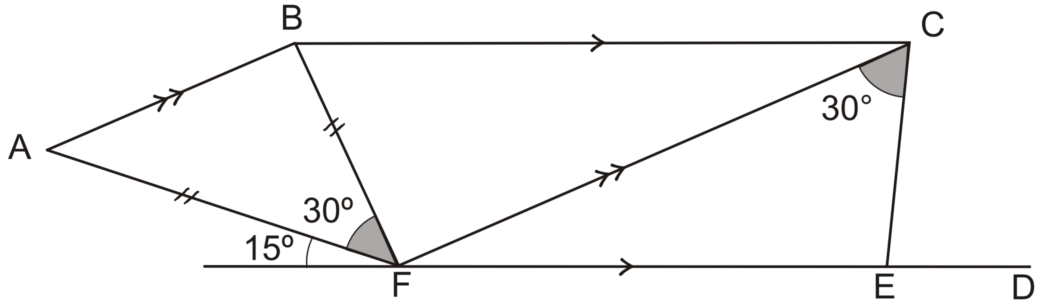


(Where n = number of sides)
This formula comes from splitting a polygon into triangles by using diagonals. There will always be 2 less triangles then there are sides.

When given a geometry problem don't just concentrate on the angle you have been asked to find. Try and find all the obvious angles according to the rules that you can remember. Sooner or later the answer you need will be found.

Geometric Reasoning Example

Find all the other angles in the diagram below.



Triangle FAB is isosceles therefore △FAB = △FBA = 75°

AB and FC are parallel. ABFC is a Z-shape.

Therefore if △FBA = 75° then △BFC = 75° (alternate or "Z" angles)

△ EFC = 60° (straight line angles sum to 180°)

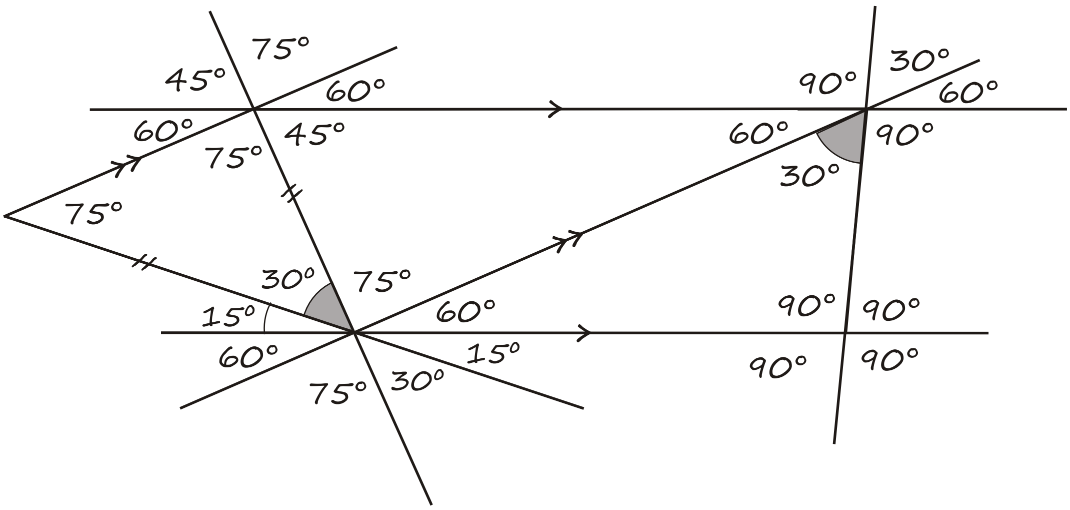
BC and FE are parallel, therefore △FCB = 60° (alternate or "Z" angles)

△FBC = 45° (triangle angles sum to 180°)

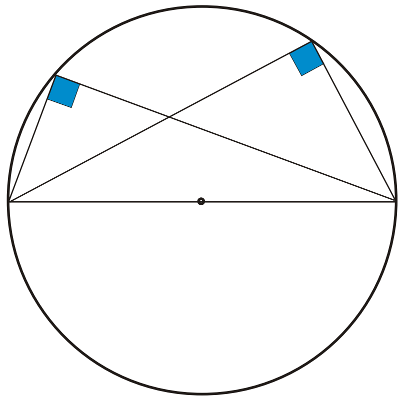
△FEC = 90° (triangle angles sum to 180°)

Angles on a straight line mean that △CED = 90° (straight line angles sum to 180°)

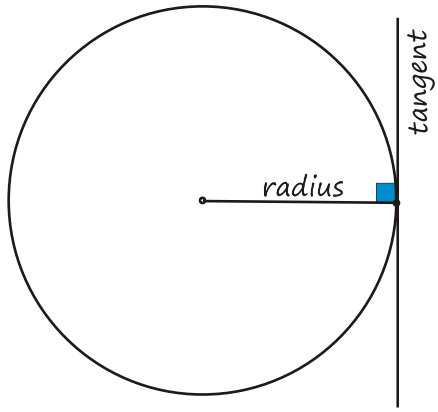
If needed extend all the lines on the diagram. This can sometimes help you see all the related angles – especially when parallel lines are concerned.



Circle Geometry

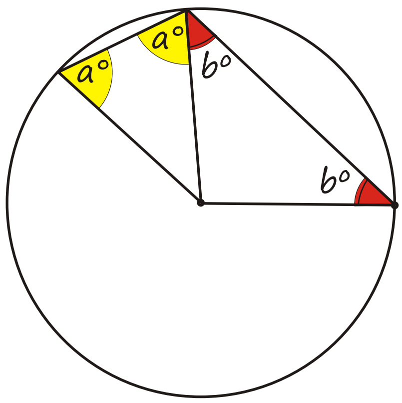


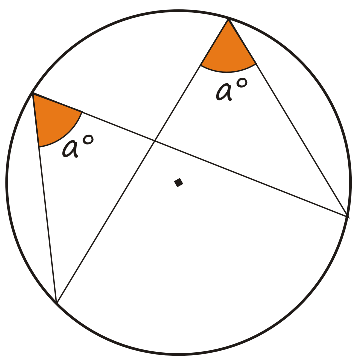
Angle in a semicircle = 90°.



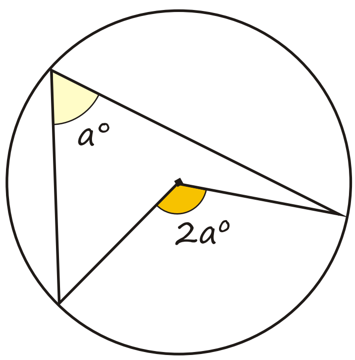
Tangent and radius meet at 90°.

A triangle formed using two radii is an isosceles triangle.

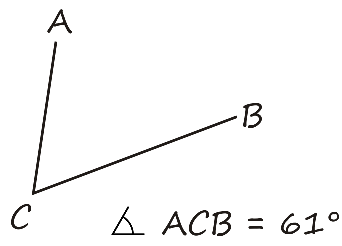




Angles in the same segment are equal.
i.e angles formed at the circumference by chords are equal.



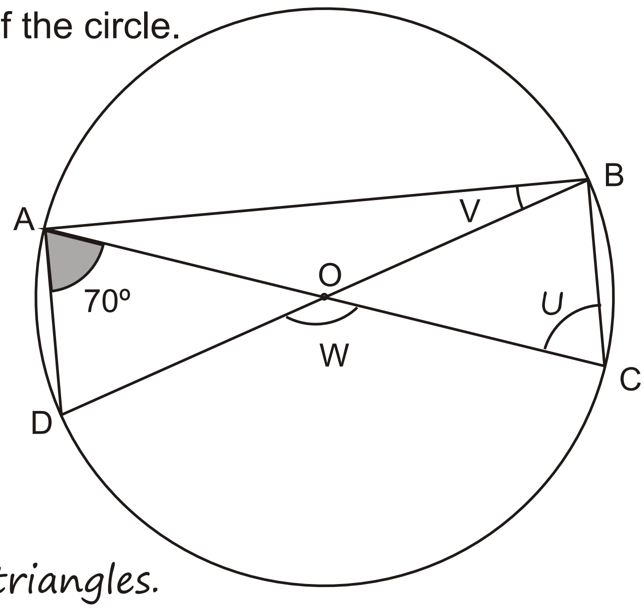
The angle at the centre is twice the angle at the circumference.



When writing angles use three letters.
The middle letter is where the angle is.

Circle Geometry Examples

1. In the diagram below, O is the centre of the circle.
   Find the angles: U, V, W.
   The diagram is not drawn to scale.



Find all the obvious angles.

Angles in the same segment

$\angle CAD = \angle CBD = 70^\circ$

Angles in a semicircle = 90°

$\angle ABC = 90^\circ$, $\angle DAB = 90^\circ$

Isosceles Triangles. Sides are radii.

AOD, BOC, BOA are all isosceles triangles.

$\angle BDA = 70^\circ$, $\angle CAB = 20^\circ$ and $\angle V = 20^\circ$

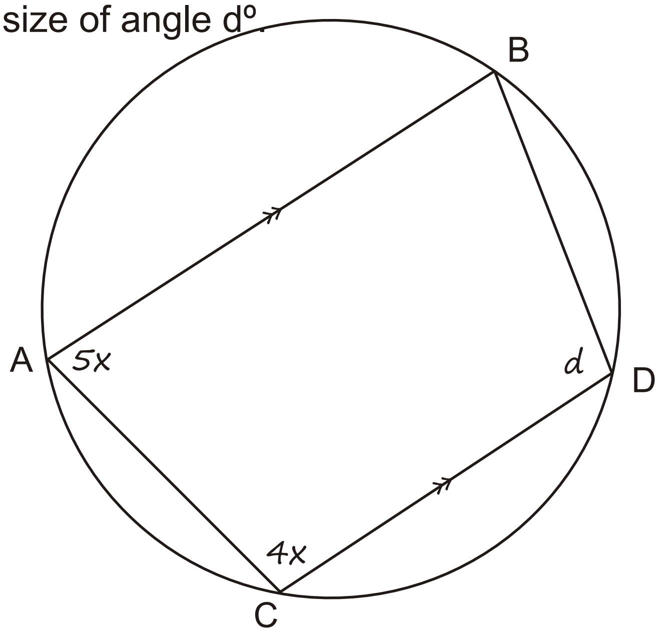
(Angle in a semicircle, angles in an isosceles triangle.)

Angle U = 70° (identical isosceles triangle to AOD)

$\angle BOC$ and $\angle AOD = 40^\circ$ (angles in a triangle = 180°)

W = 140° (straight line = 180° or isosceles triangle AOB)

2. Find, with geometric reasons, the size of angle dº.



AB and CD are parallel lines.

$4x + 5x$ add to 180°.

(co-interior angles.)

$9x = 180^\circ$ therefore $x = 20^\circ$.

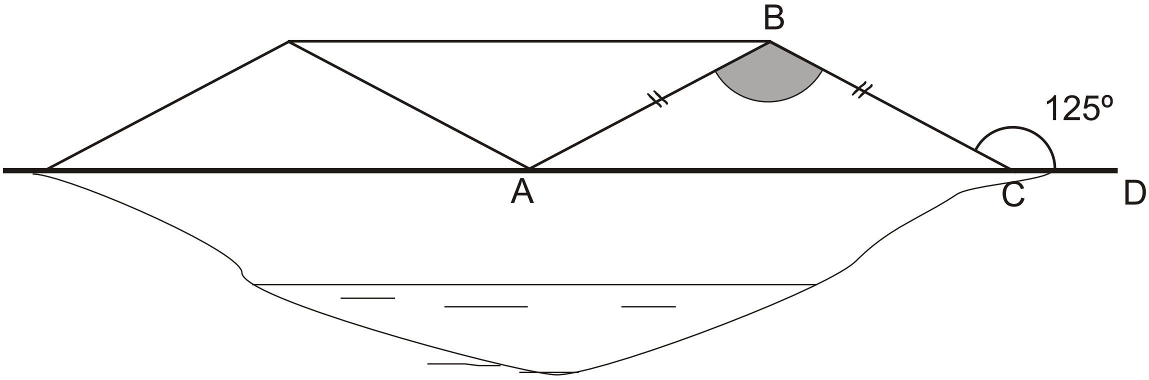
If $x = 20^\circ$, $5x = 100^\circ$.

This means that $d = 80^\circ$.

Opposite angles in a cyclic triangle are supplementary.

Geometric Reasoning - Achievement Examples

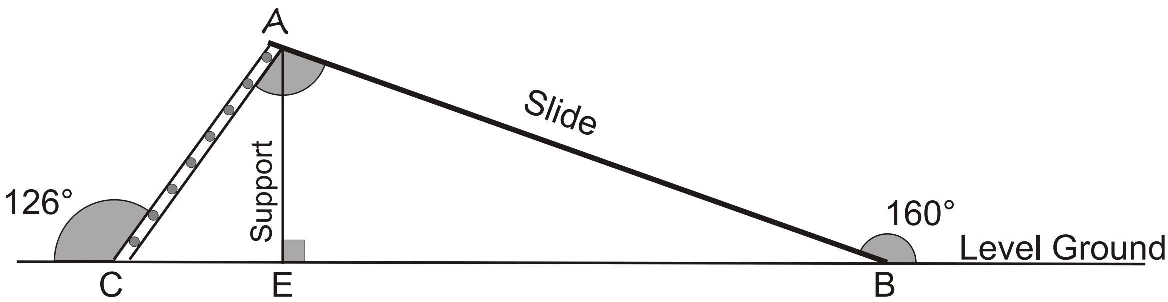
1. The diagram shows the side view of a bridge over a stream.



AB = BC, △BCD = 125º.
Calculate the size of angle ABC

△BCA = 180° – 125° (supplementary angles sum to 180°)
= 55°
△BAC = 55° (base angles of an isosceles triangle are equal)
△ABC = 70° (triangle angles sum to 180°)

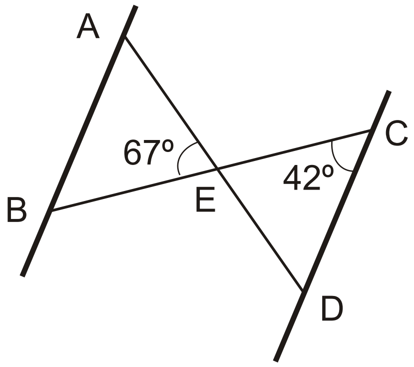
2. The diagram below shows the side view of a water slide.
Find the angle △CAB the angle between the ladder and the slide.



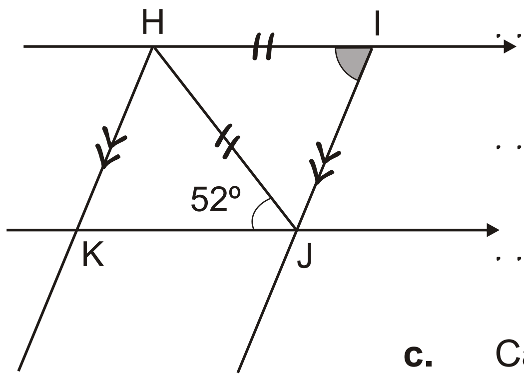
△ACE = 54° as angles on a straight line sum to 180°
△ABE = 20° as angles on a straight line sum to 180°
△CAB = 180° – 20° – 54°
= 106° as angles in a triangle sum to 180°

Exercises

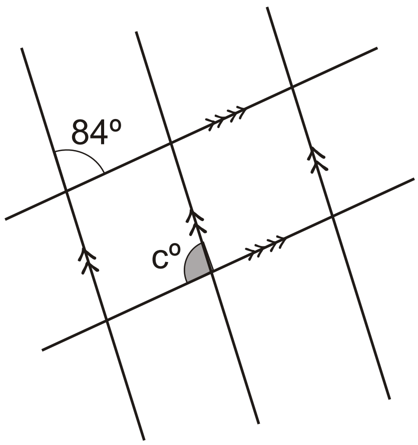
1.



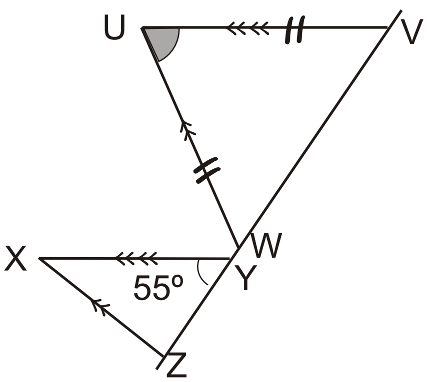
a. Calculate the size of angle ∡CDE



b. Calculate the size of angle ∡HIJ.



c. Calculate the size of angle c.

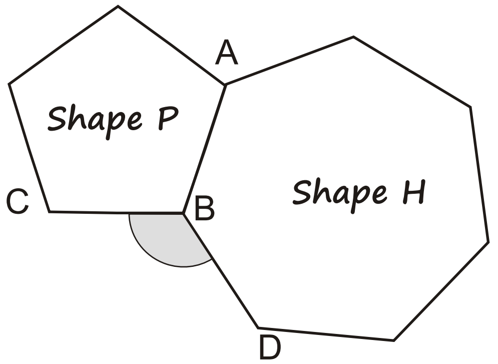


d. Calculate the size of ∡VUW

Note: Diagrams are not drawn to scale

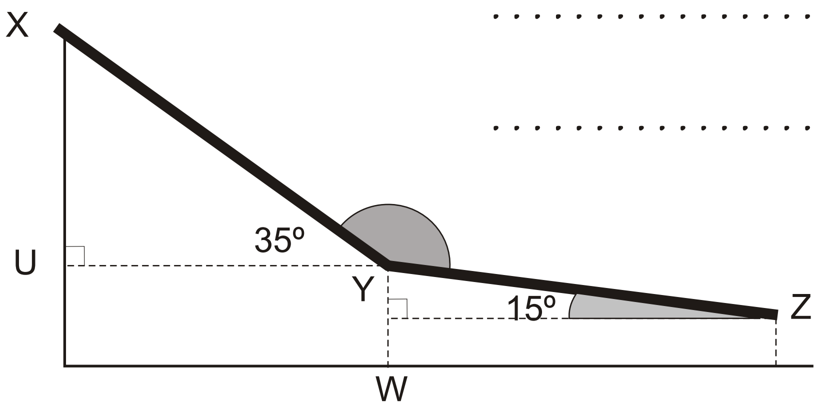
2. Shape A is a regular pentagon. Shape H is a regular heptagon. Both shapes share the side AB.

a. Calculate the size of the angle $\angle$CBD.



The diagram below gives the angles of a playground slide.

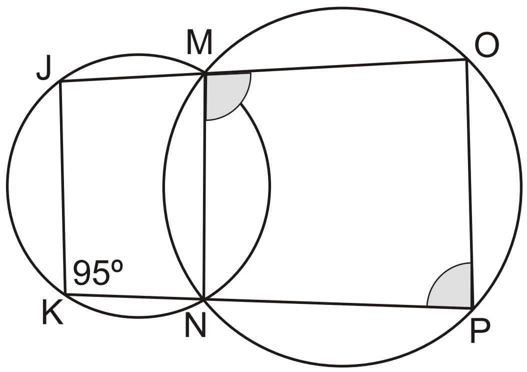
b. Calculate the size of the shaded angle $\angle$XYZ.



Two cyclic quadrilaterals are shown below. The size of angle $\angle$JKN is 95º.

c. Find the size of the angles $\angle$OMN, and $\angle$OPN.

d. How do we know that JK is parallel to OP?



3. **a.** PQRS is a cyclic quadrilateral. O is the centre of the circle.
Find the size of angles ∠PRQ and ∠PSQ.

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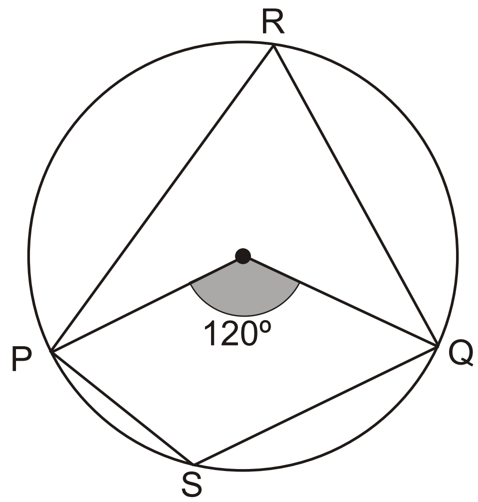
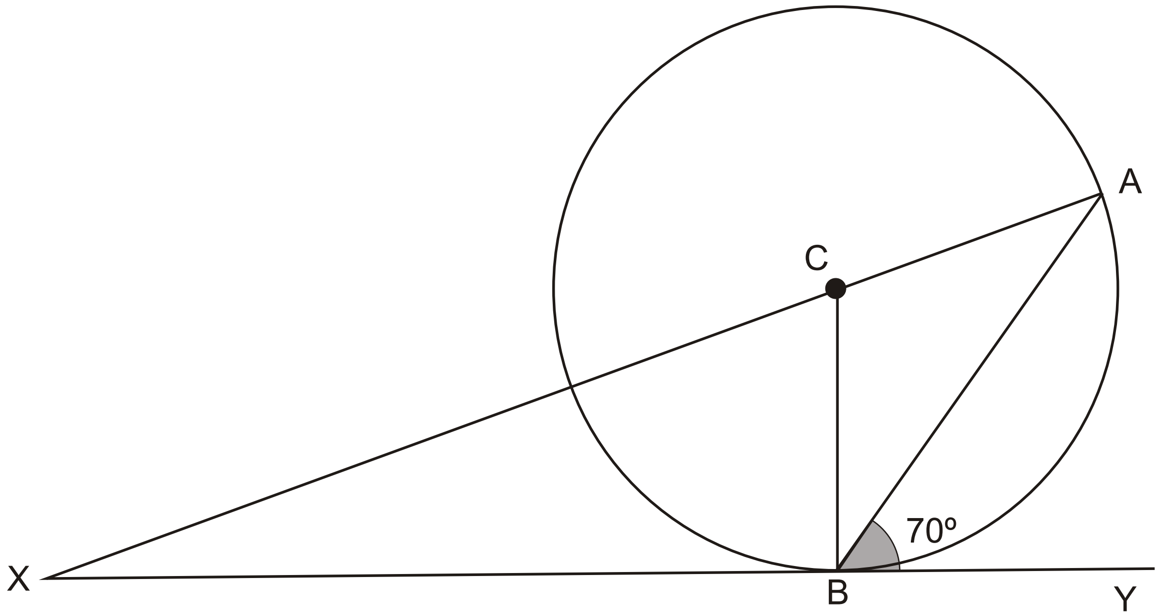


b. In the diagram below (which is not drawn to scale), C is the centre of the circle, and XY is a tangent to the circle. Angle ∠ABY = 70º. Complete the sentences below to find, in 4 logical steps the angle which equals 50º.

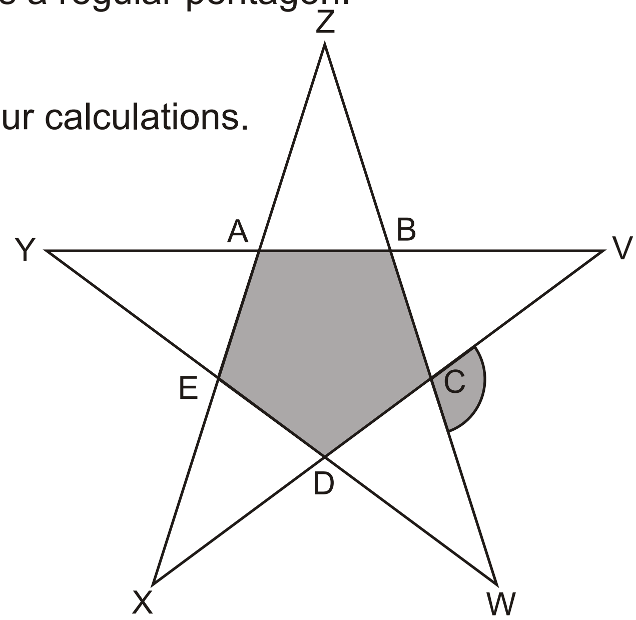


Angle	Size	Reason
XBC	90º	. . . .
CBA	. . . . . .	Adjacent angles on a line add up to 180º.
CAB	20º	. . . .
XCB	40º	. . . .
. . . . . .	50º	. . . .

Geometric Reasoning - Merit Examples

1. The diagram below shows a pentagram - a five sided star.
The shaded shape formed by ABCDE is a regular pentagon.

Calculate the angle ∠VCW.
Give geometric reasons with each of your calculations.

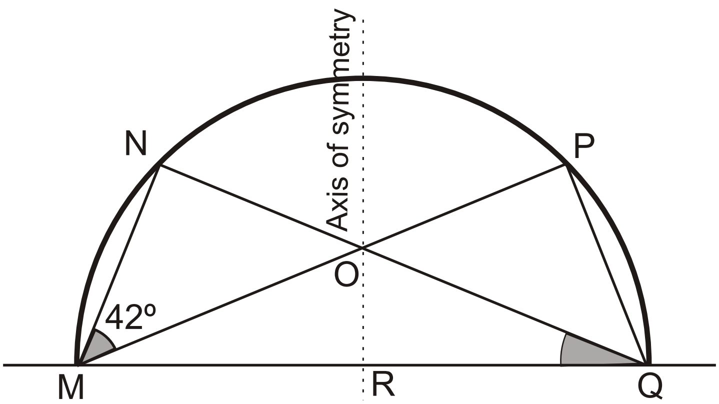


The total of all the interior angles of a regular pentagon can be calculated by the formula: (n−2) × 180 where n is the number of sides (5).

Therefore 3 × 180° = 540°

Each angle = 108° (as 540° ÷ 5 = 108°).

This means that ∠VCW = 108° as it is vertically opposite.



2. The figure above shows a semicircle with centre R.
RO is an axis of symmetry. The angle ∠NMP is 42°.
Find the angle ∠NQM. Support your answer with geometric reasons.

∠QNM = ∠MPQ = 90° (angle in a semicircle)

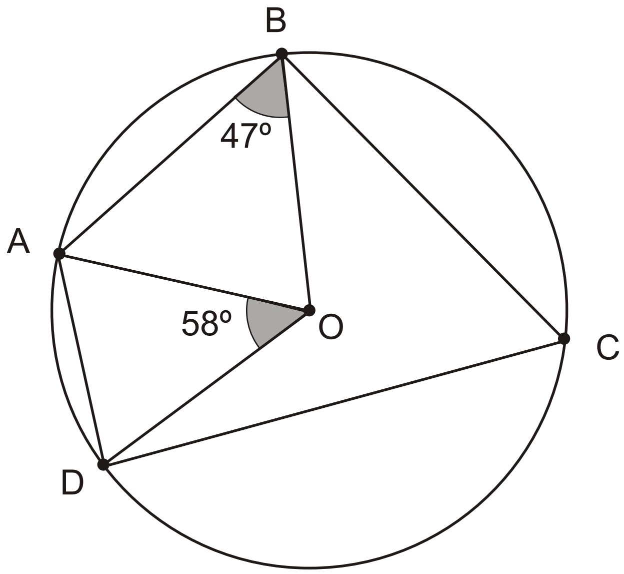
This means that ∠NOM = 48° (angles in a triangle sum to 180°)

∠NOP and ∠MOQ = 132° (angles around a point sum to 360°)

(Vertically opposite angles are equal)

∠OQM = ∠OMQ = 24° (base angles of an isosceles triangle)

4. In the diagram below the points A, B, C and D lie on a circle with centre O. Calculate the size of angle △BCD. Give geometric reasons for your answer.



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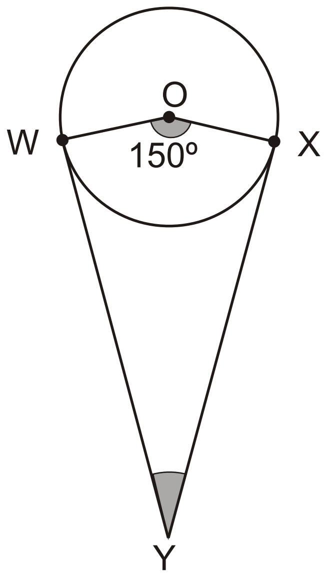
5. The diagram below shows a circle with two tangents at W and X.
O is the centre of the circle.
Calculate the angle △WYX. Give geometric reasons for your answer.



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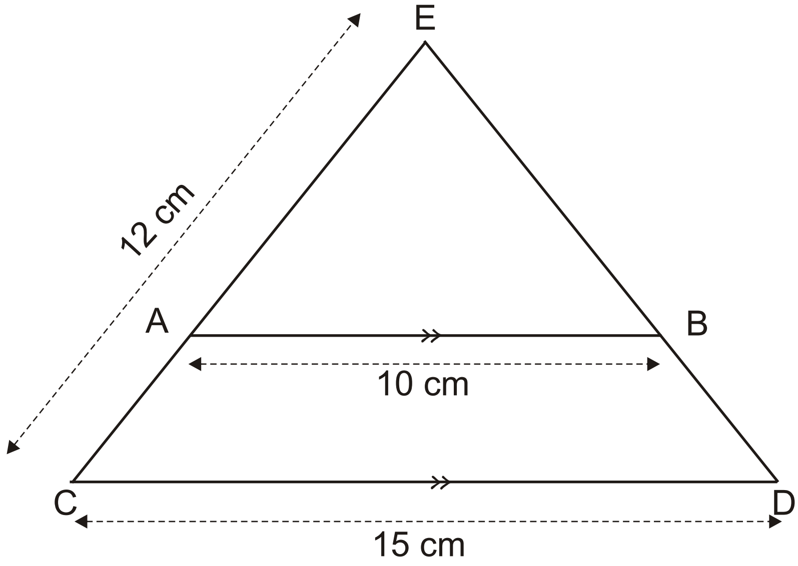
6. In the figure below ABDC is an isosceles trapezium.
Find the length of BD. Give geometric reasons for your answer.



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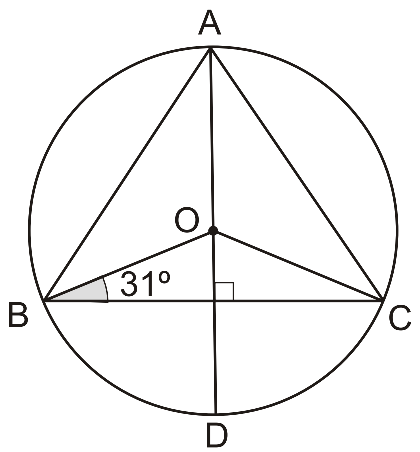
7. In the diagram below ABC is an isosceles triangle.
O is the centre of the circle. The angle $\angle OBC = 31^\circ$

a. Calculate the size of $\angle BAC$.

b. Calculate the size of $\angle ODC$.
Give reasons for your answers.



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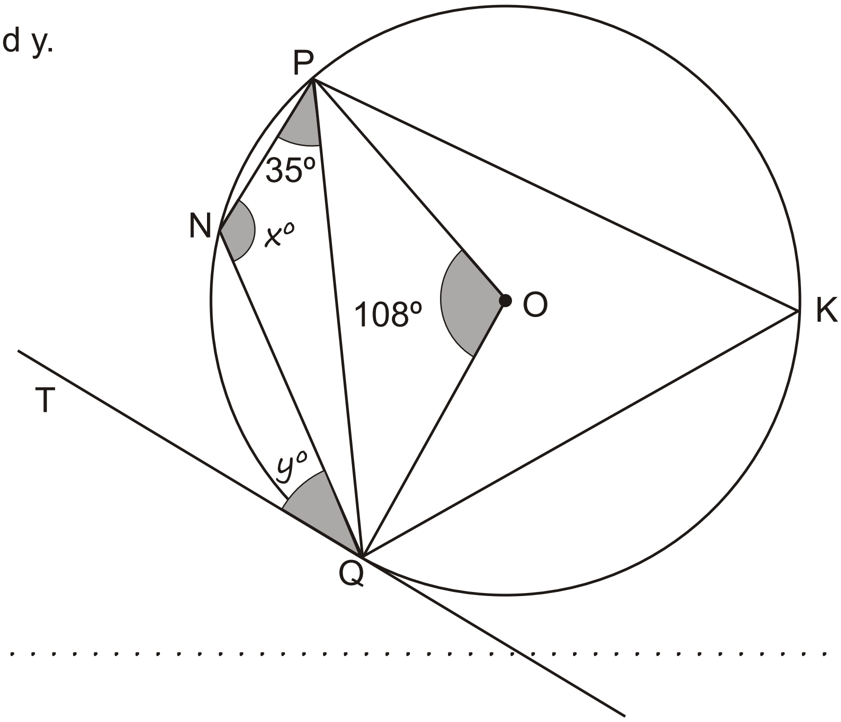
8. The diagram below is NOT drawn to scale.
O is the centre of the circle.
P, K, Q and N are points on the circumference.
QT is the tangent to the circle at Q.

Angle POQ = 108°
Angle NPQ = 35°

Calculate the angles x and y.



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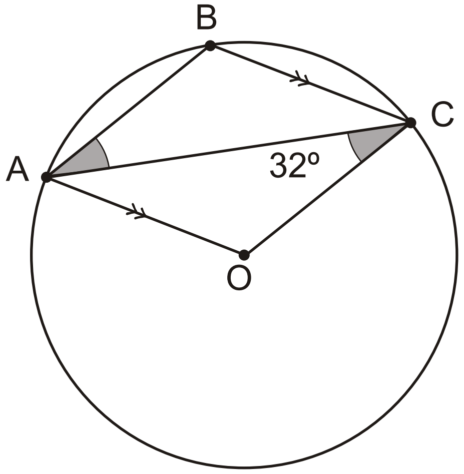
Geometric Reasoning - Excellence Examples

1. The diagram below shows a circle with points A, B and C on the circumference. Centre of the circle is O.
Calculate the size of∠CAB.
Give geometric reasons for your answer.



Triangle AOC is isosceles (OA = OC)
∠OAC = 32°
(base angles of an isosceles triangle)

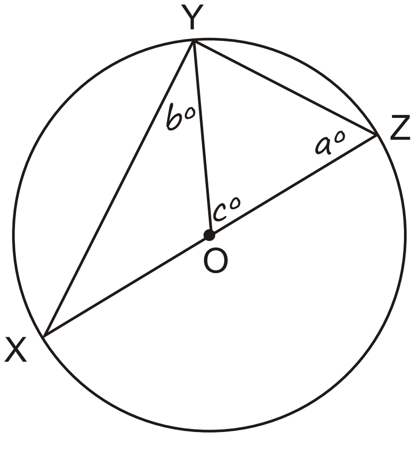
∠AOC = 116°
(angle sum of a triangle = 180°)

The reflex angle about the centre is 360° – 116° = 244°
∠ABC = 122° (angle at the circumference = ½ angle at centre)

∠BCA = 32°. (alternate angles are equal i.e. ∠BCA and ∠OAC)

∠BAC = 26° (angle sum of a triangle = 180°)

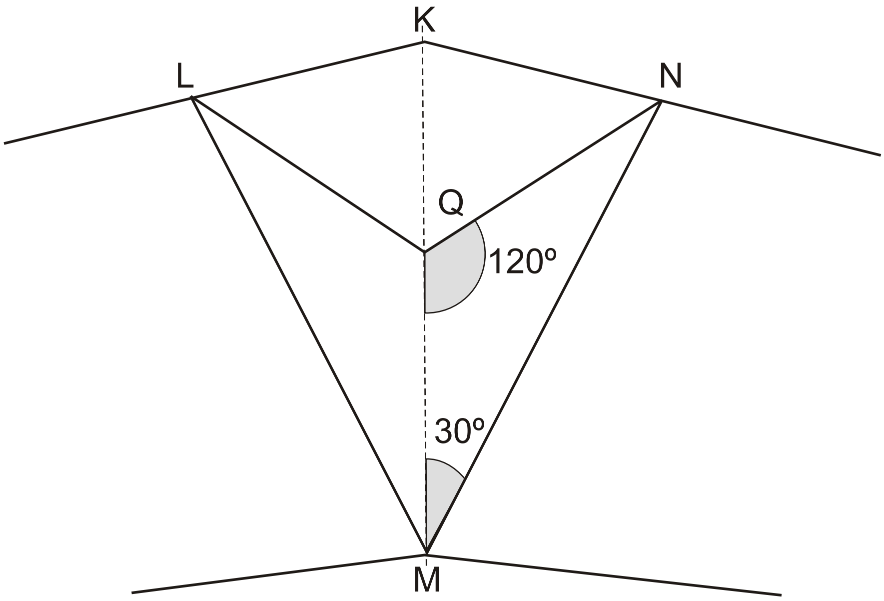
2. In the figure below, O is the centre of the circle. XZ is the diameter.
Prove that c = 2b. Give geometric reasons for your answer.



∠XYZ = 90° (angle in a semicircle)
OX, OZ and OY are radii therefore there are two isosceles triangles formed, OYZ and OYX. This means that ∠OYZ = a° and ∠OXY = b°.
b = 90 – a (angle in a semicircle)
c = 180 – 2a (angles in a triangle sum)
Therefore c = 2b

9. In the figure below, line KM forms an axis of symmetry.
Length QN = Length QK = Length QL.
Angle NQM = 120°.
Angle NMQ = 30°.

Prove that the quadrilateral KLMN is cyclic.



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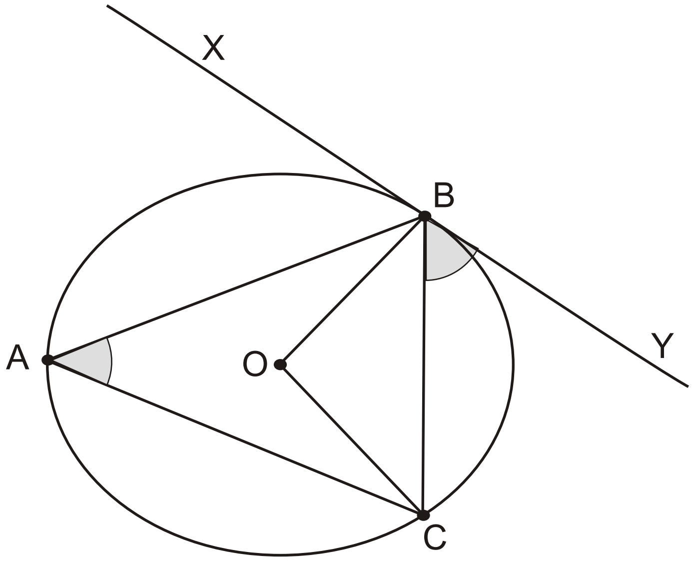
10. The circle below has a centre O and a tangent XY at point B.
The points A and D lie on the circle.

Prove that $\angle YBC = \angle BAC$



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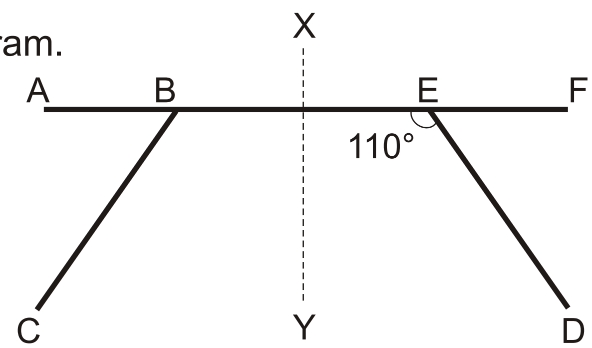
You are advised to spend 25 minutes answering the questions in this section.

QUESTION ONE

Pauline's garden table is illustrated in the diagram.
ABEF is a straight and horizontal table top.
XY is an axis of symmetry.
Angle BED = 110°

Diagram is not to scale.



Calculate the size of angle ABC giving reasons for your answers.

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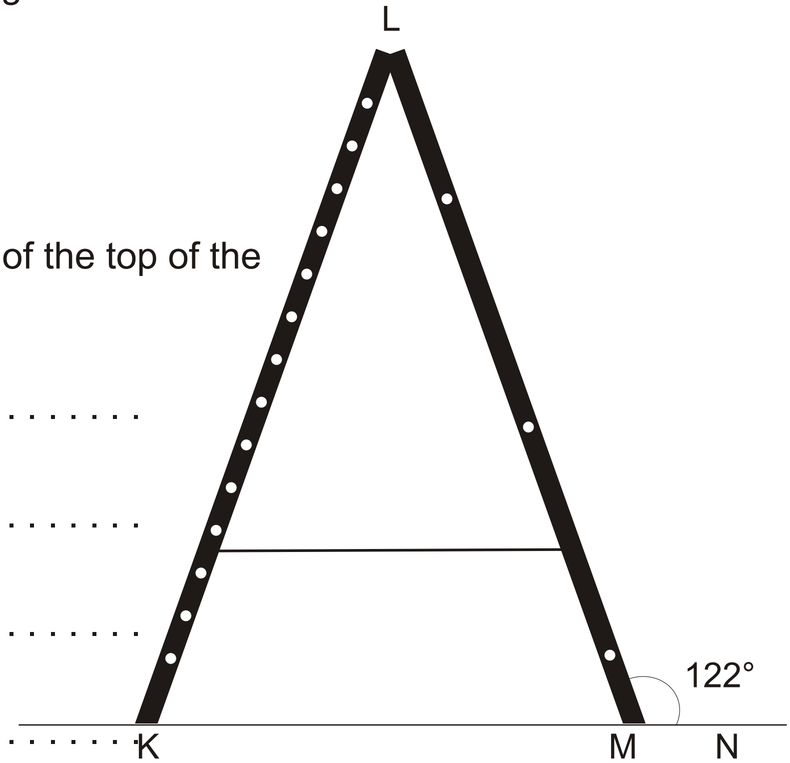
QUESTION TWO

Stan's step-ladder is shown in the diagram.

Angle LMN = 122°
KL = LN

Calculate the size of the inside angle of the top of the step-ladder KLM giving reasons.



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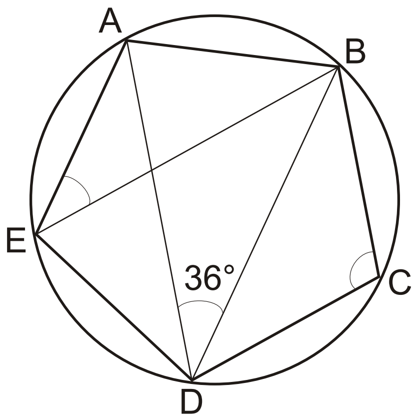
QUESTION THREE

The diagram shows a regular pentagon inside a circle.



Calculate the size of BEA and DCB giving reasons for your answers.

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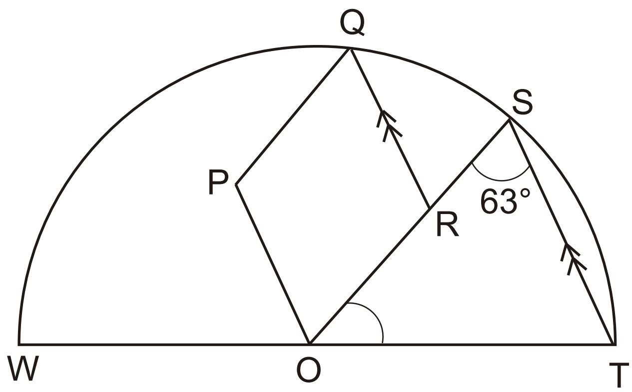
QUESTION FOUR

In the diagram, PQRO is a rhombus QR is parallel to ST.

Angle TSR = 63°. Calculate the angle of ROT.

Give a geometric reason for each step leading to your answer.



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QUESTION FIVE

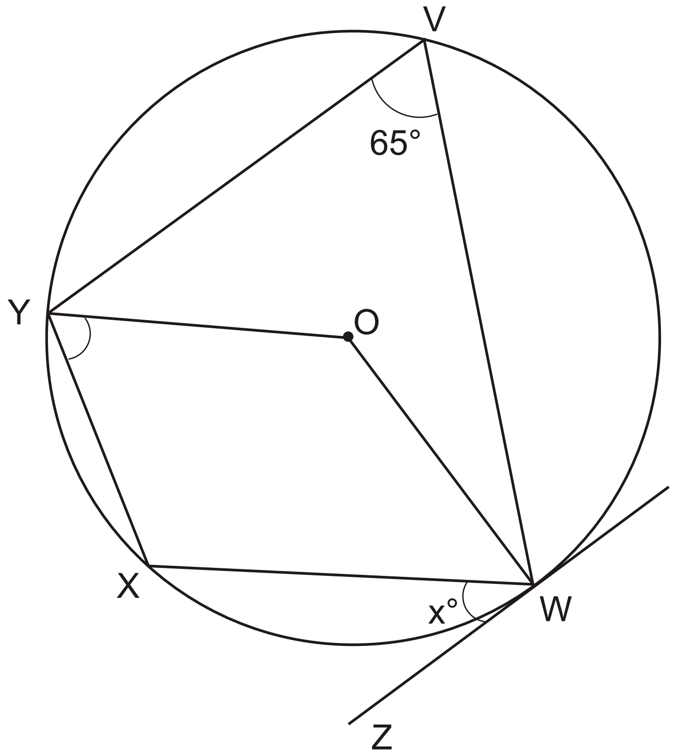
In the diagram V, W, X and Y lie on a circle, centre O.

ZW is a tangent to the circle at W.

Angle YVW = 65°

Angle XWZ = x°

Find the size of angle OYX.



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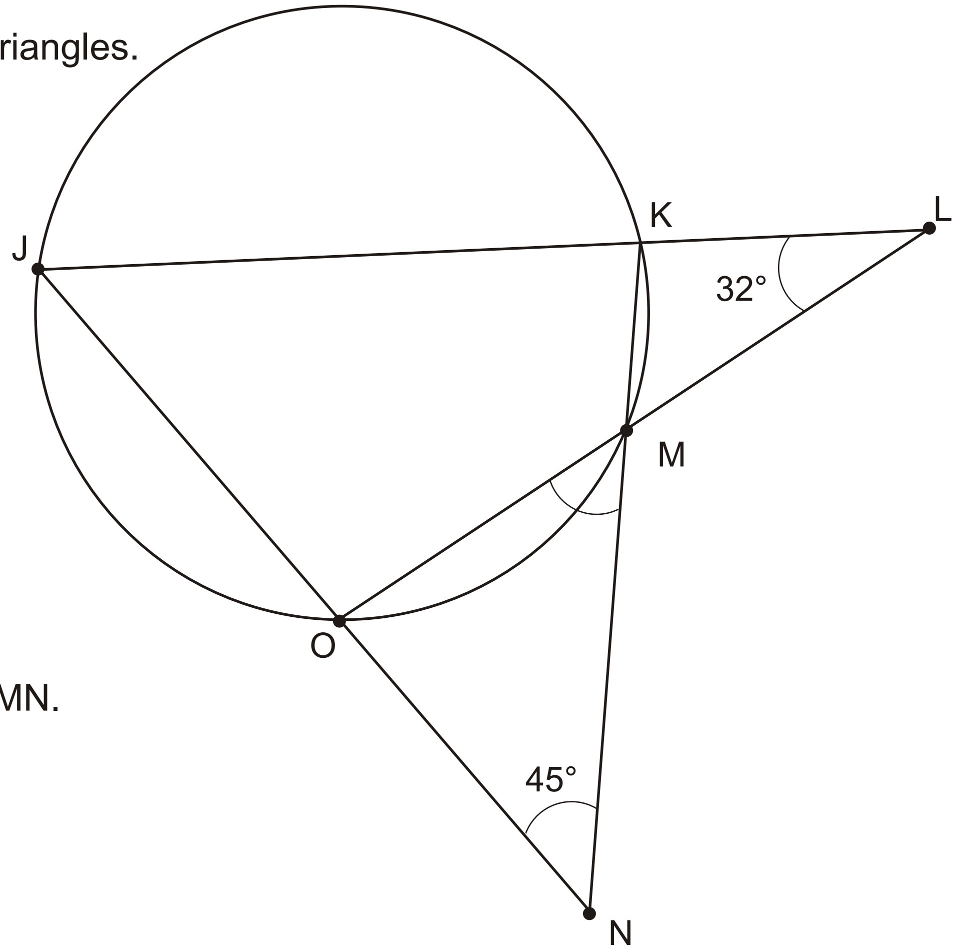
QUESTION SIX

KLM and MNO are two triangles.

Angle KLM = 32°

Angle MNO = 24°



Find the size of angle OMN.

Give geometric reasons for each step leading to your answer.

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The Answers

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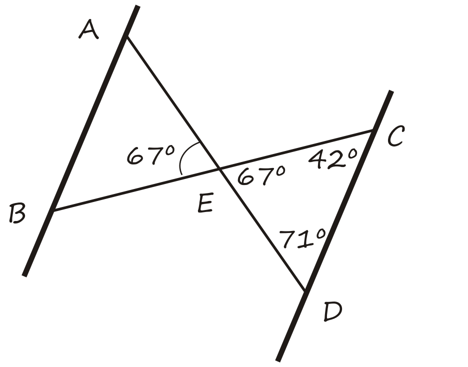
1. a.

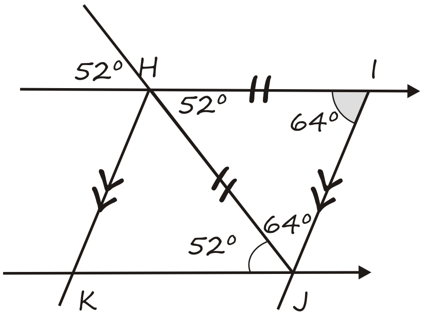


CED = 67° – vertically opposite

Angles in a triangle sum to 180°

180° – 67° – 42° = 71°

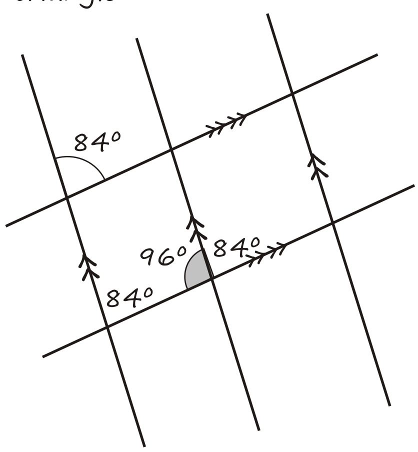
b.



Angle IHJ = 52° alternate angles, parallel lines.

HIJ = 64° – Angles in an isosceles triangle.

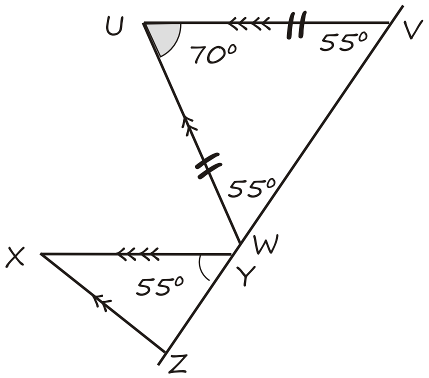
c.



See the diagram above for all the corresponding angles.

c = 96° angles on a straight line

d.



See diagram above for the corresponding angles. The 70° is then calculated from the isosceles triangle WUV.

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2. a.

Sum interior angles = (n–2) × 180°

Sum interior angles of pentagon

(5 – 2) × 180° = 540°

Each angle = 108°

Sum interior angles heptagon

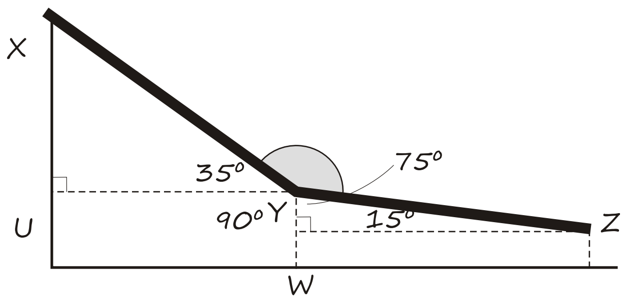
(7–2) × 180° = 900°

Each angle = 128.57°

Angle CBD = 360 – (108 + 128.57)

= 123.43°

b.



Angle UYW = 90° right angled rectangle.

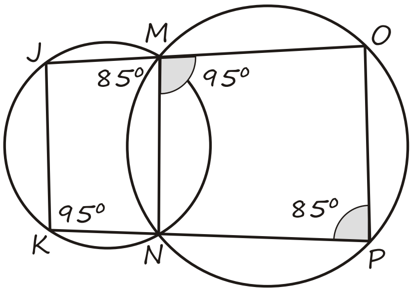
The angle ZYW = 75° angle sum of a triangle.

XYZ = 360° – (75° + 90° + 35°)

= 160°

Page 15 (cont)

2. **c.**



NMJ = 85° opposite angles of a cyclic triangle sum to 180°
This means angle NMO = 95° angles on a straight line and NPO = 85° as it is the opposite angle in the cyclic triangle.

d. JK is parallel to OP as all the angles indicate that they are corresponding.

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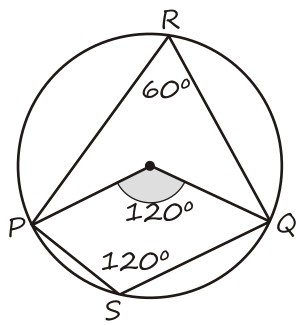
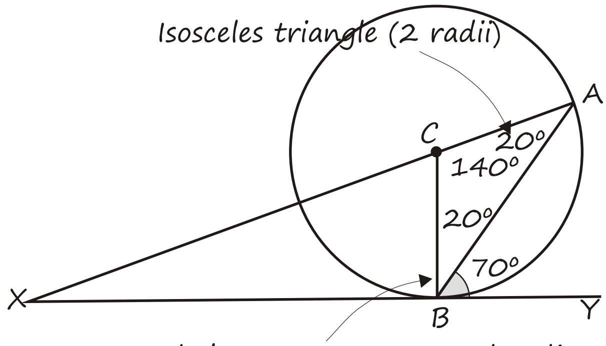


3. **a.** PRQ = 60° angle at the centre
PSQ = 120° opposite angles in a cyclic quadrilateral.

B.



b. XBC = Angle between a tangent and a radius
CBA = 20°
CAB = Base angles of an isosceles triangles are equal
XCB = 40° angles on straight line
CXB = Angles in a triangle sum to 180°

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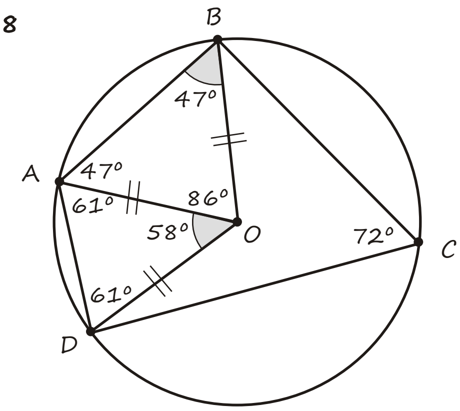
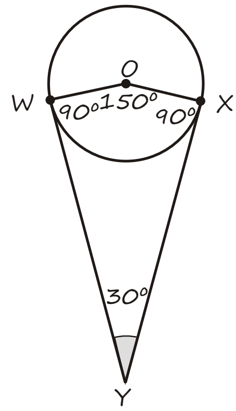
4.



OB, OA and OD are all radii.
This means that there are 2 isosceles triangles AOB (47°, 47°, 86°) and AOD (61°, 61°, 58°)
BAO + DAO: 61° + 47° = 108°
BCD = 72° opposite angles cyclic quad.
or BOA + AOD: 86° + 58° = 144°
BCD = 72° angle at circumference

5.



OWY AND OXY are 90° as the angle between the tangent and radius are perpendicular.
WYX = 30° interior angles of a quadrilateral sum to 360°

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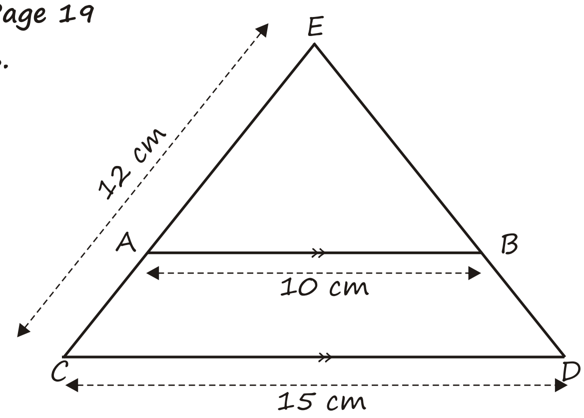
6.



EAB and ECD are similar triangles.

This means $\frac{EA}{12} = \frac{10}{15}$

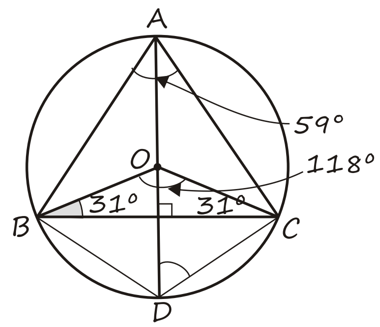
This means that EA = 8 cm

BD = CE – EA (CE = DE, AE = BE)

= 12 cm – 8 cm

= 4 cm

7.



BCO = 31° base angles isosceles triangle.

COB = 118° angle sum of a triangle

BAC = 59° angle at circumference is half that at the centre.

BAC is isosceles triangle

BDC is an isosceles triangle

Angle BDC = 121° opposite angles cyclic quadrilateral. ODC is half this 60.5°.

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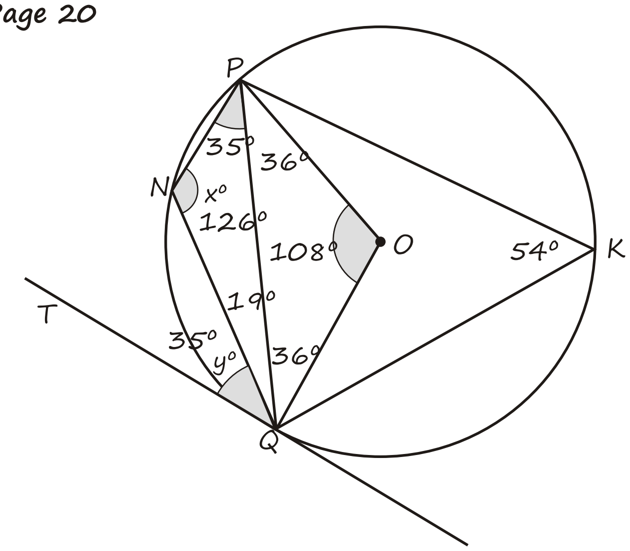

8. Lines OP and OQ are radii.

This means that OPQ is an isosceles triangle and the base angles OPQ and OQP = 36°

Angle PKQ = 54° as it is angle at the circumference (½ POQ)

Angle x (PNQ) = 126° as opposite angles of a cyclic quadrilateral sum to 180°.

OQT is the angle between a radius and a tangent = 90°.

NQP = 19° angle sum of a triangle

NQT (y°) = 90° – 36° – 19°

= 35°

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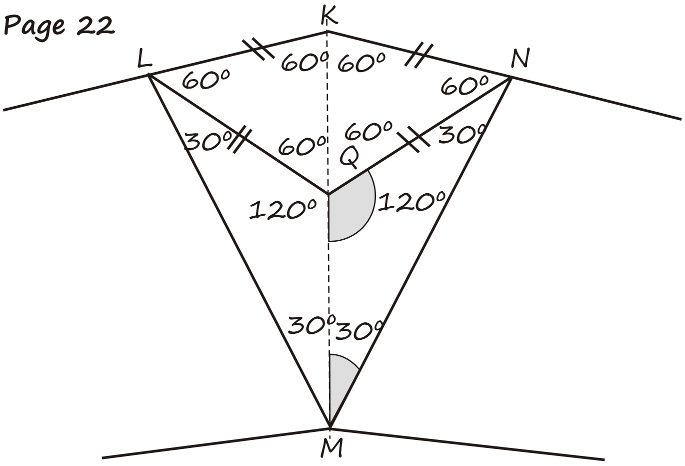

9. KQN = 60° angles on a straight line
QKN = 60° isosceles triangle QN = QK
LKN = 120° line KM is a line of symmetry
LKN = 2 × QKN
LMN = 60° due to line of symmetry KM
LKN + LMN = 180°
KLMN = a cyclic triangle (opposite angles of a cyclic quadrilateral sum to 180°)

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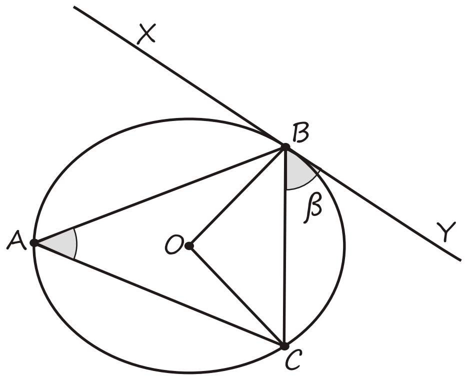
10.



OB is a radius and the tangent XY is 90° to the tangent.
Let YBC = β
OBC = 90 – β tangent 90° to radius
OCB = 90 – β base angles of isos triangle OB = OC
BOC = 180 – (90 – β) – (90 – β)
= 2β
BAC = β angle at circumference is half that at the centre
BAC = β, YBC = β
Therefore YBC = BAC

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Question One

FED = 70° angles on a straight line
ABC = 70° symmetry through XY

Question Two

LMK = 58° angles on a straight line
LKM = 58° as triangle is isosceles
KLM = 64° angle sum of a triangle

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Question Three

BEA = 36° angles in the same segment
DCB = 108° interior angles of a regular pentagon.

Question Four

OTS is an isosceles triangle
STO = 63° base angles isosceles triangle
ROT = 180° – 2 × 63°
= 54°

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Question Five

OW is at 90° to WZ
It is a radius and tangent
This means that OWX = 90° – x
YXW = 115° opposite angles of a cyclic quadrilateral
WOY = 130° angle at centre is twice the angle at the circumference
Angle sum of a quadrilateral is 360°
OYX = 360 – 130 – 115 – (90 – x)
= (25 + x)°

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Question Six

Let KML = x°, then OMN = x° as they are vertically opposite.
JOM = x + 45°, JKM = x + 32° as the exterior angles of a triangle = the sum of the two opposite interior angles
JOM + JKM = 180° (cyclic quadrilateral)
x° + 45° + x° + 32° = 180°
x = 51.5°